

Simulation of dynamic processes of shell structures with viscoelastic elements

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Abstract. The paper presents the choice and method of calculating thin-walled structures that adequately reflect deformation in the compensator example. When choosing a design model, the influence of structural elements and their mechanical characteristics on its behavior in various operating conditions is taken into account since sometimes small changes in the design model can significantly impact the design analysis results. The complete design scheme of most designs, particularly the compensator, leads to statically indeterminate systems. Taking into account the energy dissipation in the shell material; the boundary value problems are reduced to a system of ordinary differential equations of the 12th order. The forms of natural oscillations for the first four natural frequencies are given. It is established that the inertia forces acting in the axial direction do not significantly affect the stress-strain state of shell structures.

1 Introduction

Even though a large number of published works on calculating the statics and dynamics of shells, taking into account the influence of surrounding or filling media, the design with thin-walled elements is little studied. Based on mathematical modeling, when solving problems of a complicated shell structure, they are usually divided into several elementary parts consisting of shell elements (in particular cylindrical, conical, and spherical shells of rotation). For each element, a calculation scheme is drawn up; based on this scheme, the initial equations of statics or dynamics are drawn up. Then they are generalized, and, in general, dynamic characteristics or stress-strain states are determined for the structure under consideration.

Article [1] presents the development of a geometric integrator for shell structures, which preserves important qualitative features of the basic equations and is equipped with high-frequency numerical dissipation. The efficiency and reliability of the proposed approach are illustrated by specific numerical examples, which also demonstrate the need for integration schemes with high-frequency numerical dissipation.

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Article [2] is devoted to developing qualitative methods in the theory of non-conservative systems arising in the dynamics of a solid interacting with a resisting medium, the theory of oscillations, etc. A full range of cases of complete integrability of non-conservative dynamical systems with nontrivial symmetries is obtained. New families of phase portraits of systems with variable dissipation on manifolds of smaller and larger dimensions are obtained.

Article [3] presents a comprehensive review and analysis of various theoretical models of elastic and viscoelastic bases in oscillatory systems. The review covers foundation models from the simplest to the most complex and fully tracks the latest theories on mechanical foundations. In addition, the paper considers several important practical applications related to linear and nonlinear fundamentals. This article presents detailed theoretical foundations and a physical understanding of various foundations with applications in structural mechanics, nanosystems, bio-devices, composite structures, and mechanical aerospace-based systems.

Article [4] studies oscillations of functionally graded cylindrical shells based on Winkler and Pasternak bases. The shell equations are refined by introducing Winkler and Pasternak base modules. The wave propagation method is used to solve the shell dynamics equations. The method is based on approximating the eigenvalues of the characteristic functions of the beam.

In article [5] provides an overview of experimental work on determining the dynamics of smooth and reinforced cylindrical shells in contact with the ground environment under various non-stationary loads. The results of the study of three-layer shells of rotation, the equations of motion obtained within the hypotheses of the geometrically nonlinear Timoshenko theory, are presented. Numerical results for shells with piecewise or discrete filler allow us to analyze the impact of geometric and physico-mechanical parameters of structures on their dynamics and identify new mechanical effects. Based on the classical theory of shells and rods, the influence of the discrete arrangement of the ribs and the coefficients of the elastic base of Winkler or Pasternak on the normal frequencies and modes of rectangular flat cylindrical and spherical shells is investigated. Numerical algorithms are developed using the integral-interpolation method, and the corresponding non-stationary problems are solved. Special attention is paid to the formulation and solution of related problems on the dynamic interaction of cylindrical or spherical shells with a soil water-saturated medium of various compositions.

In articles [6-7], an approach to identifying the main domain of dynamic instability for systems consisting of shells of rotation of various shapes under axisymmetric periodic loading is described. The original problem is reduced to one-dimensional eigenvalue problems along the meridional coordinate. The results of calculations for a specific shell system are presented.

In article [8], a method is proposed for calculating the natural frequencies of ribbed cylindrical shells with local axisymmetric deflections. The influence of the initial imperfections of two types on the minimum vibration frequencies is analyzed.

In article [9] presents a comprehensive review and analysis of studies in which the main provisions of the theory and methods for calculating the stress-strain state of discrete rib shells are formulated. The main results of the research of the Department of Mechanics of Thin-walled structures under the leadership of I.Ya. Amiro at the Institute of Mechanics, named after S. P. Timoshenko of the National Academy of Sciences of Ukraine, are presented.

In article [10], a dynamic analysis of free vibrations of shell structures is performed and compared with solutions obtained using the finite element method. The effectiveness and reliability of the proposed isogeometric analysis are justified by comparing different theories; for example, some shortcomings of the formulation of Kindle shells are

investigated, the general basis of the theory of Nagdi shells is investigated, and the method of bending the strip is investigated.

The article [11] presents an analytical method for studying the free vibrations of cylindrical shells made of functional gradient material (FGM) with arbitrary intermediate ring supports. The material's properties continuously change in the direction of thickness following the four-parameter power distribution of the volume fractions of the components. To verify the reliability of the developed model, the natural frequencies and waveforms of several homogeneous and cylindrical shells made of functionally gradient material with and without ring supports are compared with the corresponding analytical solutions. Here, the natural frequencies and waveforms of several cylindrical shells made of functionally gradient material with and without ring supports are calculated by the finite element method (FEM), which demonstrates high accuracy and a wide range of applications of the proposed method. The results show that the containment of radial or circumferential displacements on the ring supports significantly affects the natural frequencies.

In articles [12-15], mathematical models with dynamic effects have been developed based on experimental and theoretical studies. Dynamic characteristics are determined, considering the thickness variability according to the shell's viscoelastic theory.

Articles [16-21] are devoted to studying the dynamics of structurally inhomogeneous, multi-connected shell structures, considering the influence of liquid and viscoelastic elements. Based on the laws of mechanics and the Lagrange principle, a mathematical model of the structure has been developed. The three-parameter Rzhantsyn-Koltunov kernel was used as the relaxation kernel.

It can be argued in advance that there are no analytical methods for solving the problems of complex shell systems. Therefore, numerical methods are used. The development of numerical calculation methods for solving the problems of studying the statics and dynamics of complex, structurally heterogeneous, multi-connected shells is an urgent task of our time. In this case, the shell elements of the structure can be multi-layered and structurally heterogeneous, with viscoelastic properties and bonds.

2 Methods

As is known, the analysis of any mechanical structure must begin with constructing a computational model that adequately reflects the deformation of the real structure [22].

When choosing a design model as a whole, it is necessary to assess the influence of a particular structural element on its behavior under real loading since sometimes small changes in the design model can significantly impact the design results analysis. The complete design scheme of the vast majority of aircraft structures, underground and aboveground structures, structures in shipbuilding, and other branches of engineering leads to statically indeterminate systems. One of such systems is an arbitrary axisymmetric design of shells of rotation and circular frames. As an example, consider the constructions shown in Fig. 1. These are shell multi-connected structures, which are an arbitrary composition of multilayer shells of rotations and circular frames.

By analogy with [23], we present the construction (Fig. 1) as an arbitrary composition of nodes connected by shell elements. The nodal elements, in this case, are: end and intermediate frames (nodes 3, 5, and 6), free and supported ends of shells (node 6), parallels of direct connection connections or lines along which geometric and mechanical parameters or components of applied loads are torn (nodes 2 and 4), poles of the structure, in which the shell generators intersect with the axis of rotation (node 1). Shell elements are the shells of rotation connecting the nodal elements. Elements of the "link" type carry out the connection between the nodal elements of the structure or between the nodal elements and the fixed support and are elements of the spring type with some real stiffness characteristics.

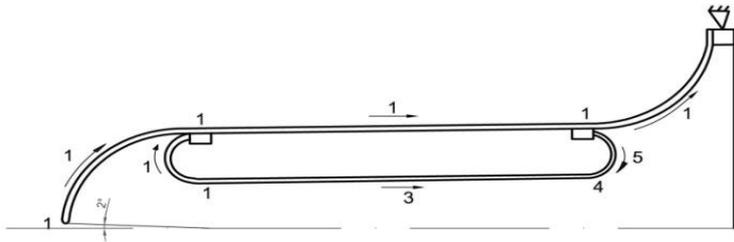


Fig.1. Design scheme of the shell structure.

Shell elements are cylindrical shells of non-circular cross-section connecting the nodal elements. Each shell element of the considered classes of structures can be isotropic, orthotropic, or structurally orthotropic and have elastic viscoelastic properties with significantly different functions of the heredity of the material in the structure of the element.

Shell elements in structures of the first class can have variable stiffness and mechanical characteristics of the generatrix, and shell elements in structures of the second class can have variables along the guide. The Kirchhoff-Lyve hypotheses must be valid for each shell element. No restrictions are imposed on the geometry of the forming shells of rotation and the geometry of the guiding cylindrical shells.

Each circular frame may have elastic or viscoelastic properties with significantly different rheological characteristics. Cross sections of frames and stringers are considered non-deformable, i.e., frames are considered according to the classical scheme of a circular ring, and stringers are considered according to the classical scheme of a rectilinear rod. The bonds' stiffness characteristics can be elastic and visco-elastic, as described by the hereditary Boltzmann-Volter relations. It is assumed that a system of external dynamic loads acts on the structure.

In the special case, when there are no external mechanical influences, free damped vibrations of the structure are considered in the presence of periodic influences - steady forced vibrations.

Suppose that the connection between nodes i and j is carried out using N_{ij} shell elements, each of which, as well as all values related to it, we assign a triple index ijs ($1 \leq s \leq N_{ij}$). In total, the design will have shell elements $N_s = \sum_{i=1}^{N_r-1} \sum_{j=i+1}^{N_2} N_{ij}$.

Let's also assume that the connection between nodes i and j is carried out using M_{ij} visco-elastic hereditary bonds, each of which, as well as all the quantities related to it, we assign a triple index ijs ($1 \leq s \leq M_{ij}$) of all the construction will have $N_e = \sum_{i=1}^{N_r-1} \sum_{j=i+1}^{N_2} M_{ij}$ visco-elastic bonds. To denote quantities related to a shell element or a viscoelastic bond, we will use, where this does not cause misunderstanding, the ordinal number of the p ($1 \leq p \leq N_s$) element or the ordinal number of the p ($1 \leq p \leq N_e$) bond.

By analogy with [23-27], for each shell element, we introduce a local coordinate system $0 \alpha_1 \alpha_2 r$. To do this, we define a certain surface inside the shell element, which we call a coordinate surface. The position of the points on this surface will be determined by the

Gaussian curvilinear coordinates α_1 and α_2 directed along the lines of the main curvature. In this case, α_1 is directed along the guide, and α_2 is directed along the generatrix of the cylindrical element. The Z coordinate, which determines the distance from some point of the shell element to the coordinate surface, is directed so that the coordinate system $O\alpha_1\alpha_2Z$ forms a right orthogonal coordinate system.

Consider next a thin-walled axisymmetric shell instruction. We associate the global right rectangular coordinate system Ox_1x_2r with this construction. The x_1 axis will be directed along the structure's rotation axis. Let's imagine this construction as an arbitrary composition of N_r annular nodal elements, N_s shells of rotation, and N_c visco-elastic bonds.

The numbering of nodes, shell elements, and connections, as well as the indexing of all values related to nodes, shell elements, and connections, will be carried out by analogy with prismatic structures. It is known that the internal geometry of the coordinate surface can be characterized by the first quadratic shape. Suppose the coordinates α_1 and α_2 correspond to the lines of the main curvature. In that case, the differentials of the arcs of the coordinate lines can be expressed in terms of the differentials of the curvilinear coordinates:

$$dS_1 = A_1 d\alpha_1, dS_2 = A_2 d\alpha_2, \quad (1)$$

where A_1 and A_2 are the Lamé coefficients.

The external geometry of the surface in the selected coordinate system is characterized by the main radii of curvature R_1 and R_2 (or the main curvatures):

$$H_1 = 1/R_1, H_2 = 1/R_2, \quad (2)$$

The values $A_1(\alpha_1, \alpha_2)$, $A_2(\alpha_1, \alpha_2)$, $H_1(\alpha_1, \alpha_2)$, $H_2(\alpha_1, \alpha_2)$, must satisfy the Gauss-Codazzi relations known from the theory of surfaces:

$$\frac{\partial}{\partial \alpha} (K_2 A_2) = K_1 \frac{\partial A_2}{\partial A_3} \quad (1 \Rightarrow Z) \quad , \quad (3)$$

$$\frac{\partial}{\partial \alpha} \left(\frac{1}{A_1} \frac{\partial A_2}{\partial \alpha_1} \right) + \frac{\partial}{\partial \alpha_2} \left(\frac{1}{A_2} \frac{\partial A_1}{\partial \alpha_2} \right) = K_1 K_2 A_1 A_2 \quad , \quad (4)$$

In the case of axisymmetric shells, the values A_1, A_2, K_1, K_2 , depend on the coordinate α_2 and depend only on the coordinate α_1 directed along the generatrix. The Gauss-Codazzi relations are simplified:

$$\frac{1}{A_1} \frac{\partial}{\partial \alpha_1} = (K_1 - K_2) \frac{1}{A_1 A_2} \frac{\partial A_2}{\partial \alpha_2}, \quad (5)$$

$$\frac{1}{A_1} \frac{\partial}{\partial \alpha_1} = \left(\frac{1}{A_1} \frac{\partial A_2}{\partial \alpha_1} \right) = -K_1 K_2 A_2, \quad (6)$$

As the main equations characterizing the natural oscillations of shell systems, we take the system of differential-algebraic equations with complex coefficients obtained in [28-30].

Thus, in the general case, the problem is reduced to solving systems of integro-differential-algebraic equations with complex coefficients [22-23].

The solution to the problem of natural oscillations of structurally inhomogeneous shell structures is reduced to finding complex values for which a system of algebraic differential equations with complex coefficients:

$$y_p^1 = f^p(\alpha_1^p, n, y_p) + f_\omega^p(y_p), (p = 1, \dots N_s), \quad (7)$$

$$[G_i] - \omega^2 [\tilde{G}_\omega] \Delta_i = \sum_j \sum_s \xi_i^{ijs} [\varphi_i^{ijs}] Q_i^{ijs} + \sum_j \sum_s \xi_{ci}^{ijs} [\varphi_{ci}^{ijs}] Q_{ci}^{ijs} \quad (8)$$

$$(i = 1, \dots N_r)$$

Equation (8) has a nontrivial solution.

The components of the vector $f^p = (\alpha_1^p, n, y_p)$ in equations (7) are calculated by the formulas:

$$f_{\omega_1}^p = -\tilde{\omega}^2 \rho^{-p} y_5^p, f_{\omega_2}^p = -\tilde{\omega}^2 \rho^{-p} y_6^p, f_{\omega_4}^p = -\tilde{\omega}^2 \rho^{-p} y_8^p, \quad (9)$$

The value ω_k^* at which there is a nontrivial solution of the system of differential-algebraic equations (7) - (8) determines the spectrum of oscillation frequencies of the structure under consideration.

3 Results and Discussion

In a concrete example, let's consider a structurally inhomogeneous structure - a bellows consisting of a set of plates, cylindrical and toroidal shells (Fig.2).

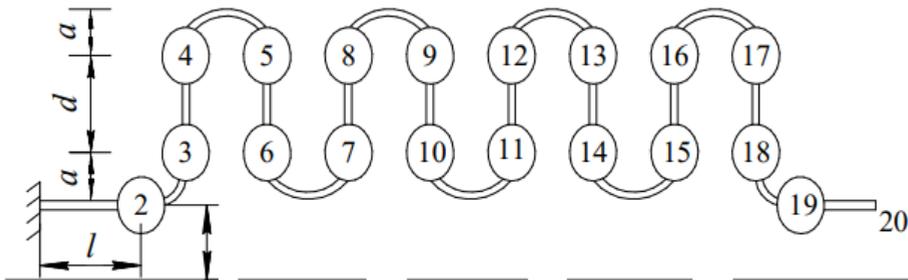


Fig.2. Design scheme of the bellows

The bellows consist of two bearing layers and a visco-elastic filler. The thicknesses of all layers are fixed, the thicknesses of the bearing layers are 0.2 sm, and the thickness of the viscoelastic layer is 1 cm ($r = 20$ cm; $l = 20$ cm; $a = 5$ cm; $d = 20$ cm). The load-bearing layers of the structure have the following mechanical characteristics:

$$E = 2 \cdot 10^5 \text{ MPa}; \nu = 0.3; \rho = 8 \cdot 10^{-6} \quad (10)$$

The visco-elastic layer of the structure has the following mechanical characteristics:

$$E = 2 \cdot 10^4 \text{ MPa}; \nu = 0.2; \rho = 1 \cdot 10^{-6}; A = 0.1; \beta = 0.05; \alpha = 0.2 \quad (11)$$

We investigate the natural oscillations of the bellows under two boundary conditions:

1. In nodes I and 20: I - node is fixed, 20 - free;
2. Both nodes are fixed.

Table 1 shows the values of natural frequencies and damping coefficients for two types of boundary conditions and the first four vibration tones.

Table 1. Values of natural frequencies and damping coefficients

№	I			II		
	ω_{RI}	ω_R	ω_I	ω_y	ω_R	ω_I
1	94.19	93.96	0.077	511.76	510.81	0.302
2	365.29	364.56	0.235	869.96	868.44	0.476
3	749.71	748.46	0.410	1136.54	1134.80	0.565
4	1064.56	1062.9	0.540	1782.60	1777.60	1.120

Fig.3 and Fig.4 show the forms of natural oscillations for the first four natural frequencies, respectively, with boundary conditions I and II.

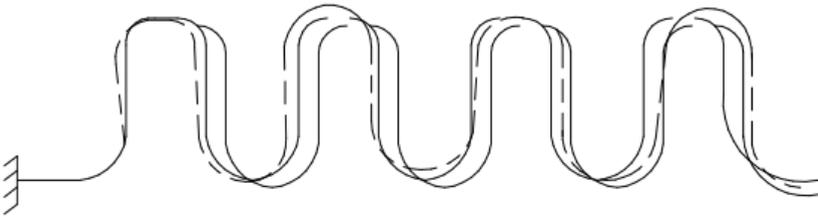


Fig.3. The forms of natural oscillations for the first four natural frequencies with boundary conditions I.

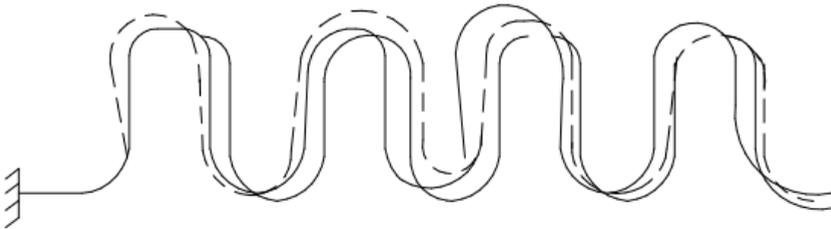


Fig.4. The forms of natural oscillations for the first four natural frequencies with boundary conditions II.

The solution to the boundary value problem consists of two stages. In the first stage, the missing initial conditions are determined by the " shooting " method at the second - the usual Cauchy problem is solved.

The *sbval* (v, t_1, t_2, D, l, s) function is designed to find the missing initial conditions by shooting in Mathcad. Here v is the vector of missing initial conditions, $[t_1, t_2]$ is the integration interval, D is the vector of the right parts of the system of differential equations, and l is the vector function of the initial values. The missing elements are defined as components of the vector v , s is a vector in which the difference between the exact and calculated values of the functions at the ends of the segment is stored.

If we consider the energy dissipation in the shell material, then the boundary value problems (6) will be of the 12th order. Numerical results show that taking into account the inertia forces acting in the axial direction does not significantly affect the stress-strain state.

4 Conclusions

1. It has been established that applying the developed techniques for determining the natural frequencies and vibration patterns of shell structures on the example of a compensator allows this technique to calculate more complex structures.
2. A technique of computer simulation technology of dynamic processes is proposed, using the example of natural oscillations of shell systems in the form of a compensator.
3. The developed technique makes it possible to determine the amplitude-frequency characteristics of real shell systems with viscoelastic properties.

References

1. Bauchau, O.A., Choi, Yu.Yu. and Bottasso, K.L. On modeling shells in the dynamics of several bodies. *Dynamics of multibody systems*. 2002. Pp. 459-489.
2. Shamolin, M.V. Dynamic systems with variable dissipation: approaches, methods and applications. *J Math Sci*. 162. 741. 2009. <https://doi.org/10.1007/s10958-009-9657-y>
3. Yunesyanyan, D., Hosseinhani A., Askari H. Elastic and viscoelastic bases: a review of linear and nonlinear vibration modeling and applications. *Nonlinear Dyn*. Pp. 853-895. 2019. <https://doi.org/10.1007/s11071-019-04977-9>
4. Shah, A.G., Mahmud, T., Naim, M.N. Vibrations of functionally graded cylindrical shells on elastic bases. *Acta Mech*. 2010. Pp. 293-307.
5. Lugovoy, P.Z., Meish, V.F. Dynamics of inhomogeneous shell systems under unsteady loading (Review). *Int Appl Mech* 53. 2017. Pp. 481-537.
6. Beshpalova, E.I., Urusova, G.P. Determination of the areas of dynamic instability of inhomogeneous shell systems under periodic loads. *Int Appl Mech*. 2011.
7. Beshpalova, E.I., Urusova, G.P. Vibrations of shells of rotation with a branched meridian. *Int Appl Mech* 52, Pp.82-89. 2016. <https://doi.org/10.1007/s10778-016-0735-9>
8. Gavrilenko, G.D., Matsner, V.I., Kutenkova, O.A. Free oscillations of ribbed cylindrical shells with local axisymmetric deflections. *Int Appl Mech*. Pp.1006-1014. 2008. <https://doi.org/10.1007/s10778-009-0116-8>
9. Zarutsky, V.A. Theory and methods of analysis of the stress-strain state of ribbed shells. *International Applied Mechanics* 36. 2000. Pp.1259-1283.
10. Atri, H.R., Shoji, S. Analysis of free vibrations of thin-walled structures using finite elements based on an isogeometric approach. *Iran. Doctor of Technical Sciences. Technol.Trans. Civ. Eng.* 40. 2016. Pp.85-96. <https://doi.org/10.1007/s40996-016-0011-6>
11. Xie, K., Chen, M. Analytical method for free oscillations of functionally graded cylindrical shells with arbitrary intermediate ring supports. *J Braz. Soc. Mechanic*. 2021. <https://doi.org/10.1007/s40430-021-02829-5>
12. Mirsaidov M. and Troyanovsky I. E. Forced axisymmetric oscillations of a viscoelastic cylindrical shell. *Polymer Mechanics* 11(6). 1975. Pp. 953-955.
13. Mirsaidov, M.M., Khudainazarov, Sh.O. Spatial natural vibrations of viscoelastic axisymmetric structures. *Magazine of Civil Engineering*. No.04. 2020. 96(4). Pp.118-128. DOI: 10.18720/MCE.96.10
14. Mirsaidov M., Abdikarimov R., Khudainazarov S., Sabirjanov T. Damping of vibrations of high-rise structures by viscoelastic dynamic dampers. *E3S Web Conference*. 2020. 02020. /DOI10.1051/e3sconf/202022402020

15. Sherzod Khudainazarov, Talibjan Sabirjanov and Alisher Ishmatov. Assessment of Dynamic Characteristics of High-Rise Structures Taking into Account Dissipative Properties of the Material. *Journal of Physics*. 1425. 2020. 012009 doi:10.1088/1742-6596/1425/1/012009
16. Tulkin Mavlanov, Sherzod Khudainazarov and Islomjon Khazratkulov. Natural Vibrations Of Structurally Inhomogeneous Multi-Connected Shell Structures With Viscoelastic Elements. *Journal of Physics*. 1425 012017. 2020. doi:10.1088/1742-6596/1425/1/012017
17. Mavlanov T and Khudainazarov Sh. Calculation of structural-inhomogeneous multiply connected shell structures with viscoelastic elements E3S Web of Conferences. Vol. 97. No 040542. 2019. DOI:10.1051/e3sconf/20199704054
18. Mirsaidov, M., Safarov, I.I., Teshae, M.K. Dynamic instability of vibrations of thin-wall composite curvilinear viscoelastic tubes under the influence of pulse pressure. *E3S Web of Conferences* 164(5), 2020, № 1401320
19. Mirsaidov, M.M., Safarov, I.I., Teshae, M.K., Boltayev, Z.I. Dynamics of structural - Inhomogeneous coaxial-multi-layered systems "cylinder-shells". *Journal of Physics: Conference Series*, 1706(1), 2020, № 0120331
20. Bagheri, H., Kiani, Y., Eslami, M.R. Free vibration of joined conical–cylindrical–conical shells. *Acta Mechanica* 229 (7). 2018. 2751-2764. DOI: 10.1007/s00707-018-2133-3
21. Tian, L., Ye, T., Jin, G. Vibration analysis of combined conical-cylindrical shells based on the dynamic stiffness method. *Thin-Walled Structures* 159. 2021. №107260. DOI: 10.1016/j.tws.2020.107260
22. Mavlyanov, T. Development of methods and algorithms for calculating shell structures, taking into account structural heterogeneity and interaction with various media. *Monograph. –T.:TIHAME. -2019. p.217.*
23. Myachenkov V.I., Maltsev V.P. Methods and algorithms for calculating spatial structures on a computer. - M.: Mechanical Engineering, 1984. - 278 p.
24. Ilyushin, A.A. Mechanics of Elastic and Plastic Strains of Solids. 2003. *Collection of Works. Vol. 1 (1935- 1945)*, Pp.232-272.
25. Ilyushin, A.A., Vasin, R.A., Mossakovskii, P.A. Theory of Elastoplastic Processes under Large Plastic Strains. *Applied Problems of Mechanics of Thin-Walled Structures*. 2000. Pp. 128-137.
26. Georgievskii, D.V., Pobedrya, B.E. Asymptotic analysis of evolution of a neck in extended thin rigid plastic solids. *Russian Journal of Mathematical Physics*. 23 (2). 2016. Pp. 200-206.
27. Pobedria, B.E., Georgievskii, D.V. Two thermodynamic laws as the fourth and the fifth integral postulates of continuum mechanics. *Studies in Systems. Decision and Control*. 69. 2016. Pp. 317-325.
28. Maltsev, L.E. Replacing the exact equation of the dynamic viscoelasticity problem with an approximate one. *Mechanics of Polymers*. No.3. 1977. Pp.408-416.
29. Koltunov M.A. Creep and relaxation - M.: Vysshaya shkola. 1976. 277 p.
30. Kravchuk, A.S., Mayboroda, V.P., Urzhumtsev, Yu.S. Mechanics of polymer and composite materials.-M.: Nauka. 1985. p. 304.