Combined soil foundation analysis based on ultimate limit state

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Abstract. The paper defines an updated method for “soil – reinforced concrete foundation” calculation that takes into account their mutual interaction. In this method, both the foundation structure and the footing structure are represented by a rigid-plastic model, which makes it possible to determine the bearing capacity of the system. This method eliminates the possibility of destruction of the reinforced concrete foundation from punching to the exhaustion of its bearing capacity. It further presents the results of the test conducted by the authors at Novocherkassk School for Soil Mechanics Research to validate the theoretical calculations attained with the use of the proposed method, as well as the results of tray testing of reinforced concrete foundation models. The calculation of the bearing capacity by the modernized method gives values close to the results of experimental studies. The authors discuss the challenge of determining the strength of sand base, which is known to affect the calculation of load-bearing capacity, and further propose a solution that uses piecewise linear limit line of shear strength of soil and has been developed in an attempt to increase the accuracy of calculations.

1 Introduction

In the Russian Federation, the method for calculating the load-bearing capacity of “reinforced concrete foundation – soil” system involves two independent calculations, but the fact remains: under load, there occurs an interaction between the foundation and the ground base. Pursuant to the current Construction Regulation 22.13330.2016 “Foundations of Buildings and Structures”, soil basement analysis should follow the formula (1), which relies on ultimate limit state and is based on Ludwig Prandtl ultimate load formula, later supplemented by K. Terzaghi.

\[
N_u = b' \cdot L' \cdot (N_y \cdot \xi_y \cdot b' \cdot \gamma_f + N_q \cdot \xi_q \cdot \gamma_q \cdot d + N_c \cdot \xi_c \cdot c_i) \quad (1)
\]

At the same time, the above formula is valid only for rigid foundations and, in fact, is not used in computational practice, since the maximum load values obtained with its use appear to exceed the experimental ones by 2 to 4 times. In fact, any calculation of centrally loaded foundation is performed with account of condition that the pressure under the foundation base does not exceed the calculated soil resistance R, and it is assumed that the contact stress

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curve beneath the foundation base is evenly distributed. The numerous experimental studies have shown that the contact stress curve under the foundation base changes from a saddle-shaped form at the initial stage of loading to an elongated shape along the foundation axis under maximum load.

As prescribed by regulations, an ultimate limit state-based analysis of centrally loaded concrete should involve the calculation of maximum bursting pressure and of the area of reinforcement, based on the maximum bending moment.

As can be seen from the above, the strength of reinforced concrete foundation appears missing in the calculation of the load-bearing capacity of a soil basement, while the calculation of the load-bearing capacity of a reinforced concrete foundation fails to take account of the strength of soil basement. Therefore, there is a need for ultimate limit state-based calculation method that would taking into account the mutual interaction between the reinforced concrete foundation and the soil.

In EN 1997-1:2004 Eurocode 7: Geotechnical design, the analytical method for calculating the load-bearing capacity of soil in drained conditions uses the formula (2) below, which takes into account neither the soil-foundation interaction, nor its characteristics.

\[
\frac{R}{A} = 0.5\gamma'B'N_y b'_y s'_y l'_y + q'N_q b'_q s'_q l'_q + c'N_c b'_c s'_c l'_c
\]

It can thus be concluded that the European Union is faced with a similar challenge with regard to calculation of “soil – reinforced concrete foundation” load-bearing capacity. Moreover, the studies that are being conducted into the bearing capacity of foundation soils in Europe and in Russia, led by Stefan Van Baars[1, 2], R. L. Michalowski (1997) in Europe and by V.G. Fedorovsky[3] in Russia, seem to be nearly identical, coming down to updating the coefficients in Terzaghi formula or introducing in it extra elements.

2 Theoretical studies

A study was conducted at Novocherkassk School of Soil Mechanics Research, led by V.P. Dyba, that developed a method for determining the maximum load for “soil – reinforced concrete foundation” system within the framework of the theory of ideal plasticity [4]. According to the limit analysis of plastic systems, the plotting of statically permissible stress fields in soil and reinforced concrete foundation generates a lower estimate of the load-bearing capacity of “soil – reinforced concrete foundation” system. Such plotting made use of Prandtl generalized solution for strip load with surcharge. Subsequently, the method was improved by Matvienko for a rectangular iron-concrete foundation [5]. The peculiarity of the method consists in the fact that it does not neglect the base in calculations by way of replacing it with contact stresses of different shapes, as was the case with predecessor methods, but, instead, takes it into account together with the foundation without detriment to strength characteristics of both the soil and the foundation. The method further takes into account the unevenness of contact stress beneath the foundation base, which was formalized in the form of a curve with rapture in the point which had been considered fixed when under ultimate load and which separates the active and passive limit states of soil (Fig. 1).
For calculating a rectangular reinforced concrete foundation, the method is as follows:
Let us calculate the ultimate bending moments (3) and (4):

\[ M_B = 0.9 \cdot A_B \cdot R_S \cdot h_0 \]  \hspace{1cm} (3)
\[ M_L = 0.9 \cdot A_L \cdot R_S \cdot h_0 \]  \hspace{1cm} (4)

Let us calculate the dimensionless coefficients for the system of equations (5):

\[ \mu_B = \frac{2 \cdot M_B}{N_y \cdot \gamma \cdot b \cdot L} \]
\[ \mu_L = \frac{2 \cdot M_L}{N_y \cdot \gamma \cdot b \cdot L^2} \]
\[ \alpha = \frac{N_a \cdot d}{N_y \cdot \gamma \cdot b} \]
\[ \beta = \frac{N_c}{N_y \cdot \gamma \cdot b} \]
\[ \eta = \frac{b_D}{L} \]  \hspace{1cm} (5)

Let us solve the system of equations (6):

\[ \mu_B \cdot \xi_b^2 \cdot (1 + 2 \cdot \xi_L) \cdot \left[ (1 + 2 \cdot \xi_b) \cdot \left( 1 - 0.25 \cdot \eta \cdot \frac{1 + \xi_b}{1 + \xi_L} \right) + \right. \]
\[ \alpha \cdot \left( 1 + 1.5 \cdot \eta \cdot \frac{1 + \xi_b}{1 + \xi_L} \right) + \beta \cdot \left( 1 + 0.3 \cdot \eta \cdot \frac{1 + \xi_b}{1 + \xi_L} \right) \]
\[ \mu_L \cdot \xi_L^2 \cdot (1 + 2 \cdot \xi_b) \cdot \left[ (1 + 2 \cdot \xi_b) \cdot \left( 1 - 0.25 \cdot \eta \cdot \frac{1 + \xi_b}{1 + \xi_L} \right) + \right. \]
\[ \alpha \cdot \left( 1 + 1.5 \cdot \eta \cdot \frac{1+ \xi_b}{1+ \xi_L} \right) + \beta \cdot \left( 1 + 0.3 \cdot \eta \cdot \frac{1+ \xi_b}{1+ \xi_L} \right) \]  \hspace{1cm} (6)

Let us calculate the breadth (7) and length (8) of foundation:

\[ b_1 = b_D \cdot (1 + 2 \cdot \xi_b) \]  \hspace{1cm} (7)
\[ L_1 = L_D \cdot (1 + 2 \cdot \xi_L) \]  \hspace{1cm} (8)

Let us calculate the coefficients specific to the foundation, for the load limit formula (9):

\[ \eta_1 = \frac{L_1}{b_1} \quad \xi_Y = 1 - \frac{0.25}{\eta_1} \quad \xi_q = 1 + \frac{1.5}{\eta_1} \quad \xi_c = 1 + \frac{0.3}{\eta_1} \]  \hspace{1cm} (9)

Let us calculate the load limit (10):

\[ P = b_1 \cdot L_1 \cdot (N_Y \cdot \xi_Y \cdot b_D \cdot \gamma_D + N_q \cdot \xi_q \cdot \gamma_1 \cdot d + N_c \cdot \xi_c \cdot c) \]  \hspace{1cm} (10)
Subsequently, the method was updated by V. P. Dyba and M.P. Matvienko, based on the assumption that a statically permissible stress field exists in the body of a reinforced concrete foundation, if the maximum bending moment in the slab part of the foundation does not exceed the maximum moment, and the shear and tensile forces on the surface of the pushing prism do not exceed the potential retaining forces [6]. This method has allowed us to solve the problem of determining the reinforcement of the slab part of the foundation, at which rupture by bending or pushing occurs under one and the same load, taking into account the interaction with the soil.

Let us calculate the limit load according to Prandtl generalized formula (11):

\[
P = A \cdot e^{\frac{\pi (A - 1)}{2\pi}} \cdot \left( q + \frac{C}{A - 1} \right) - \frac{C}{A - 1}
\]  

(11)

where the strength characteristics of the soil are determined with account of Mohr-Coulomb strength condition according to the following formulas (12):

\[
A = \frac{1 + \sin \phi}{1 - \sin \phi}, \quad C = \frac{2c \cdot \cos \phi}{1 - 5 \sin \phi}
\]  

(12)

Let us calculate the working height of the slab part (13):

\[
h_0 = -0.25 \cdot (b_{gr} + L_{gr}) + 0.5 \frac{N_p}{R_{bt} + P}
\]  

(13)

Let us calculate the pushing force limit N (14):

\[
F = R_{bt} \cdot 2\sqrt{2} \cdot h_0
\]

\[
N = F + P \cdot (L + 2h_0)
\]  

(14)

Let us calculate the distance between the rigid part of the foundation and the fixed point in the slab part (15):

\[
L_x = \frac{P + 2 \cdot P \cdot h_0 - 2L_1 q}{2 \cdot (P - q)}
\]  

(15)

Let us calculate the maximum bending moment for the slab part of the foundation (16):

\[
M_{np} = \frac{1}{2} P \cdot L_x^2 + \frac{1}{2} q \cdot (L_1^2 - L_2^2)
\]  

(16)

Let us calculate the value of reinforcement at which force N is bending limit (17):

\[
A_s = \frac{R_{bt} h_0 - \sqrt{(R_{bt} - h_0)^2 - 2 \cdot M_{np} \frac{R_{bt}^2}{R_{bt}^2}}}{\frac{R_{bt}^2}{R_{bt}^2}}
\]  

(17)

Now that the non-uniformity of the contact stress curve has been taken into account, the bending moment in the slab part of the foundation is found to be decreasing under the foundation base, which makes it possible to reduce the cross-sectional area of reinforcement.

The accuracy of the load-bearing capacity calculations largely depends on the quality of determining the strength characteristics of the soil. To determine the strength characteristics of sand, Yu.N. Murzenko performed a series of shear tests, using the trays of MF-1 testing machine and standard single-plane shear devices with round and rectangular confining elements (Murzenko Yu. N., 1972). Based on the combined results of the shear tests, the
linear dependence $\tau(\sigma)$ was determined using least squares method and the strength characteristics of sand $c$ and $\varphi$ were found. The results are presented in Table 1.

![Fig. 2. The limit line of sandy soil shear resistance.](image)

**Table 1. Shear tests results.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Area and shape of shear fracture</th>
<th>Combined processing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50 cm$^2$, round</td>
<td>200 cm$^2$, rectangular</td>
</tr>
<tr>
<td>$\varphi$, degree</td>
<td>41°40'</td>
<td>41°30'</td>
</tr>
<tr>
<td>$c$, kg/cm$^2$</td>
<td>0.22</td>
<td>0.46</td>
</tr>
</tbody>
</table>

The shear tests results show a wide interval between the values of the specific adhesion obtained on different devices. The cloud of experimental points indicates the curvature of the limit line $\tau(\sigma)$ (Fig. 2).

Based on his experimental data, Yu.N. Murzenko has determined the following values of the strength characteristics of the sand base: $\varphi=40^\circ$, $c=0.107$ kg/cm$^2$ (MurzenkoYu.N., 1972). It is also necessary to point out the imperfection of the standard shear devices, which, when testing dense sandy soils, determine the maximum peak shear force, whereas the parameter which is much closer to reality is that of the shear force that decreases and stabilizes after the peak is passed.

For a more accurate calculation of the strength characteristics of the sand soil, V.P. Dyba proposed a piecewise linear limit line [4], with the following characteristics for the sand in the tray of MF-1 machine: $\varphi_1=48^\circ 49'$, $c_1=0$, $\varphi_2=40^\circ 04'$, $c_2=0.0541$ kg/cm$^2$. Consequently, the calculation method has been changed with account of the following.

Coefficients for the limit pressure formula (18):

$$A_1 = \frac{1+\sin\varphi_1}{1-\sin\varphi_1}, \quad A_2 = \frac{1+\sin\varphi_2}{1-\sin\varphi_2}, \quad C_2 = \frac{2-c_2\cos\varphi_2}{1-\sin\varphi_2}$$  \hspace{1cm} (18)

Maximum pressure on the soil foundation with account of bilinear limit line (19):
\[ P = A_2 \cdot e^{\frac{\pi (A_2 - 1)}{2\sqrt{A_2}}} \cdot \frac{C_2 \cdot (A_1 - 1)}{(A_2 - 1) \cdot (A_1 - A_2)} \times \left[ \frac{(A_1 - A_2)^2}{C_2} \right]^{\frac{1}{M_2(A_1 - 1)}} - \frac{C_2}{A_2 - 1} \] \tag{19}

Ultimate bending moments (20) and (21):

\[ M_b = 0.9 \cdot A_b \cdot R_s \cdot h_0 \tag{20} \]
\[ M_L = 0.9 \cdot A_L \cdot R_s \cdot h_0 \tag{21} \]

System of equations (22):

\[
\begin{align*}
\left\{ x_b^2 \cdot (x_b + 2 \cdot x_b) - \frac{2 \cdot M_b}{P} = 0 \\
\left( x_L^2 \cdot (b_g + 2 \cdot x_L) - \frac{2 \cdot M_L}{P} = 0 \right.
\end{align*}
\tag{22}
\]

Breadth (23) and length (24) of the foundation:

\[ b_1 = b_g + 2 \cdot x_b \tag{23} \]
\[ L_1 = L_g + 2 \cdot x_L \tag{24} \]

Conditions (25) and (26) testing:

\[
\begin{align*}
b_1 &= \begin{cases} b_1 & \text{if } b_1 < b \\
b & \text{otherwise} \end{cases} \tag{25} \\
L_1 &= \begin{cases} L_1 & \text{if } L_1 < L \\
L & \text{otherwise} \end{cases} \tag{26} 
\end{align*}
\]

Load limit (27):

\[ N = b_1 \cdot L_1 \cdot P \tag{27} \]

With the piecewise linear limit line for soil shear resistance, developed by V.P. Dyba, the values of the load-bearing capacity appear to be more accurate.

### 3 Experimental studies

To verify the accuracy of the load-bearing capacity of a rectangular reinforced concrete foundation, calculated using the newly proposed method, a series of experimental studies were conducted in the Laboratory of Foundations of Foundations of Plavo South Russian State Polytechnic University Department of Industrial and Civil Engineering, Geotechnics and Foundation Engineering, using test machine MF-1, designed by Yu.N. Murzenko, the central element of research automation system (RAS) for foundation modeling. SIIT-3 strain-gauge system and various stress sensors, engineered by Plavo University and the team at Research Institute for Chemical Rubber Studies, were used as measuring equipment.
The experiment involved the manufacturing of a gypsum model with dimensions of the slab part, 60x45x3.5 cm, reinforced with wire mesh of 2 mm diameter with a cell of 5x5 cm and the 20x15x14 cm column footing (Fig. 3). The model was placed in the center of a 3x3x2 m tray and buried to the depth of 20 cm. The load was applied to the model without any misalignment, transmitted by the central jack of MF-1 test machine (the maximum force of one jack equaling 500 kN).

To create surcharge, a metal box was placed around the model and sanded to a height of 20 cm (Fig. 4).

To measure the pressure at the base of the model, load cells were used, installed both at the contact point between the foundation model and the soil and at the depth of 60 cm. The load cells were arranged along and across the model, passing through its center (Fig. 5).
Fig. 5. The layout of the load cells at the base of the model along the Y-axis.

The load application was performed in 2kN stages with time intervals until complete stabilization of deformations and stresses. The rupture of the model occurred at a load of 68 kN, caused by the collapse of the slab part from the bending moment (Fig. 6).

Fig. 6. Foundation base fracture diagram.
4 Results

The maximum sediment of the foundation base under the model was 6.2 mm, and a graph showing the dependence of the sediment on the load was plotted according to the results of the experiment (Fig. 7).

The load cells installed on the contact point between the foundation model and its base made it possible to determine contact stresses. The measurements were taken at every stage of loading. Figure 8 shows the contact stresses at loads 0.25 Rpr(18 kN), 0.5 Rpr(34 kN), 0.75 Rpr(52 kN), Rpr(68 kN). As can be seen on Figure 8, the contact stress curve is not equally distributed and stretches along Z axis with an increase in load.

Fig. 7. Foundation settlement vs. load curve.

Fig. 8. Contact stress diagrams obtained experimentally at different loading stages.

The experiment has shown that when calculating the bending moments that occur in the slab part of the foundation, it is necessary to take into account the uneven distribution of the contact stresses under the foundation base. The rupture load, found as a result of the
experiment and equaling 68 kN, differs from the rupture load calculated with the use of the updated method by 22.1% (rupture load equals 87.2 kN) and by 1.9% (rupture load equals 69.4 kN) from the rupture load calculated by the same method, but with account of the piecewise linear limit line.

Let us first calculate the models of reinforced concrete foundations on a sandy soil, which have been tested at different times at Novocherkassk School for Soil Mechanics Research, using the updated method, as well as the same method but with account of the piecewise linear limit line, and then compare the results with values of the rupture load obtained experimentally. Let us consider the studies by Tsesarsky A. A. (Tsesarsky A.A. & Murzenko, Yu.N., 1969), Yevtushenko S. I. [7,8], Murzenko A. Yu. [9], as well as the test experiment conducted by the authors [5]. The calculation results are summarized in Table 2.

Table 2. Rupture load calculation results for models of reinforced concrete foundations on sandy soil.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Model dimensions, mm</th>
<th>Experimentally obtained rupture load, kN</th>
<th>Rupture load according to updated method, kN</th>
<th>Load-bearing capacity under the pressure prism according to Regulation 5.32 formula, kN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Models by A.A. Tsesarsky</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>500x500x35</td>
<td>73.1</td>
<td>91.1 (19.7%)</td>
<td>98.2 (25.5%)</td>
</tr>
<tr>
<td>2</td>
<td>500x500x50</td>
<td>93.9</td>
<td>109.4 (14.2%)</td>
<td>130.7 (28.1%)</td>
</tr>
<tr>
<td>3</td>
<td>500x500x75</td>
<td>121.2</td>
<td>135.1 (10.3%)</td>
<td>195.2 (37.9%)</td>
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<tr>
<td>Models by S.I. Yevtushenko</td>
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<td></td>
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<tr>
<td>1</td>
<td>1200x900x90</td>
<td>824</td>
<td>906.6 (9.1%)</td>
<td>1257 (34.4%)</td>
</tr>
<tr>
<td>3</td>
<td>1200x900x70</td>
<td>598</td>
<td>780.3 (23.4%)</td>
<td>1116 (30.1%)</td>
</tr>
<tr>
<td>Models by A.Yu. Murzenko</td>
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</tr>
<tr>
<td>1.1</td>
<td>600x450x35</td>
<td>76.4</td>
<td>85.1 (10.1%)</td>
<td>141.7 (46.1%)</td>
</tr>
<tr>
<td>1.9</td>
<td>600x450x30</td>
<td>61.23</td>
<td>72.4 (15.5%)</td>
<td>129.9 (52.8%)</td>
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<tr>
<td>2.3</td>
<td>1200x1000x110</td>
<td>460.7</td>
<td>608.7 (24.3%)</td>
<td>805.1 (42.8%)</td>
</tr>
<tr>
<td>2.4</td>
<td>900x790x90</td>
<td>459.1</td>
<td>545.0 (15.8%)</td>
<td>805.1 (42.9%)</td>
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<tr>
<td>Models by Matvienko</td>
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</tr>
<tr>
<td>1</td>
<td>600x450x35</td>
<td>68</td>
<td>87.21 (22.1%)</td>
<td>171.4 (60.3%)</td>
</tr>
</tbody>
</table>

The values in brackets show the difference, %, from relative rupture load obtained experimentally.

**5 Discussion**
Our analysis of the obtained results has shown the discrepancy between the load-bearing capacity obtained by the updated method and the experimentally obtained data, the former exceeding the latter by 10% to 20%, with the values obtained by the standard method for the area bounded by the prism of pushing exceeding the experimentally obtained data by 25% to 60%. It should be also noted that it wouldn’t be correct to compare the load-bearing capacity of the “soil-foundation” system and that of individual foundations. Nor is it correct to suppose that the standard load-bearing capacity is considered only under the pushing pyramid, without taking into account the flexible section of the foundation slab. At the same time, it is obvious that the updated method gives a more accurate value of the load-bearing capacity.

6 Conclusions

The updated calculation method makes it possible to estimate the load-bearing capacity of the “soil – reinforced concrete foundation” system. In this method, the environment of foundation and that of its base are represented by a rigid-plastic model, which makes it possible to determine the system’s load-bearing capacity.

The updated method rules out the probability of pushing-induced rupture in the reinforced concrete foundation until exhaustion of its bearing capacity.

References