

Propagation of skew-symmetric unsteady shear waves from thick-walled shell in elastic space

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Abstract. Problems of propagation and diffraction of nonstationary waves in elastic bodies are of great theoretical and practical importance in such fields of science and technology as aircraft construction, shipbuilding, seismic exploration of minerals, seismic resistance of structures, and many others. The paper considers the problem of the propagation of skew-symmetric unsteady shear waves from a thick-walled spherical shell in elastic space. To solve the problem, the integral Laplace transforms in dimensionless time, and the method of incomplete separation of variables was used. In the image space, the problem is reduced to an infinite system of linear algebraic equations, the solution of which is sought in the form of an infinite exponential series. Formulas for the displacement vector components and the stress tensor are obtained. The transition to the originals is carried out using the theory of residues. Numerical experiments have been carried out, the results of which are presented in graphs. The obtained results of the work can be used in geophysics, seismology, and design organizations in the construction of structures and in the design of underground reservoirs.

1 Introduction

The study of unsteady wave processes in continuous media is a difficult complex and, at the same time, an important area of wave dynamics in the mechanics of a deformable body. In particular, among the many questions put forward by practice, one of the urgent is problems in which the nonstationary interaction of deformable bodies with the environment is considered.

The relevance of the problems of the dynamics of deformable bodies is due to the development of various fields of technology, the creation of new structures operating under dynamic loads, as well as the problems of geophysics, seismology, gas exploration, oil exploration, the extractive industry, the construction of civil and industrial structures.

Currently, there are a large number of publications in the periodical literature [1-5, 10, 16-21], as well as monographs [6-9, 11-13, 15] devoted to the study of the propagation and diffraction of steady and transient waves in elastic and acoustic media.

The diffraction of a stationary plane elastic shear wave by cylindrical cavities in an isotropic half-space is considered in [1]. The boundary value problem is reduced to solving

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an infinite system of linear algebraic equations for the amplitude coefficients of the scattered waves.

In the first part of work [2], the propagation in an elastic spherical layer of SH-type waves generated by a rotational action is studied. The exact solution of the problem is transformed into the sum of interference waves, each of which is represented by a Fourier-type integral. In the second part [3], a nonstationary displacement field arising due to the interference of SH waves reflected from the boundaries of an elastic spherical layer is studied. The conditions are found under which the propagation of an interference wave in a spherical layer occurs in the same way as in a layer by plane-parallel boundaries.

In [4], the problem of diffraction by a stationary sphere of a plane shear torsional wave (SH-wave) in an infinite elastic medium is considered. The exact solution to the problem is found in the form of a convolution integral. With the help of the Watson transform, a physical interpretation of the components of the diffraction field is given. The problem of the diffraction of shear waves by cavities and rigid inclusions in a half-space with a clamped and force-free boundary was studied in [10]. Calculations were carried out for cylindrical cavities and rigid inclusions of the elliptical section.

Propagation of unsteady shear waves from a spherical inclusion in an elastic half-space in [5]. The problem is reduced to solving an infinite system of algebraic equations, the solution of which is sought in the form of an infinite exponential series. Formulas for the displacement vector components and the stress tensor are obtained. The results of systematic studies on the problem of unsteady interaction of thin-walled and solid deformable spherical bodies with elastic and acoustic media are presented in the monograph [6]. The theory of sound, shock and kinematic waves and oscillatory motions in two-phase media, hydraulics and thermal physics of gas-liquid flows, the theory of heat transfer crises, critical outflows, filtration of multiphase liquids are given in [11]. Experimental methods and their results are presented.

The monograph [12] is devoted to applied problems of seismodynamics of structures interacting with soil. The formulations of the problems are stated, and effective methods for their solution are developed. The questions of the influence of the mechanical properties of the environment on the behavior of the underground structure and the contact forces of interaction with the soil on the formation of the wave field in the body of the structure are investigated.

In [17], the Watson transform method is used to solve the problem of diffraction of plane transverse waves by a sphere, and the short-wavelength asymptotics for displacements in scattered waves in different regions of elastic space is found. The axisymmetric problem of the diffraction of nonstationary waves was studied in [16]. An absolutely rigid stationary ball in an elastic half-space was chosen as an obstacle. An infinite system of linear algebraic equations is obtained in the image space of the Laplace transform in time. The stress-strain state of the medium in the vicinity of the sphere is investigated. The paper [18] investigates the interaction of spherical elastic SH-waves of the harmonic type with a spherical layer when the source is placed outside the layer. The exact solution to this scattering problem is investigated in detail after establishing the generalized Debye expansion.

In [19], the problem of the propagation of shear perturbations from a spherical cavity to an infinite elastic medium was considered. In this case, the Fourier transform in time was used. Expressions are obtained for the displacement and stress in time caused by an axially symmetric shear stress applied to the inner surface of a spherical cavity in an infinite isotropic elastic medium. The problem of plane wave diffraction by a system of two concentric spherical shells surrounded by acoustic media was studied in [20]. In this case, the representation of the solution in the form of a superposition of elementary waves is used.

The propagation of SH-waves of small amplitude in an infinite elastic plate subjected to primary normal stress was investigated in [21]. It is determined that the effect of stress can be represented by changing the scale of the plate thickness, provided that one elastic constant is also redefined accordingly.

This work is devoted to studying the problem of the propagation of skew-symmetric unsteady waves from a thick-walled spherical shell in elastic space.

The work aims to develop an algorithm for solving the problem and to study nonstationary wave shear processes in a thick-walled shell and elastic space.

2 Methods

2.1 Formulation of the problem

Let a thick-walled elastic spherical shell with inner and outer radius R_1 and R_2 ($R_2 > R_1$) be located in a linearly elastic homogeneous isotropic space. The thickness of the thick-walled shell is equal to h ($h = R_2 - R_1$). Axisymmetric kinematic or force loads are applied to its inner surface. The motion of media is considered in a spherical coordinate system (r, θ, ϑ) with the origin at the center of the shell. At the initial moment of time $\tau = 0$, the shell is in an unperturbed state.

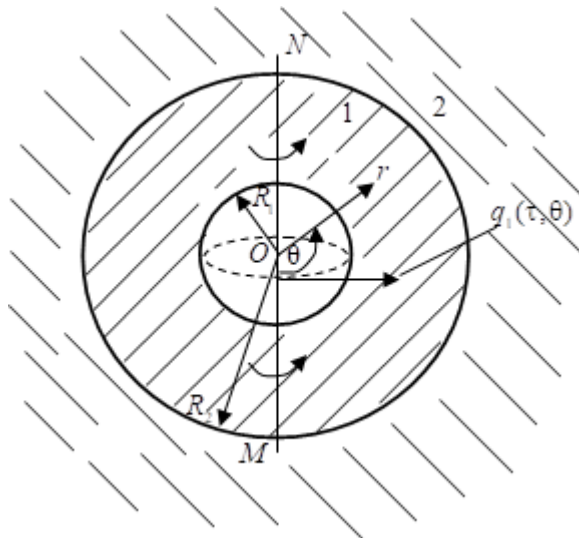


Fig. 1.

Taking into account the axial symmetry, the problems of motion of the shell and the environment relative to the elastic potentials are described by the wave equations (l is the number of the medium)

$$\gamma_l^2 \frac{\partial^2 \psi_l}{\partial \tau^2} = \Delta \psi_l - \frac{\psi_l}{r^2 \sin^2 \theta}, \quad \Delta = \frac{1}{r^2} \frac{\partial}{\partial \tau} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right), \quad (1)$$

and the initial conditions are homogeneous

$$\Psi_l|_{\tau=0} = \frac{\partial \Psi_l}{\partial \tau} \Big|_{\tau=0} = 0. \quad (2)$$

There is no disturbance at infinity

$$\lim_{r \rightarrow \infty} \Psi_2 = 0 \quad (3)$$

On the inner surface of a thick-walled spherical shell, two types of boundary conditions are considered.

Task A. An axisymmetric specified tangential surface load $q_1(\tau, \theta)$ is applied to the inner surface of the shell, which forms a rotational motion of the medium around the NM axis passing through the centers of the spheres (see Fig. 1)

$$\sigma_{r\theta}^{(1)} \Big|_{r=R_1} = q_1(\tau, \theta). \quad (4)$$

Task B. On the inner surface of the shell, the tangential displacement $V_1(\tau, \theta)$:

$$w_1 \Big|_{r=R_1} = V_1(\tau, \theta). \quad (5)$$

The conditions of contact with the media, consisting of the continuity of displacement and stress, can be written in the following form:

$$\sigma_{r\theta}^{(1)} \Big|_{r=R_2} = \sigma_{r\theta}^{(2)} \Big|_{r=R_2}, \quad w_1 \Big|_{r=R_2} = w_2 \Big|_{r=R_2}. \quad (6)$$

In these cases, the functions w_l , $\sigma_{r\theta}^{(l)}$, $\sigma_{\theta\theta}^{(l)}$ and Ψ_l are interconnected by the dependencies

$$\begin{aligned} \sigma_{r\theta}^{(l)} &= \frac{\beta_l}{\eta_l^2} \left(\frac{\partial w_l}{\partial r} - \frac{w_l}{r} \right), \quad \sigma_{\theta\theta}^{(l)} = \frac{\beta_l}{\eta_l^2 r} \left(\frac{\partial w_l}{\partial \theta} - w_l \cot \theta \right), \\ w_l &= \frac{\partial \Psi_l}{\partial r} + \frac{\Psi_l}{r}. \end{aligned} \quad (7)$$

The statement of the problem is given in the following dimensionless quantities (a prime means a dimensionless quantity):

$$\begin{aligned} r' &= \frac{r}{R_1}, \quad \tau = \frac{c_2^{(1)} t}{R_1}, \quad \beta_l = \frac{\lambda_l + 2\mu_l}{\lambda_1 + 2\mu_1}, \quad \sigma_{\alpha\beta}^{(l)'} = \frac{\sigma_{\alpha\beta}^{(l)}}{\lambda_1 + 2\mu_1}, \quad w_l' = \frac{w_l}{R_1}, \\ \Psi_l' &= \frac{\Psi_l}{R_1^2}, \quad R_2' = \frac{R_2}{R_1}, \quad \eta_l = \frac{c_1^{(l)}}{c_2^{(l)}}, \quad \gamma_l = \frac{c_2^{(1)}}{c_2^{(l)}}, \end{aligned}$$

where λ_l , μ_l are elastic parameters of Lamé; $c_2^{(l)} = \sqrt{\mu_l/\rho_l}$ is velocity of propagation of elastic waves of shape change (S - wave); ρ_l is the density of the medium; t is time; ($l=1, 2$). Further, the strokes in the designation of dimensionless quantities will be omitted.

2.2 Solution method

The initial-boundary value problem (1) - (6) is solved using the integral Laplace transform over the dimensionless time τ . In the image space, the potentials ψ_l^L , the components w_l^L of the displacement vector and $\sigma_{r\theta}^{(l)L}$ of the stress tensor, as well as the given functions $q_1^L(s, \theta)$, $V_1^L(s, \theta)$, can be represented as infinite series in Gegenbauer polynomials $C_{n-1}^{3/2}(x)$ [6], and the representation of the infinite series for the component $\sigma_{\theta\theta}^{(l)L}$ of the stress tensor has the following form:

$$\sigma_{\theta\theta}^{(l)L} = \sum_{n=1}^{\infty} \sigma_{\theta\theta n}^{(l,1)L}(r, s) P_n(\cos \theta) + \cos \theta \sum_{n=1}^{\infty} \sigma_{\theta\theta n}^{(l,2)L}(r, s) C_{n-1}^{3/2}(\cos \theta), \quad (8)$$

where $P_n(\cos \theta)$ are Legendre polynomials.

The coefficients of these series, according to the dependence of functions (7), are related to each other as follows:

$$w_{l,n}^L = \frac{\partial \psi_{l,n}^L}{\partial r} + \frac{\psi_{l,n}^L}{r}, \quad \sigma_{r\theta n}^{(l)L} = \frac{\beta_l}{\eta_l^2} \left(\frac{\partial w_{l,n}^L}{\partial r} - \frac{w_{l,n}^L}{r} \right), \quad (9)$$

$$\sigma_{\theta\theta n}^{(l,1)L}(r, s) = -\frac{n(n+1)\beta_l}{\eta_l^2 r} w_{l,n}^L(r, s), \quad \sigma_{\theta\theta n}^{(l,2)L}(r, s) = \frac{2\beta_l}{\eta_l^2 r} w_{l,n}^L(r, s).$$

Then the statement of the initial-boundary value problem (1) - (6) concerning the coefficients of the series has the following form:

$$\gamma_l^2 s^2 \psi_{ln}^L = \Delta_n \psi_{ln}^L, \quad (10)$$

$$\sigma_{r\theta n}^{(1)L} \Big|_{r=R_1} = q_{1n}^L(s) \text{ or } w_{1n}^L \Big|_{r=R_1} = V_{1n}^L(s), \quad (11)$$

$$\sigma_{r\theta n}^{(1)L} \Big|_{r=R_2} = \sigma_{r\theta n}^{(2)L} \Big|_{r=R_2}, \quad w_{1n}^L \Big|_{r=R_2} = w_{2n}^L \Big|_{r=R_2}, \quad (12)$$

$$\lim_{r \rightarrow \infty} \psi_{2n}^L = 0 \quad (13)$$

In the image space for $l=1$, the solution to equation (10) is sought in the form [5, 6]:

$$\psi_{1n}^L(r, s) = \frac{1}{\sqrt{r}} A_n^L(s) K_{n+1/2}(r\gamma_1 s) + \frac{1}{\sqrt{r}} B_n^L(s) I_{n+1/2}(r\gamma_1 s), \quad (14)$$

where $I_{n+1/2}(x)$ and $K_{n+1/2}(x)$ are modified Bessel functions of the first and second kind; $A_n^L(s)$ and $B_n^L(s)$ are unknown functions of the parameter s .

Taking into account the expressions for the Bessel functions in terms of elementary ones [14], we write the image of the solution (14) as follows:

$$\psi_{1n}^L(r, s) = \frac{1}{r^{n+1}(\gamma_1 s)^n} \left[R_{n0}(r\gamma_1 s) A_n^L(s) e^{-r\gamma_1 s} + (-1)^n G_{n0}(r\gamma_1 s) B_n^L(s) \right]. \quad (15)$$

Further, according to dependence (9), we obtain the following expressions for the coefficients of the displacement components and the stress tensor for the shell

$$\begin{aligned} w_{1n}^L(r, s) &= -\frac{1}{r^{n+2}(\gamma_1 s)^n} \left[R_{n3}(r\gamma_1 s) A_n^L(s) e^{-r\gamma_1 s} + (-1)^n G_{n3}(r\gamma_1 s) B_n^L(s) \right] \\ \sigma_{r\theta n}^{(1)L}(r, s) &= \frac{\beta_1}{r^{n+3}\eta_1^2(\gamma_1 s)^n} \left[R_{n4}(r\gamma_1 s) A_n^L(s) e^{-r\gamma_1 s} + (-1)^n G_{n4}(r\gamma_1 s) B_n^L(s) \right], \quad (16) \\ \sigma_{\theta\theta n}^{(1,1)L}(r, s) &= \frac{n(n+1)\beta_1}{r^{n+3}\eta_1^2(\gamma_1 s)^n} \left[R_{n3}(r\gamma_1 s) A_n^L(s) e^{-r\gamma_1 s} + (-1)^n G_{n3}(r\gamma_1 s) B_n^L(s) \right], \end{aligned}$$

$$\sigma_{\theta\theta n}^{(1,2)L}(r, s) = -\frac{2\beta_1}{r^{n+3}\eta_1^2(\gamma_1 s)^n} \left[R_{n3}(r\gamma_1 s) A_n^L(s) e^{-r\gamma_1 s} + (-1)^n G_{n3}(r\gamma_1 s) B_n^L(s) \right], \quad (17)$$

$$G_{ni}(s) = R_{ni}(-s) \exp(s) - R_{ni}(s) \exp(-s).$$

where $R_{ni}(s)$ are polynomials [5, 6], $i = \overline{0, 2}$.

Taking into account the condition that there is no disturbance at infinity (13), for $l = 2$, we represent the solution of equation (10) in the following form [6]:

$$\psi_{2n}^L(r, s) = \frac{1}{\sqrt{r}} C_n^L(s) K_{n+1/2}(r\gamma_2 s)$$

or, taking into account the expressions for the Bessel functions in terms of elementary ones [14], we arrive at the expression

$$\psi_{2n}^L(r, s) = \frac{1}{r^{n+1}(\gamma_2 s)^n} R_{n0}(r\gamma_2 s) C_n^L(s) e^{-r\gamma_2 s}, \quad (18)$$

where $C_n^L(s)$ are unknown functions of the parameter s .

Then, by virtue of dependencies (9), we find the following expressions for the coefficients of the displacement components and the stress tensor for the environment:

$$w_{2n}^L(r, s) = -\frac{1}{r^{n+2}(\gamma_2 s)^n} R_{n3}(r\gamma_2 s) C_n^L(s) e^{-r\gamma_2 s} \quad (19)$$

$$\sigma_{r\theta n}^{(2)L}(r, s) = \frac{\beta_2}{r^{n+3}\eta_2^2(\gamma_2 s)^n} R_{n4}(r\gamma_2 s) C_n^L(s) e^{-r\gamma_2 s}. \quad (20)$$

Now, satisfying the contact conditions (12) and excluding the unknown functions $C_n^L(s)$, we obtain expressions for the coefficients of the displacement components and the stress tensor for the environment concerning the unknown functions $A_n^L(s)$, $B_n^L(s)$:

$$w_{2n}^L(r, s) = -\frac{R_{n3}(r\gamma_2 s)(\gamma_1 s)^{-n} e^{-r\gamma_2 s}}{r^{n+2} R_{n3}(R_2 \gamma_2 s) e^{-R_2 \gamma_2 s}} \left[R_{n3}(R_2 \gamma_1 s) A_n^L(s) e^{-R_2 \gamma_1 s} + (-1)^n G_{n3}(R_2 \gamma_1 s) B_n^L(s) \right], \quad (21)$$

$$\sigma_{r9n}^{(2)L}(r, s) = \frac{\beta_1 R_{n4}(r\gamma_2 s)(\gamma_1 s)^{-n} e^{-r\gamma_2 s}}{r^{n+3} \eta_1^2 R_{n4}(R_2 \gamma_2 s) e^{-R_2 \gamma_2 s}} \left[R_{n4}(R_2 \gamma_1 s) A_n^L(s) e^{-R_2 \gamma_1 s} + (-1)^n G_{n4}(R_2 \gamma_1 s) B_n^L(s) \right]$$

Further, using expressions (16), (21), satisfying boundary (11) and contact conditions (12), we obtain an infinite system of linear algebraic equations for unknown functions $A_n^L(s)$, $B_n^L(s)$, which we write in matrix form [5, 16]:

$$\begin{cases} \mathbf{M}\mathbf{A}y^2 + \mathbf{F}_1\mathbf{B} - \mathbf{F}_2\mathbf{B}y^2 = \mathbf{k}y \\ \mathbf{N}\mathbf{A}x^2 + \mathbf{T}_1\mathbf{B} - \mathbf{T}_2\mathbf{B}x^2 = 0 \end{cases} \quad (22)$$

$$\mathbf{k}(s) = \|k_1(s), k_2(s), \dots\|^T, \quad \mathbf{A}(s) = \|A_1^L(s), A_2^L(s), \dots\|^T, \quad \mathbf{B}(s) = \|B_1^L(s), B_2^L(s), \dots\|^T,$$

$$x = e^{-R_2 \gamma_1 s}, \quad y = e^{-R_1 \gamma_1 s}.$$

Here, $\mathbf{F}_l$, $\mathbf{T}_l$, $\mathbf{M}$, $\mathbf{N}$ are infinite diagonal matrices with the elements $F_{ln}(s)$, $T_{ln}(s)$ ($l=1,2$), $M_n(s)$, $N_n(s)$ and $\mathbf{k}(s)$, respectively - an infinite column vector with elements $k_n(s)$, which have the following forms:

$$N_n(s) = \beta_2 \eta_1^2 R_{n4}(R_2 \gamma_2 s) R_{n3}(R_2 \gamma_1 s) - \beta_1 \eta_2^2 R_{n3}(R_2 \gamma_2 s) R_{n4}(R_2 \gamma_1 s),$$

$$T_{1np}(s) = (-1)^n [\beta_2 \eta_1^2 R_{n4}(R_2 \gamma_2 s) R_{n3}(-R_2 \gamma_1 s) - \beta_1 \eta_2^2 R_{n3}(R_2 \gamma_2 s) R_{n4}(-R_2 \gamma_1 s)],$$

$$T_{2np}(s) = (-1)^n [\beta_2 \eta_1^2 R_{n4}(R_2 \gamma_2 s) R_{n3}(R_2 \gamma_1 s) - \beta_1 \eta_2^2 R_{n3}(R_2 \gamma_2 s) R_{n4}(R_2 \gamma_1 s)].$$

- in the case of task **A**

$$M_n(s) = R_{n4}(R_1 \gamma_1 s), \quad F_{1np}(s) = (-1)^n R_{n4}(-R_1 \gamma_1 s),$$

$$F_{2np}(s) = (-1)^n R_{n4}(R_1 \gamma_1 s), \quad k_n(s) = R_1^{n+3} \eta_1^2 (\gamma_1 s)^n q_n^L(s).$$

- in the case of task **B**

$$M_n(s) = R_{n3}(R_1 \gamma_1 s), \quad F_{1np}(s) = (-1)^n R_{n3}(-R_1 \gamma_1 s),$$

$$F_{2np}(s) = (-1)^n R_{n3}(R_1 \gamma_1 s), \quad k_n(s) = -R_1^{n+2} (\gamma_1 s)^n V_n^L(s).$$

Note that the elements of all the indicated matrices and vectors are rational functions of the transformation parameter s .

We represent solutions of the system of matrix equations (22) by the following infinite series

$$\begin{pmatrix} \mathbf{A} \\ \mathbf{B} \end{pmatrix} = \sum_{j=0}^{\infty} \begin{pmatrix} \mathbf{a}_{ij}(s) \\ \mathbf{b}_{ij}(s) \end{pmatrix} x^i y^{-j-1}, \quad \mathbf{a}_{ij}(s) = \|a_{ij}^{(1)}(s), a_{ij}^{(2)}(s), \dots\|^T, \quad \mathbf{b}_{ij}(s) = \|b_{ij}^{(1)}(s), b_{ij}^{(2)}(s), \dots\|^T. \quad (23)$$

Substituting (23) into system (22) and equating the coefficients in the left and right sides at the same degrees of the variables, we obtain the following recurrence relations for the coefficients $a_{ij}^{(n)}(s)$, $b_{ij}^{(n)}(s)$, ($n=1, 2, \dots$) and the initial conditions for them.

$$a_{0,0}^{(n)}(s) = \frac{k_n(s)}{M_n(s)}, \quad \mathbf{a}_{0,j}(s) = 0, \quad j \geq 1; \quad \mathbf{b}_{0,j}(s) = 0, \quad \mathbf{b}_{1,j}(s) = 0, \quad j \geq 0; \quad (24)$$

$$\mathbf{a}_{i,0}(s) = \mathbf{G}_2 \mathbf{b}_{i,0}(s), \quad i \geq 1; \quad \mathbf{a}_{i,1}(s) = \mathbf{G}_2 \mathbf{b}_{i,1}(s), \quad i \geq 0; \quad (25)$$

$$\mathbf{a}_{i,j}(s) = \mathbf{G}_2(s) \mathbf{b}_{i,j}(s) - \mathbf{G}_1(s) \mathbf{b}_{i,j-2}(s), \quad i \geq 0, \quad j \geq 2; \quad (26)$$

$$\mathbf{b}_{i,j}(s) = \mathbf{H}(s) \mathbf{b}_{i-2,j}(s) - \mathbf{D}(s) \mathbf{a}_{i-2,j}(s), \quad i \geq 2, \quad j \geq 0; \quad (27)$$

$$\mathbf{G}_i(s) = \left\| \frac{F_m(s)}{M_n(s)} \delta_{nm} \right\|, \quad \mathbf{H}(s) = \left\| \frac{T_{2n}(s)}{T_{1n}(s)} \delta_{nm} \right\|, \quad \mathbf{D}(s) = \left\| \frac{N_n(s)}{T_{1n}(s)} \delta_{nm} \right\|,$$

where δ_{nm} is the Kronecker delta, $n, m=1, 2, \dots$.

These recurrence relations allow one to determine all images of the elements $a_{ij}^{(n)}(s)$, $b_{ij}^{(n)}(s)$ ($n=1, 2, \dots$) of infinite vectors $\mathbf{a}$, $\mathbf{b}$ in the form of rational functions, which allows one to determine the originals of the sought functions using the theory of residues [6].

Substituting series (23) into formulas (16) and (21), in the image space we find the following expressions for the coefficients of the series of displacement components and the stress tensor:

$$\begin{aligned} w_{1n}^L(r, s) &= -\frac{1}{r^{n+2}(\gamma_1 s)^n} \sum_{i,j=0}^{\infty} \left[R_{n3}(r\gamma_1 s) a_{ij}^{(n)}(s) e^{-r\gamma_1 s} + (-1)^n G_{n3}(r\gamma_1 s) b_{ij}^{(n)}(s) \right] x^i y^{-j-1}, \\ \sigma_{r9n}^{(1)L}(r, s) &= \frac{\beta_1}{r^{n+3} \eta_1^2 (\gamma_1 s)^n} \sum_{i,j=0}^{\infty} \left[R_{n4}(r\gamma_1 s) a_{ij}^{(n)}(s) e^{-r\gamma_1 s} + (-1)^n G_{n4}(r\gamma_1 s) b_{ij}^{(n)}(s) \right] x^i y^{-j-1}, \\ \sigma_{99n}^{(1,1)L}(r, s) &= \frac{n(n+1)\beta_1}{r^{n+3} \eta_1^2 (\gamma_1 s)^n} \sum_{i,j=0}^{\infty} \left[R_{n3}(r\gamma_1 s) a_{ij}^{(n)}(s) e^{-r\gamma_1 s} + (-1)^n G_{n3}(r\gamma_1 s) b_{ij}^{(n)}(s) \right] x^i y^{-j-1}, \\ \sigma_{99n}^{(1,2)L}(r, s) &= -\frac{2\beta_1}{r^{n+3} \eta_1^2 (\gamma_1 s)^n} \sum_{i,j=0}^{\infty} \left[R_{n3}(r\gamma_1 s) a_{ij}^{(n)}(s) e^{r\gamma_1 s} + (-1)^n G_{n3}(r\gamma_1 s) b_{ij}^{(n)}(s) \right] x^i y^{-j-1}, \\ w_{2n}^L(r, s) &= -\frac{R_{n3}(r\gamma_2 s) e^{-(r-R_2)\gamma_2 s}}{r^{n+2}(\gamma_1 s)^n R_{n3}(R_2\gamma_2 s)} \times \end{aligned}$$

$$\begin{aligned}
& \times \sum_{i,j=0}^{\infty} \left[R_{n3}(R_2\gamma_1s)a_{ij}^{(n)}(s)e^{-R_2\gamma_1s} + (-1)^n G_{n3}(R_2\gamma_1s)b_{ij}^{(n)}(s) \right] x^i y^{-j-1}, \\
\sigma_{r\theta n}^{(2)L}(r,s) &= \frac{\beta_1 r^{-n-3} R_{n4}(r\gamma_2s)e^{-(r-R_2)\gamma_2s}}{\eta_1^2(\gamma_1s)^n R_{n4}(R_2\gamma_2s)} \sum_{i,j=0}^{\infty} \left[R_{n4}(R_2\gamma_1s)a_{ij}^{(n)}(s)e^{-R_2\gamma_1s} + (-1)^n G_{n4}(r\gamma_1s)b_{ij}^{(n)}(s) \right] x^i y^{-j-1}, \\
\sigma_{\theta\theta n}^{(2,1)L}(r,s) &= \frac{n(n+1)\beta_2 R_{n3}(r\gamma_2s)e^{-(r-R_2)\gamma_2s}}{r^{n+3}\eta_2^2(\gamma_1s)^n R_{n3}(R_2\gamma_2s)} \sum_{i,j=0}^{\infty} \left[R_{n3}(R_2\gamma_1s)a_{ij}^{(n)}(s)e^{-R_2\gamma_1s} + (-1)^n G_{n3}(R_2\gamma_1s)b_{ij}^{(n)}(s) \right] x^i y^{-j-1}, \\
\sigma_{\theta\theta n}^{(2,2)L}(r,s) &= -\frac{2\beta_2 R_{n3}(r\gamma_2s)e^{-(r-R_2)\gamma_2s}}{r^{n+3}\eta_2^2(\gamma_1s)^n R_{n3}(R_2\gamma_2s)} \sum_{i,j=0}^{\infty} \left[R_{n3}(R_2\gamma_1s)a_{ij}^{(n)}(s)e^{-R_2\gamma_1s} + (-1)^n G_{n3}(R_2\gamma_1s)b_{ij}^{(n)}(s) \right] x^i y^{-j-1}.
\end{aligned}$$

3 Results and Discussion

The results of numerical experiments are presented as graphs of changes in the components $\sigma_{r\theta}$, $\sigma_{\theta\theta}$ the stress tensor, and the displacement vector over the dimensionless time τ . As an example, we consider a thick-walled steel ($E_1 = 20.104 \cdot 10^4$ MPa, $\nu_1 = 0.3$, $\rho_1 = 7800 \text{ kg/m}^3$) shell with the inner and outer radius $R_1 = 1$, $R_2 = 2.5$, respectively, in the aluminum space ($E_2 = 6.8 \cdot 10^4$ MPa, $\nu_2 = 0.33$, $\rho_2 = 2700 \text{ kg/m}^3$), which corresponds to the following dimensionless parameters: $\beta_1 = 1$, $\beta_2 = 0,37046$, $\gamma_1 = 1$, $\gamma_2 = 1.026$, $\eta_1 = 1.87083$, $\eta_2 = 1.9853$. The Heaviside function $q_1(\tau, \theta) = q_0 H(\tau)$, $q_0 = 1$, was chosen as the law of variation of the given load concerning time. Numerical results are obtained considering seven terms of the series in Gegenbauer polynomials. In fig. 2 (a) presented the graphs of the tangential $\sigma_{r\theta}^{(1)}$ stress over time at the points of the thick-walled shell: $r = 1$, $\theta = \pi/4$ (curve 1, which corresponds to a given load on the inner surface of the shell), $r = 1.5$, $\theta = \pi/4$ (curve 2) $r = 1.5$, $\theta = \pi/2$ (curve 3) and $r = 2.5$, $\theta = \pi/4$ (curve 4). The graphs shown in fig. 2 (b), demonstrate the change in the component w_1 of the displacement in time at the points of the thick-walled shell: $r = 1$, $\theta = \pi/4$ (curve 1), $r = 1.5$, $\theta = \pi/4$ (curve 2), $r = 2.5$, $\theta = \pi/4$ (curve 3) and $r = 1.5$, $\theta = \pi/2$ (curve 4).

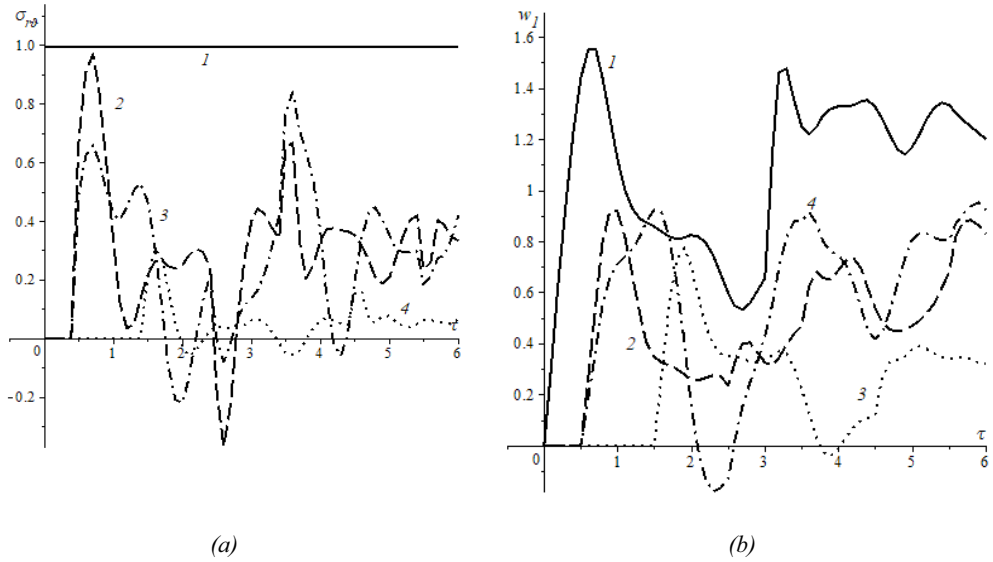


Fig. 2.

In fig. 3 (c), curves are plotted characterizing the changes in the stress $\sigma_{r\theta}^{(2)}$ component with time at various points of the environment: $r = 2.5$, $\theta = \pi/4$ (curve 1), $r = 3.0$, $\theta = \pi/4$ (curve 2) and $r = 4.0$, $\theta = \pi/4$ (curve 3). Fig. 3 (d) shows graphs of changes in the component w_2 of displacement at points in the environment: $r = 2.5$, $\theta = \pi/4$ (curve 1), $r = 3.0$, $\theta = \pi/4$ (curve 2) and $r = 4.0$, $\theta = \pi/4$ (curve 3).

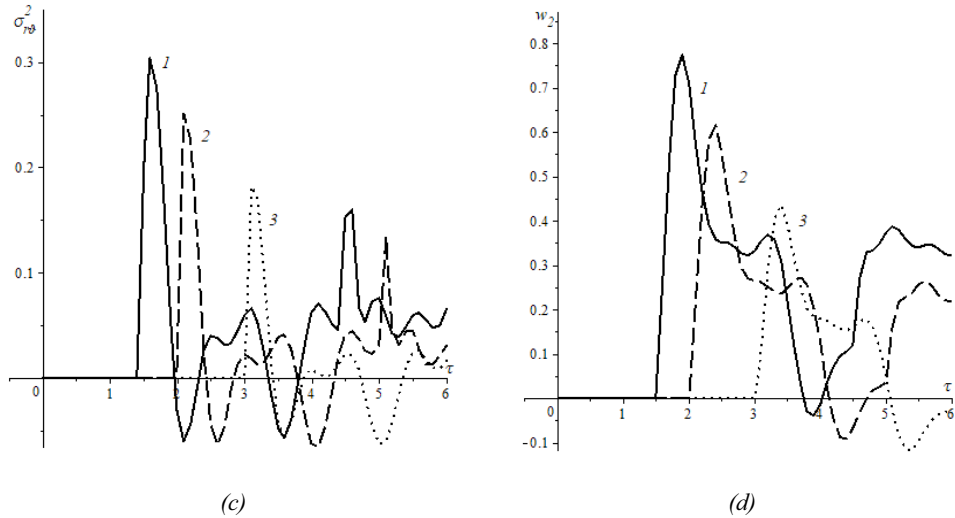


Fig. 3.

4 Conclusion

An algorithm is developed for solving the problem of the propagation of skew-symmetric unsteady shear waves from a thick-walled spherical shell in elastic space. In the Laplace image space, an infinite system of linear algebraic equations is obtained, the solution of which is constructed in the form of an infinite series in exponentials, which made it possible to obtain recurrence relations and initial conditions for them. Numerical results are obtained. The graphs of the time dependence of the stress $\sigma_{r\theta}$ and w of displacement show that the waves reflected from the contact boundary affect the stress-strain state of the shell and the environment.

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