Effect of non-stationary external forces on vibrations of composite pipelines conveying fluid

A A Verlan[1], *B A* Khudayarov[2], *F Zh* Turaev[2*], *N N* Yuldashev[2], and *K* Ruzmetov[3]

[1]Norwegian University of Science and Technology, Trondheim, Norway
[2]"Tashkent institute of irrigation and agricultural mechanization engineers" National Research University, Tashkent, Uzbekistan
[3]Tashkent State Agrarian University, Tashkent, Uzbekistan

Abstract. The effect of non-stationary external forces on the vibration of pipelines made of composite materials is investigated in the paper. A mathematical model of composite pipeline vibration is developed, considering the viscosity properties of the structure and pipeline base material, axial forces, internal pressure, resistance forces, and external disturbances. A mathematical model of viscoelastic pipelines conveying fluid under vibrations is constructed based on the Boltzmann-Volterra integral model. The mathematical model to study a pipeline is based on the Euler-Bernoulli beam theory. Considering the physicomechanical properties of the pipeline material, the mathematical model of the problems under consideration presents a system of integro-differential equations (IDE) in partial derivatives with corresponding initial and boundary conditions. The nonlinear partial differential equations, obtained using the Bubnov-Galerkin method under considered boundary conditions, are reduced to solving the system of ordinary integro-differential equations. A computational algorithm is developed based on eliminating features of integro-differential equations with weakly singular kernels, followed by using quadrature formulas.

1 Introduction

Pipeline transport is the main and most frequently used means to convey various fluids over long distances. It is the only type of transport that conveys the transported product while remaining in a stationary position. Despite the advantages and the widespread prevalence of this type of transport, trunk pipelines have all the signs of a source of increased danger, and the unexpected vibration of the pipeline caused by various external and internal factors limits their use. Therefore, the vibration of pipelines is of scientific and practical interest and attracts the attention of scientists and specialists [1-10].

In [11], pipeline vibration was investigated, and its reaction to external forces was analyzed. The dynamic stability of an elastic pipeline was calculated in [12]. The stress-strain state of a pipeline under external load caused by sea water forces was studied. In

* Corresponding author. t.fozil86@mail.ru

[13], the Euler – Bernoulli equations were used to model pipe vibrations. Using the finite element method, a discrete model was obtained. Numerical calculations were carried out at various flow rates and under unstable flutter conditions. In [14], vibration in pipelines caused by external forces was studied. The effect of vibration on the pipeline stress-strain state was analyzed using the finite element method. In [15], the nonlinear vibration of a pipeline conveying fluid was studied under the effect of harmonic external excitation. The equations of motion were described by integro-differential equations. The study of such equations using the Galerkin method is reduced to solving the systems of nonlinear integro-differential equations in ordinary derivatives.

In [16], the influence of slug flow on the behavior of pipelines conveying multiphase flows was studied. In the Euler-Bernoulli theory of beams, a carrying flow of variable density is used to simulate the pipeline motion. The regions of parametric resonance in the supercritical mode of pipe vibration were determined.

In [17], the parametric resonance of a combined pipe made of soft and hard materials was studied based on an elastic beam. It has been established that with an increase in the rigidity coefficient, a composite pipe made of hard materials effectively improves the region of system stability.

In [18], a dynamic model of a bent pipe conveying fluid was obtained. To discretize a physical model in space, a differential-quadrature method was used. It is shown that the pipe's vibrations are significantly affected by its unstrained configuration.

One of the important tasks of pipeline transport development is to increase the durability of the elements of these systems, their resistance to aggressive environments, and, as a result, to increase the environmental friendliness of projects. Pipelines made of composite materials are widely used in the oil and gas industry when creating modern oil pipeline systems for transporting oil, and gas, constructing prefabricated oil reservoirs, and various pipeline systems in infrastructure engineering the oil and gas industry.

Due to the severe wear of pipeline systems and the need to ensure their operational reliability, the study of pipeline transport in the oil and gas industry and the development of mathematical models of vibration of pipelines made of composite materials, as well as the creation of effective software tools based on these models, are extremely important and relevant tasks.

Modeling strain processes of a viscoelastic pipe conveying fluid, with account for geometrical nonlinearity, was proposed in [19] based on the Euler theory of beam and the generalized Hamilton principle. A nonlinear dynamic model of the equation of pipe motion of fractional order was obtained. The effect of geometrical nonlinearity parameters, internal flow rate, and mass ratio on the oscillatory process of a composite pipeline was studied.

In [20], a numerical study of the vibration of a composite pipe conveying fluid was carried out. To derive the equation of pipe oscillations, the Hamilton principle was applied. Discrete models were obtained using the finite element method. The effect of inner surface damage on the vibrational behavior of a composite pipe was studied. A mathematical model of a cantilever pipe conveying a fluid flow of variable density was proposed in [21]. The equations of fluid motion in the pipe were obtained based on the Bernoulli-Euler theory of beams taking into account the fluid internal forces acting on the pipe, according to Newton's second law. The equations of pipe motion were solved numerically using the finite difference methods. The effect of fluctuating amplitudes, wavenumber, and fluid density on the stability and dynamics of a cantilever system of pipes was studied. It was shown that a change in fluid density at high amplitude led to instability of the cantilever system of pipes.

In [22], vibrations of a single-walled carbon nanotube were studied. Based on the Tymoshenko theory and using the Hamilton principle, nanotubes' motion equations were obtained under the temperature effect. A discrete model of a pipe was obtained using the

Galerkin method. The effect of compressive axial force, rotating velocity, the ratio of gravity, and the mass of fluid on the critical velocity of flutter and divergence was investigated.

The dynamic vibration behavior of single-walled carbon nanotubes subjected to a longitudinal magnetic field was studied in [23], considering geometrical nonlinearity and nonlinear damping.

The loss of stability caused by the strain pattern: the static (divergent) and dynamic (flutter) instability of nanotubes, was studied in [24]. In combination with the Bernoulli-Euler beam model and using the Newton method, the nonlocal determining equations of a nanotube conveying fluid were obtained. It was established that the piezoelectric stress reduced the critical flow rates in the system.

In [25], the vibration of a viscoelastic pipe conveying fluid was investigated. In a geometrically nonlinear statement, mathematical models of the problem of vibration and dynamic stability of a viscoelastic pipe under the influence of an external load varying over time were constructed based on the Euler-Bernoulli theory. Kelvin's models of the hereditary theory of viscoelasticity were used to describe the strain processes in viscoelastic materials. The methods for effective rejection of pipe vibration were shown.

The development trend in pipelines vibration research follows the path of mathematical models complication when describing vibrational processes of pipelines conveying fluid. A significant amount of scientific research has been devoted to the modeling of dynamic stability and vibration of elastic pipelines. However, a comparatively small number of publications are devoted to a numerical study of composite pipeline vibration based on the Euler-Bernoulli theory of beams, considering external forces.

The existing dynamic models of pipelines are simplified and, in the study of vibration, do not take into account the properties of structural material and the totality of real loads characteristic of a real structure. Under these conditions, it is urgent to build a computational model that adequately reflects pipelines' real vibrations and develop effective software tools based on this model.

The study aims to elaborate existing methods for calculating the vibration of the pipeline; to build mathematical models, computational algorithms, and software and their design substantiation; to analyze the effect of viscoelastic properties of the pipeline material; to analyze the influence of external periodic forces on pipeline vibration.

2 Equation of motion of a composite pipe conveying fluid

Consider a composite pipeline in the form of a rod of length L lying on an earth base. Earth in these sites is modeled by a viscoelastic Winkler base with a modulus of subgrade reaction k_1 (Fig. 1). A steady-state flow of incompressible fluid of axial velocity U_f and axial force N_0 is transported through the pipeline. The origin is located at the left end at the intersection of the pipeline axes, and the pipe length is plotted on the abscissa. The displacements of the pipeline axis points along the z-axis are denoted by $w(x,t)$.

Based on the proposed assumptions [26-32], the equations of motion of bending vibrations of a rod-like viscoelastic structure in the presence of a moving mass have the form:

$$EI\left(1-R^{*}\right)\frac{\partial^{4}w}{\partial x^{4}}+2m_{f}U_{f}\frac{\partial^{2}w}{\partial t\partial x}+(m_{f}+m_{p})\frac{\partial^{2}w}{\partial t^{2}}$$

$$+\left[m_f U_f^2 \frac{\lambda L}{4d} - \frac{EA_p}{2L}\left(1-R^*\right)\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx\right]\frac{\partial^2 w}{\partial x^2} \tag{1}$$

$$+\left[m_f U_f^2 - N_0 + A_p P_i\right]\frac{\partial^2 w}{\partial x^2} + k_1\left(1-R_1^*\right)w = (m_f + m_p) f \Omega^2 \sin(\Omega t).$$

The x coordinate is counted along the pipeline axis. Here, w is the deflection; EI is the pipe stiffness; m_f is the mass of fluid per unit length of the pipe; $A_p = \pi r_1^2$ is the cross-sectional area of the pipe; r_1 is the inner radius of the pipe; N_0 is the tensile (compressive) force; R^*, R_1^* are the integral operators; k_1 is the rigidity of the Winkler base; m_p is the mass of the pipe per unit length, f and Ω are the amplitude and frequency of external force variation.

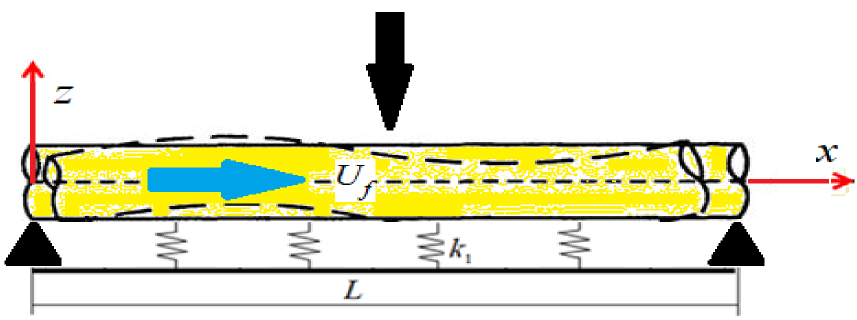

Fig. 1. The pipeline geometry

3 Solution Methods

An approximating expression for w is taken in the form:

$$w(x,t) = \sum_{n=1}^{N} w_n(t)\xi_n(x), \tag{3}$$

where $w_n(t)$ are some functions to be determined, and functions $\xi_n(x)$ are selected so that each term of the sum (3) satisfies the boundary conditions. Below consider the boundary conditions – hinged support on the pipe edges.

Conducting the Bubnov-Galerkin procedure with (3), the following equations are derived

$$\sum_{n=1}^{N} a_{kn}\ddot{w}_n + 2U_f\sqrt{\beta_{fp}}\sum_{n=1}^{N}\gamma_{nk}\dot{w}_n + \sum_{n=1}^{N} c_{kn}\left(1-R^*\right)w_n +$$
$$\left[U_f^2\left\langle 1+\gamma_{\mathrm{Re}}\right\rangle - \overline{N}_0 + \overline{P}\right]\sum_{n=1}^{N} b_{kn}w_n + k_w\sum_{n=1}^{N} a_{kn}\left(1-R_1^*\right)w_n + \qquad (4)$$
$$\gamma_p\sum_{n,i,j=1}^{N} b_{kn}\psi_{ij}w_n\left(1-R^*\right)w_iw_j = 2\gamma_k\omega^2 f\sin(\omega t).$$
$$w_n(0)=w_{0n};\quad \dot{w}_n(0)=\dot{w}_{0n};\quad k=1,2,\ldots,N.$$

Here the dimensionless parameters are:

$$x\leftrightarrow\frac{x}{L},\quad w\leftrightarrow\frac{w}{L},\quad U_f\leftrightarrow U_fL\sqrt{\frac{m_f}{EI}},\quad t\leftrightarrow\frac{t}{L^2}\sqrt{\frac{EI}{m_p+m_f}},$$

Introduce the notations:

$$\overline{N}_0 = N_0\frac{L^2}{EI};\qquad \beta_{fp}=\frac{m_f}{m_f+m_p};\quad k_w=\frac{k_1L^4}{EI};\quad \gamma_p=\frac{A_pL^2}{I};$$
$$\omega=\overline{\omega}\cdot L^2\left(\frac{m_f+m_p}{EI}\right)^{0,5};\ \overline{P}=\frac{r_1^2L^2P_i}{EI};\ \gamma_{\mathrm{Re}}=\frac{\lambda L}{4d};\ a_{nk}=\int_0^1\xi_n(x)\xi_k(x)dx;$$
$$b_{nk}=\int_0^1\xi_n''(x)\xi_k(x)dx;$$
$$c_{nk}=\int_0^1\xi_n^{IV}(x)\xi_k(x)dx;\ \psi_{ij}=\int_0^1\xi_i'(x)\xi_j'(x)dx;\ \gamma_k=\int_0^1\xi_k(x)dx;\ \gamma_{nk}=\int_0^1\xi_n'(x)\xi_k(x)dx.$$

Nonlinear IDE systems (4) are solved numerically using the method proposed in [26-33]. This system is written in integral form; using a rational transform, weakly singular features of integral operator R^* are eliminated. Assuming that $t=t_i$, $t_i=i\Delta t$, $i=1,2,\ldots$ ($\Delta t=const$) and replacing the integrals with some quadrature formulas to calculate $w_n=w_n(t)$, we obtain the following recurrence relation:

$$\sum_{n=1}^{N}a_{kn}w_{in} + 2U_f\sqrt{\beta_{fp}}\sum_{n=1}^{N}B_i\gamma_{kn}w_{in} =$$
$$\sum_{n=1}^{N}\left(\dot{w}_{0n}t_i + w_{0n}\right)a_{kn} - \sum_{j=0}^{i-1}B_j\left\{2U_f\sqrt{\beta_{fp}}\sum_{n=1}^{N}\gamma_{kn}w_{jn} +\right.$$
$$\left(t_i-t_j\right)\left\langle\sum_{n=1}^{N}c_{kn}\left[w_{jn}-\frac{A}{\alpha}\sum_{s=0}^{j}C_s\exp(-\beta t_s)w_{j-s,n}\right]+\right.$$
$$k_w\sum_{n=1}^{N}a_{nk}\left(w_{jn}-\frac{A_1}{\alpha_1}\sum_{s=0}^{j}C_{1s}\exp(-\beta t_s)w_{j-s,n}\right)+$$

$$\sum_{n=1}^{N} b_{kn}\left[U_f^2(1+\gamma_{\text{Re}})-\bar{N}_0+\bar{P}_i\right]w_{jn}+2\gamma_k\omega^2 f\sin(\omega t_j)+$$
$$+\gamma_p\sum_{n,i_1,j_1=1}^{N} b_{kn}\varphi_{i_1j_1}w_n\left(w_{i_1n}w_{j_1n}-\frac{A}{\alpha}\sum_{s=0}^{j}C_s\exp(-\beta t_s)w_{i_1-s,n}w_{j_1-s,n}\right)\Bigg\rangle\Bigg\} \quad (5)$$

$i=1,2,...;\quad n=\overline{1,N};\quad m=\overline{1,L}$; where B_j , C_s, and C_{1s} are the numerical coefficients related to the quadrature formulas of the trapezoid:

$$B_0=\frac{\Delta t}{2};\quad B_j=\Delta t,\quad j=\overline{1,i-1};\quad B_i=\frac{\Delta t}{2};$$
$$C_0=\frac{\Delta t^\alpha}{2};\quad s=j,\quad C_j=\frac{\Delta t^\alpha(j^\alpha-(j-1)^\alpha)}{2};\qquad C_s=\frac{\Delta t^\alpha((s+1)^\alpha-(s-1)^\alpha)}{2};$$
$$C_{10}=\frac{\Delta t^{\alpha_1}}{2};\quad s=j,\quad C_{1j}=\frac{\Delta t^{\alpha_1}(j^{\alpha_1}-(j-1)^{\alpha_1})}{2};\qquad C_{1s}=\frac{\Delta t^{\alpha_1}((s+1)^{\alpha_1}-(s-1)^{\alpha_1})}{2}.$$

Due to the proposed approach, the factor t_i-t_j at $j=i$ in the algorithm for the numerical solution of the problem in formula (5) takes a zero value, i.e., the last term of the sum is zero. Therefore, the summation is done from zero to i -1 ($j=\overline{0,i-1}$).

4 Discussion of results

The software has been created based on the computational algorithm (5). The results of calculations are reflected in the graphs in Figs. 2-6. The graphs show the numerical convergence of the Bubnov-Galerkin method. When calculating the deflection value by formula (3), the first five harmonics were held (N = 5). Calculations showed that a further increase in the number of terms does not significantly affect the amplitude of pipe vibration.

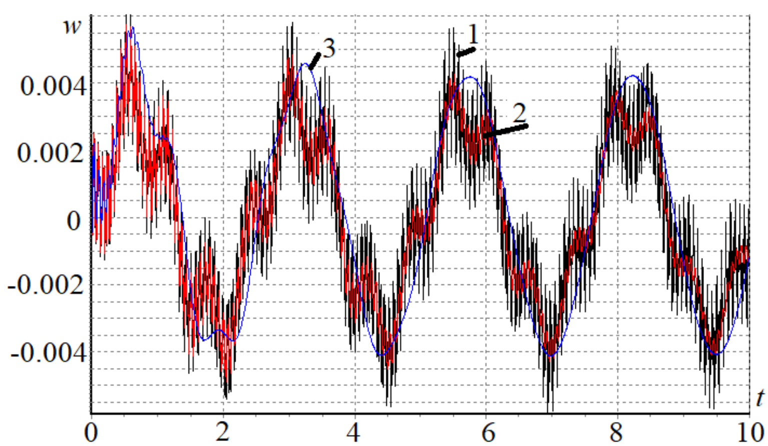

Fig.2. Forms of vibrations of the point x/L =0.5 of a pipe to elastic and viscoelastic theory: A=0 (1); A=0.005 (2); A=0.1(3).

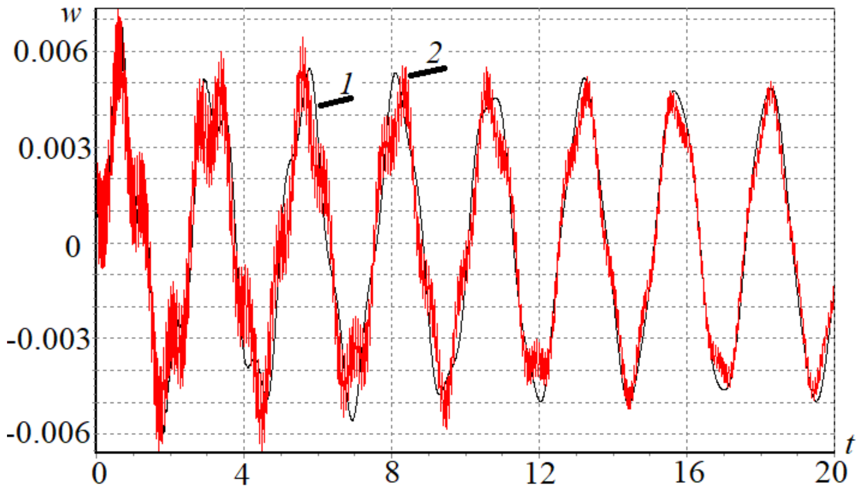

Fig. 3. The effect of singularity parameter α on the amplitudes of pipeline vibrations: α=0.1 (1); α=0.5 (2).

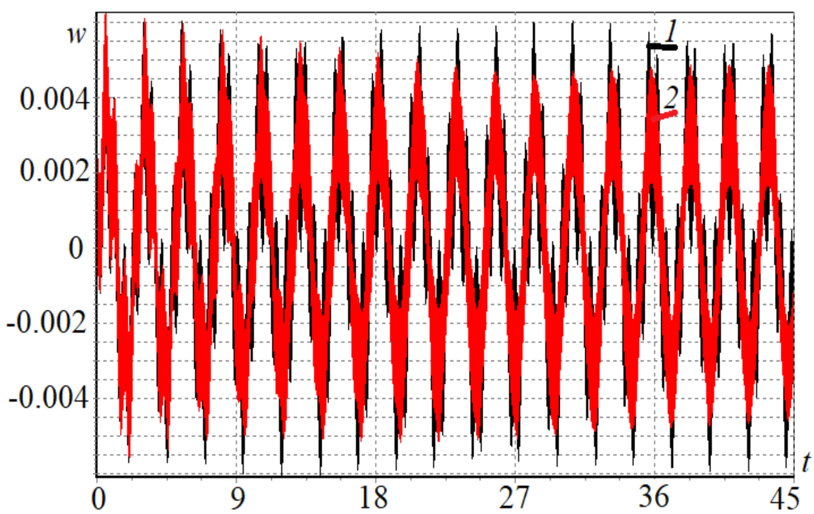

Fig. 4. A_1=0 (1); A_1=0.2 (2); A=0; α=0.25; β=0.05; α_1=0.25; β_1=0.05; β_{fp}=0.32; ω=2.5; k_w=10; $\overline{N}_o=2$; $\overline{P}=5$; f=0.004; $U_f=0.5$; $\gamma_{\mathrm{Re}}=0.1875$.

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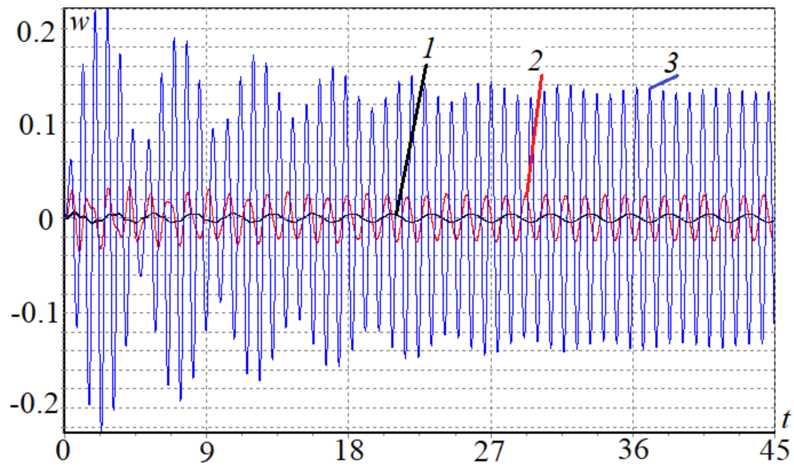

Fig. 5. ω=2.5 (1); ω=5 (2); ω=7 (3); A=0.01; α=0.25; β=0.05; A_1=0.1; α_1=0.25; β_1=0.05; β_{fp}=0.32; k_w=10; $\overline{N}_o = 2$; $\overline{P} = 5$; f=0.004; $U_f = 1.5$; $\gamma_{\mathrm{Re}} = 0.1875$.

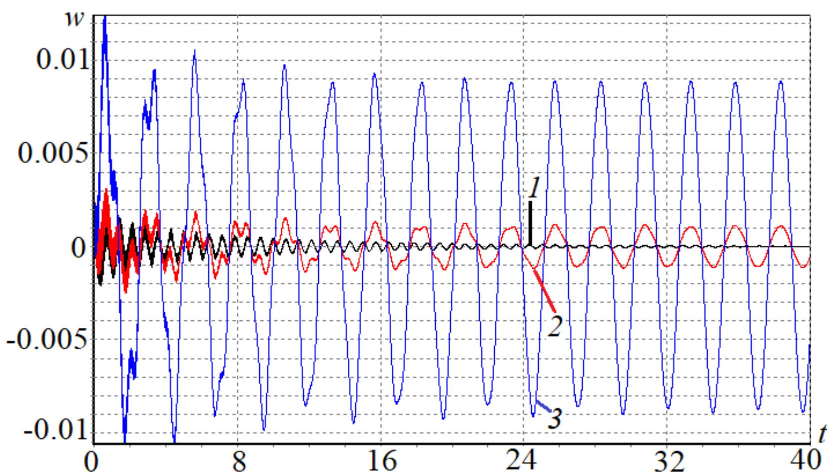

Fig. 6. The influence of the amplitude parameter of external forces: f=0 (1); f=0.001 (2); f=0.008 (3).

4.1 The effect of rheological parameters

Fig. 2 refers to the case of a pipe in a viscoelastic base A_1=0.1, under non-stationary transverse loads. Fig. 2 shows that an account for viscous resistance of the pipe material leads to amplitude attenuation and a shift in the vibration phase. The graphs show that the viscosity parameter A reduces the maximum deflection value. In the presented case, the vibration amplitudes for a viscoelastic pipe noticeably decrease compared to elastic pipes (A = 0). The following data are used in calculations: A = 0 (curve 1); A = 0.005 (curve 2); A = 0.1 (curve 3). α=0.25; β=0.05; A_1=0.1; α_1=0.25; β_1=0.05; β_{fp}=0.32; ω=2.5; k_w=10; $\overline{N}_o = 2$; $\overline{P} = 5$; f=0.004; $U_f = 0.05$; $\gamma_{\mathrm{Re}} = 0.1875$.

The effect of a singular parameter α on the behavior of a viscoelastic pipe was studied. Figure 3 presents a graph of the relationship between the deflection and time at α=0.1

(curve 1); α=0.5 (curve 2) at the flow rate $U_f = 1.5$. With the increase in this parameter, the amplitude and frequency of vibrations increase as well. In calculations, the following parameters were taken: A=0.01; β=0.05; A_1=0.1; α_1=0.25; β_1=0.05; β_{fp}=0.32; ω=2.5; k_w=10; $\overline{N}_o = 2$; $\overline{P} = 5$; f=0.004; $U_f = 1.5$; $\gamma_{\mathrm{Re}} = 0.1875$.

Fig. 4 shows the effect of the base viscosity parameter on the amplitude and frequency of vibrations of a viscoelastic pipeline conveying fluid. The effect of parameters $A_1 = 0$ (curve 1); $A_1 = 0.2$ (curve 2) was investigated. As expected, with an increase in viscosity parameter A_1 of the Winkler bases, the system vibrations damp. In this case, vibration amplitudes decrease, and the vibration phase shifts to the right.

4.2 The effect of external loads

Fig. 5 shows the calculation data referred to the pipe. A set of curves in Fig. 6 describes the motion trajectory *w (t)* of the pipe central point under unsteady load of a frequency: ω = 2.5 (curve 1); ω = 5 (curve 2); ω = 7 (curve 3). As these figures show, the oscillatory process is accompanied by a slight decrease in the deflection amplitude. With an increase in the frequency of external unsteady loads, the amplitude and frequency of pipe vibrations increase. At ω = 7 (curve 3), starting from t = 0 to 20, "beat" vibrations are observed, and later vibration amplitudes slowly decrease.

Proceed to analyze the effect of an external unsteady load on the behavior of a viscoelastic pipeline. As follows from the graphs in Fig. 6, the effect of the amplitude values of the disturbing external load (f = 0 (curve 1); f = 0.001 (curve 2); f = 0.008 (curve 3)) is expressed in a slight increase in the maximum amplitude of deflections. Without an external load, f = 0 (curve 1), vibration frequency increases, but vibration amplitude damps rapidly over time. As seen, with an increase in the external load parameter, vibration amplitudes increase, and vibration frequency decreases. The following parameters were used in the calculation:

A=0.01; α=0.25; β=0.05; A_1=0.1; α_1=0.25; β_1=0.05; β_{fp}=0.32; ω=2.5; k_w=10; $\overline{N}_o = 2$; $\overline{P} = 5$; $U_f = 1.5$; $\gamma_{\mathrm{Re}} = 0.1875$.

5 Conclusion

A computational algorithm is developed based on eliminating the features of integro-differential equations with weakly singular kernels, followed by using quadrature formulas. It is shown that the error of the method coincides with the error of the used quadrature formulas and has the same order of smallness concerning the interpolation step. It has been shown that considering the viscoelastic properties of the construction material and pipeline bases leads to a decrease in the critical flow velocity by 30-50%. It was found that considering the viscoelastic properties of the pipeline material leads to a decrease in the amplitude and frequency of vibrations by 20-40%. The effects of an external non-stationary load on the behavior of a viscoelastic pipeline are analyzed. It was found that with an increase in external load parameters, vibration amplitudes increase, and vibration frequency decreases. It is revealed that with an increase in the frequency of non-stationary external loads, the amplitude and frequency of pipeline vibration increase.

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