

Statement and general technique for solving problem of oscillation of a hereditarily deformable aircraft

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Abstract. This article provides a general statement and a technique for solving the problem of oscillations of a hereditarily deformable aircraft in a gas flow with a finite number of degrees of freedom. Using the Lagrange equations and the variational principle of the hereditary theory of viscoelasticity, the equations of motion of the problem under consideration are derived. The generalized forces acting on the aircraft in the subsonic flight mode are determined according to the stationarity hypothesis. As a result, closed interconnected weakly singular integro-differential equations are obtained that describe the mathematical model of the problem with a finite number of degrees of freedom. General schemes for the numerical solution of these equations are outlined. As an example, the flexural-torsional-aileron flutter of the transient process of a hereditarily deformable wing with a finite freedom number is considered.

1 Introduction

An algorithm for the numerical solution of a system of linear and nonlinear weakly singular integro-differential equations is constructed by eliminating weakly singular singularities of integral and integro-differential equations.

The problem of flexural-torsional-aileron flutter of a hereditarily deformable aircraft wing with finite degrees of freedom is considered. During the research, a special approach to solving this problem was developed. According to this approach, the problem's solution ultimately reduces to solving a system of homogeneous linear algebraic equations with complex coefficients. From the conditions for the existence of non-trivial solutions of this system, transcendental algebraic equations are obtained, the solution of which can be obtained numerically using applied computer programs.

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2 General statement of the problem. Numerical solution technique

A hereditarily deformable aircraft moving in a gas flow is a complex mechanical system with infinite degrees of freedom, continuously exchanging mechanical and thermal energy with the environment. The description of the motion of such a system in the most general form seems unlikely. Therefore, in most problems, the described motions of mechanical systems have to be considered as a structure consisting of a finite number of deformable and rigid elements, the relative mobility of which is limited by holonomic constraints. Thus, in this case, along with external loads, the equations of motion will also include unknown reaction forces of constraints. These forces can be eliminated from the equation of motion using the principle of possible displacements. As a result, we obtain a system of equations describing the motion of a non-free material system with holonomic constraints - the Lagrange equation of the second kind

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0, \quad i = 1, 2 \dots n \quad (1)$$

where, $L = T - \Pi - W$, T, Π -k is the kinetic and potential energy, W is the work of external forces, q_i, \dot{q}_i are the generalized coordinates and its time derivative.

The kinetic energy of a mechanical system with holonomic constraints is represented in the form of a homogeneous quadratic function of generalized velocities

$$T = \frac{1}{2} \sum_{i,j=1}^n c_{ij} \dot{q}_i \dot{q}_j \quad (2)$$

Potential energy, according to the variational principle of the hereditary theory of viscoelasticity [2], can be expressed in the form of a homogeneous quadratic function of generalized coordinates

$$\Pi = \frac{1}{2} \sum_{i,j=1}^n a_{ij} q_i (q_j - 2R^* q_j) \quad (3)$$

where, c_{ij} is inertial coefficients, a_{ij} is stiffness coefficients,

$$W = \sum_{i=1}^n Q_i q_i \quad (4)$$

where Q_i is the generalized force,

$$R^* q_j(t) = \int_0^t R(t - \tau) q_j d\tau$$

- case of transient process,

$$R^* q_j(t) = \int_{-\infty}^t R(t - \tau) q_j d\tau$$

- for steady process

$R(t - \tau)$ is the core of heredity having a weakly singular feature of the Abel type, i.e.,

$$R(t - \tau) = \varepsilon e^{-\beta(t-\tau)}(t - \tau); \quad \varepsilon > 0, \quad \beta > 0, \quad 0 < \alpha < 1$$

Substituting (2) - (4) into (1), we obtain a system of ordinary weakly singular integro-differential equations of the second order of the form:

$$\frac{d}{d\bar{q}_i} \frac{\partial T}{\partial \bar{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial \Pi}{\partial q_i} = Q_i, \quad i = 1, \bar{n} \quad (5)$$

Methods for solving such weakly singular integro-differential equations both for the transient process and for the steady process are well described in [1,3].

In particular, for a flexural-torsional-aileron flutter of a hereditarily deformable wing, expression (2)–(4) takes the form [1, 4]

$$T = \frac{1}{2} c_{11} \dot{q}_1 + \frac{1}{2} c_{22} \dot{q}_2^2 + \frac{1}{2} c_{33} \dot{q}_3^2 + c_{12} \dot{q}_1 \dot{q}_2 + c_{13} \dot{q}_1 \dot{q}_3 + c_{23} \dot{q}_2 \dot{q}_3 \quad (6)$$

$$\Pi = \frac{1}{2} a_{11} q_1 [q_1 - 2R^* q_1] + \frac{1}{2} a_{22} [q_2 - 2R^* q_2] + \frac{1}{2} a_{33} [q_3 - 2R^* q_3] \quad (7)$$

$$W = Q_1 q_1 + Q_2 q_2 + Q_3 q_3 \quad (8)$$

Integro-differential equations (5) for this problem take the form

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} = Q_{q_i}; \quad i = \bar{1}, \bar{3} \quad (9)$$

where $L = \Pi - T$,

If the external forces are aerodynamic, then according to the stationarity hypothesis, for Q_{q_1} , Q_{q_2} , Q_{q_3} we have [5,6]

$$\begin{aligned} Q_{q_1} &= -\rho t V^2 \left[a_1 q_2 + a_2 \frac{t}{V} \dot{q}_2 + a_3 q_3 + a_4 \frac{1}{V} \dot{q}_3 + a_5 \frac{1}{V} \dot{q}_1 \right] \\ Q_{q_2} &= -\rho t^2 V^2 \left[m_1 q_2 + m_2 \frac{t}{V} \dot{q}_2 + m_3 q_3 + m_4 \frac{t}{V} \dot{q}_3 + m_5 \frac{1}{V} \dot{q}_1 \right] \\ Q_{q_3} &= -\rho t^2 V^2 \left[d_{31} q_2 + d_{32} \frac{t}{V} \dot{q}_2 + d_{33} q_3 + d_{34} \dot{q}_3 + d_{35} \frac{1}{V} \dot{q}_1 \right] \end{aligned} \quad (10)$$

where coefficients a_i, m_i, d_{3i} ($i = \bar{1}, \bar{5}$) are known constants [1,2]; ρ is air density; t is wing chord; V is flight speed.

Substituting (10) into (9), we obtain a system of weakly singular integro-differential equations for determining the generalized coordinates $q_1 = h$, $q_2 = a$, $q_3 = s$, which in matrix form can be written as

$$M\ddot{U} + K(1 - R^*)U + V^2 B\dot{U} + V D\dot{U} = 0 \quad (11)$$

with initial conditions

$$U(0) = U_0; \quad \dot{U}(0) = \dot{U}_0 \quad (12)$$

that

$$U = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}; \quad M = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}; \quad K = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$$

$c_{12} = c_{21}$; $c_{13} = c_{31}$; $c_{23} = c_{32}$; V is flight speed;

$$D = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}; \quad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

The system of weakly singular integro-differential equations (11) together with the initial conditions (12) is described by a discrete mathematical model in the general formulation of the problem of the flexural-torsional-aileron flutter of a hereditarily deformable wing in a gas flow. This system is quite general. In particular, it can be obtained from it: a) if $q_3 = 0$ is the flexural-torsional flutter of a hereditarily deformable wing: b) if $q_2 = 0$ – flexural-aileron flutter and finally c) if $q_j = 0$ – torsion -aileron flutter.

Then the problem becomes a classical flutter problem with two degrees of freedom: $R^* = 0$ we obtain the problem of the flexural-torsional-aileron flutter of an ideally elastic wing together with the above-mentioned particular cases.

The exact solution of the system of weakly singular integro-differential equations (11) presents significant mathematical difficulties. Therefore, an approximate solution, according to the method of eliminating weakly singular singularities of integral and integro-differential equations [3,7], is found from the following linear systems of recurrent algebraic equations:

$$(M + VDC_n)U_n = (M + VDt_n)U_0 + M\dot{U}_0 t_n - \sum_{j=0}^{n-1} C_j(t_n - t_j) \left[KU_j + V^2 BU_j - \frac{\varepsilon}{\alpha} \sum_{m=0}^j P_m e^{-\beta t_m} KU_{j-m} \right] - V \sum_{j=0}^{n-1} C_j D U_j, \quad n = 1, 2, \dots \quad (13)$$

Where

$$C_0 = C_n = \frac{\Delta t}{2}; \quad t_n = n\Delta t; \quad C_j = \Delta t; \quad j = 1, n-1; \quad P_0 = \frac{(\Delta t)^\alpha}{2}; \quad U_n = U(t_n); \\ P_m = \frac{(\Delta t)^\alpha}{2} [(m+1)^\alpha - (m-1)^\alpha]; \quad m = 1, j-1; \quad P_j = \frac{(\Delta t)^\alpha}{2} [j^\alpha - (j-1)^\alpha]$$

Determining the critical flight speed at which the flutter phenomena of aircraft begins is one of the most important tasks of aeroelasticity. The solution is to study the oscillatory instability (flutter) of the unperturbed motion of the aircraft using the developed computational algorithm (13) and a special algorithm for finding critical speeds V_{kp} based on a computational experiment for given geometric and mechanical parameters. According to this technique, the loss of dynamic stability is determined from the conditions for the existence of undamped amplitudes (critical time, critical speed) [1].

Thus, the use of the proposed mathematical models and the numerical solution algorithm (13) makes it possible to investigate the problem of vibration of a wing with an aileron made of hereditarily deformable materials in an air flow in subsonic flight modes.

Suppose the nature of the natural vibrations of a structure is known. In that case, it is possible to judge its inherent internal properties that manifest themselves under the action of external disturbances. On fig. 1. bending, torsional, and aileron oscillations are shown on an elastic setting.

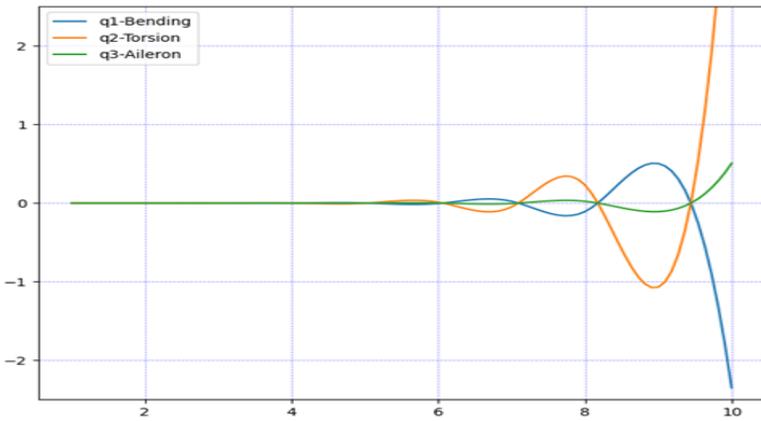


Fig. 1. $V = 320, \alpha = 0.25, \beta = 0.05, \varepsilon = 0.1$

On fig. 2. bending, torsional, and aileron vibrations are shown in a viscous-elastic setting.

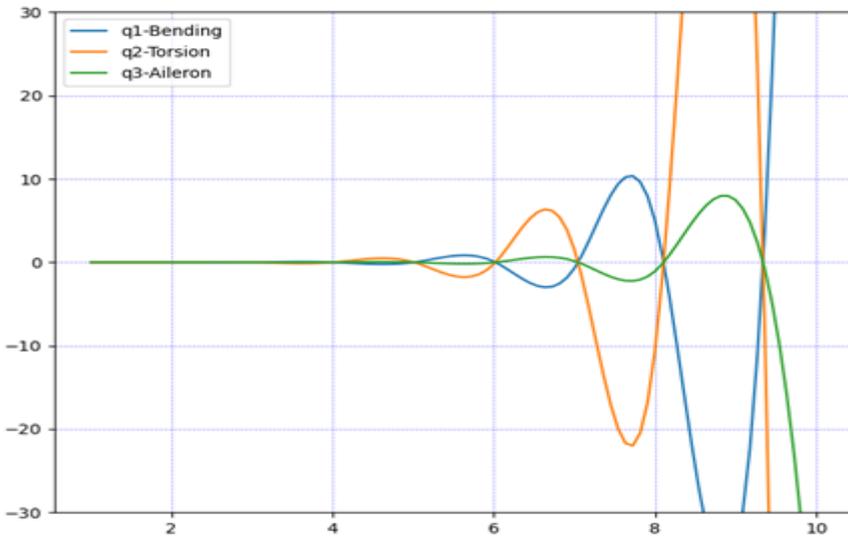


Fig. 2. $V = 320, \alpha = 0.25, \beta = 0.05, \varepsilon = 0.0$

3 Conclusions

1. Computational experiments have shown that a slight decrease in the singularity parameter or a slight increase in the viscosity parameter leads to a significant decrease in the critical flutter speed. Accounting for aerodynamic damping in the viscoelastic case leads to an increase in the critical flutter velocity. This can be explained by the fact that in the viscoelastic case, the damping terms of the aerodynamic forces turn out to be a destabilizing factor that causes oscillatory instability of hereditarily deformable systems.

2. It has been found that the damping rate of free flexural-torsional vibrations essentially depends on the rheological parameters of the construction material. A decrease in the

singularity parameter leads to an increase in the coefficient of internal energy absorption of the system. Thus free oscillations of the system in practice will disappear after a certain period, which provides a new opportunity to optimize the damping properties of the material of vibrating structures used in aerospace technology.

3. It is determined that at low flight speeds, natural damped oscillations occur near the equilibrium position, then with an increase in flight speed, the oscillatory process occurs slightly above the equilibrium position with slowly increasing amplitudes.

4. It has been established that the excess of the critical flutter velocity in the ideal elastic case does not mean the immediate destruction of the structure, the destruction occurs only after a certain period and is of a fatigue nature. To find the expected service life of a structure, it is necessary to determine the amplitude of its oscillations in the flutter region, taking into account the structure material's nonlinear and hereditarily deformable properties.

References

1. Badalov F.B. Ganikhanov Sh.F. Vibrations of hereditary - deformable structural elements of aircraft. Tashkent. TGAI.2002. 230s.
2. Rzhantsyn A.R. Creep theory. M. Nauka 1968. 416s
3. Matyash V.I. Flutter of an elastic-viscous plate. Mechanics of polymers. 1971. No. 6. p. 1077-1083
4. Usmonov B.Sh. Oscillations of a hereditarily deformable wing with an aileron. Problems of mechanics. Tashkent. 2006. No. 4. p.19-22
5. Keldysh M.V., Grossman E.P., Parkhomovsky Ya.M. Vibrations of the wing with aileron. Proceedings of TsAGI 1937. no . 337. p.1-98.
6. Keldysh M.V. Selected works. Mechanics. M. science. 1985.348s.
7. Usmonov, B., Rakhimov, Q., Akhmedov, A. The study of the influence of the gamma function on the flutter velocity. International Conference on Information Science and Communications Technologies: Applications, Trends and Opportunities, ICISCT 2019.