The importance of a three-dimensional formulation of the thermal conductivity problem in assessing the effect of a temperature shock on the rotational motion of a small spacecraft

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Abstract. The paper analyzes the conditions under which a two-dimensional formulation of the thermal conductivity problem for a correct assessment of the large elastic elements temperature shock effect on the rotational motion of a small spacecraft is insufficient. Numerical simulation was carried out for a scheme of a small "Aist-2D" spacecraft. The results of this work can be used in modeling the rotational motion of a small spacecraft taking into account the temperature shock of large elastic elements.

1 Introduction

To successfully implement a number of space missions, a significant limitation of the spacecraft rotational motion parameters is required [1, 2]. As illustrative examples, we can cite spacecraft for remote sensing of the Earth (Fig. 1) [3, 4], technological spacecraft [5, 6], as well as tug spacecraft for cleaning space debris [7, 8].

In the first case, it is required to implement a high accuracy of orientation by angular position at the time of the target task. Thus, the confirmed characteristics of the small «Aist-2D» spacecraft for remote sensing of the Earth shown in Figure 1 show that the accuracy of its angular orientation in the uniaxial solar orientation mode is 0.5°, and in the triaxial orientation mode in the orbital coordinate system is 2° [9]. At the same time, restrictions are imposed on the angular velocity of rotation. The accuracy of angular velocity in the uniaxial solar orientation mode of 0,01°/s and in the triaxial orientation in the orbital coordinate system of 0.0005 °/s has been experimentally confirmed [9]. When the spacecraft immerses into the Earth's shadow or exits it, natural oscillations of large elastic elements may occur as a result of the temperature shock [10, 11]. These oscillations can degrade the above characteristics of the angle orientation accuracy and angular velocity. Therefore, it is necessary to take into account the temperature shock for small spacecraft.

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Fig. 1. A small «Aist-2D» spacecraft for remote sensing of the Earth.

For technological spacecraft, restrictions on the rotational motion parameters are associated with restrictions on the level of micro-accelerations in the area of technological equipment for the implementation of gravity-sensitive processes [12, 13]. Thus, with additional vibration-proof devices that reduce the level of micro-accelerations in the protected zone, it is possible to achieve sufficiently low values of micro-accelerations [14]. Fig. 2 shows the results of micro-accelerations measurements on the International Space Station [15].

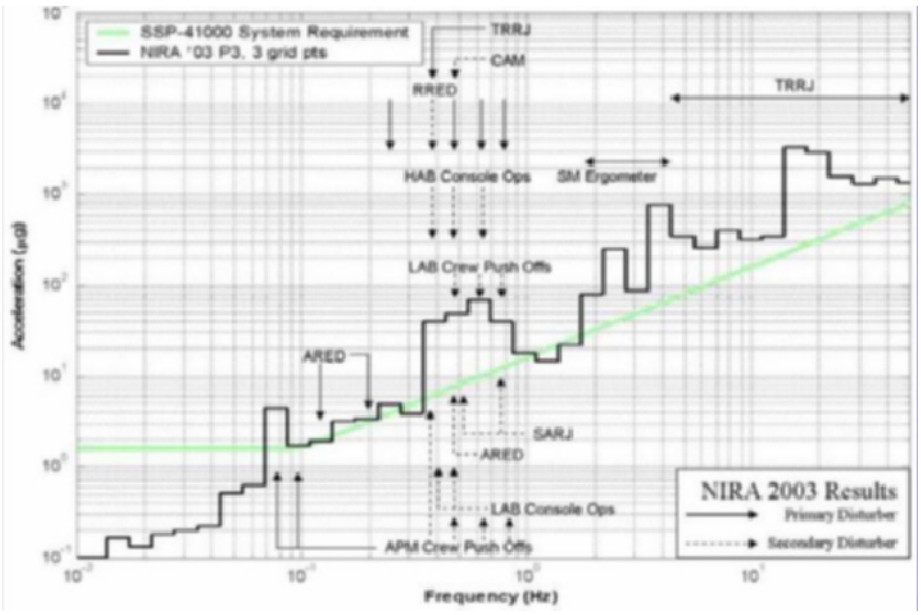

Fig. 2. Assessment of the micro-accelerations level in the protected area of the vibration-proof device [15].

However, the temperature shock has a significant effect only on small technological spacecraft, its contribution to the micro-accelerations field of the internal environment is insignificant for other classes of spacecraft [16].

When transporting space debris using cable systems, the problem of temperature shock is also relevant. If the space debris is a small spacecraft, which contains large elastic elements, then significant disturbances may occur during the temperature shock due to natural

oscillations of elastic elements. In a certain situation, such disturbances can disrupt the connection between the tug spacecraft and space debris and interfere with the successful implementation of the space debris transportation mission.

Thus, studying and taking into account the temperature shock when modeling the rotational motion of a small spacecraft is an important and topical task. Its solution will make it possible to solve the target tasks of a small spacecraft more effectively.

2 Mathematical model of the temperature shock

The paper considers three different problems of thermal conductivity - one-dimensional, two-dimensional and three-dimensional. These problems relate to the classical third initial boundary value problem of thermal conductivity. In the considered formulation, natural oscillations of the elastic element under temperature shock are not taken into account for all three problems. It is believed that the period of natural oscillations is much longer than the duration of the temperature shock. One-dimensionality, two-dimensionality or three-dimensionality of the problem is associated with the initial shape of the elastic element at the time of the temperature shock. Fig. 3 shows the shapes of the elastic element that meet various problems of thermal conductivity.

Let us suppose that at the moment of the temperature shock, the elastic element had a flat shape (the initial deflection $w_0=0$). Moreover, the direction of the heat flux is perpendicular to its plane (Fig. 3 case 1). In this case, it is sufficient to consider the one-dimensional problem of thermal conductivity, assuming that the surface layer has a uniform temperature distribution. The following system of equations and initial and boundary conditions is valid for it.

$$\begin{cases} \dfrac{\partial T(z;t)}{\partial t} = a^2 \dfrac{\partial^2 T(z;t)}{\partial z^2}, -\dfrac{h}{2} \le z \le \dfrac{h}{2}, t > 0; \\ \lambda \dfrac{\partial T\left(\frac{h}{2};t\right)}{\partial z} = Q - \varepsilon\sigma\left[T\left(\dfrac{h}{2};t\right)^4 - T_c^4\right], z = \dfrac{h}{2}, t > 0; \\ \lambda \dfrac{\partial T\left(-\frac{h}{2};t\right)}{\partial z} = -\varepsilon\sigma\left[T\left(-\dfrac{h}{2};t\right)^4 - T_c^4\right], z = -\dfrac{h}{2}, t > 0; \\ T(z,0) = T_0, -\dfrac{h}{2} \le z \le \dfrac{h}{2}, t = 0. \end{cases} \quad (1)$$

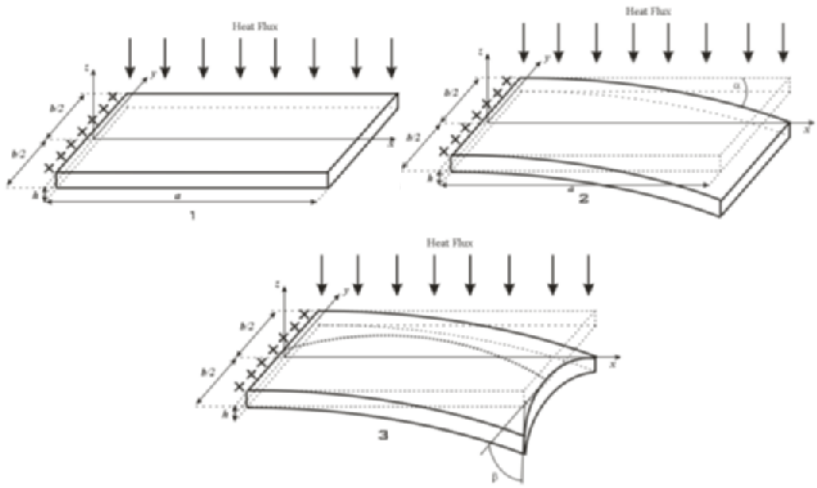

Fig. 3. Various forms of the elastic element at the time of the temperature shock onset, corresponding to: 1 one-dimensional model of thermal conductivity; 2 two-dimensional model of thermal conductivity; 3 three-dimensional model of thermal conductivity.

In (1), the following designations are used: a - the coefficient of temperature conductivity; ε - the degree of blackness of the elastic element material; σ - the Stefan-Boltzmann constant; λ - the coefficient of thermal conductivity; T_c - the ambient temperature; h - the thickness of the elastic element; T_0 - some given initial temperature distribution at the time of the temperature shock, which is considered uniform in this work.

In formulation (1), the heat fluxes inside the elastic element move only inward the plate parallel to the z axis (Fig. 3), so its heating is the most intense. Moreover, it is assumed that the heat flux is perpendicular to the plane of the plate surface all the time. This assumption also contributes to the implementation of the most intense heating of the elastic element scenario.

Let us further consider the case when the initial deflection of the elastic element at the moment of the temperature shock is significant (due to natural oscillations of elastic elements). However, the segments parallel to the y axis can be considered rectilinear (Fig. 3 case 2). Due to the initial deflection ($w_0=w_0(x)$), the temperature distribution of the surface layer cannot be considered uniform, since not all sections of the elastic element are perpendicular to the heat flux. In this case, it is necessary to consider the two-dimensional problem of thermal conductivity (the criteria for two-dimensionality are given in [17]). For a two- dimensional model of thermal conductivity, the following system of equations and initial and boundary conditions is valid:

$$\begin{cases} \dfrac{\partial T(x,\,z;\,t)}{\partial t} = a^2\left[\dfrac{\partial^2 T(x,\,z;\,t)}{\partial x^2} + \dfrac{\partial^2 T(x,\,z;\,t)}{\partial z^2}\right], \\ 0 \le x \le a,\ -\dfrac{h}{2} \le z \le \dfrac{h}{2},\ t > 0; \\ \lambda \dfrac{\partial T\left(x,\,\frac{h}{2};\,t\right)}{\partial z} = Q\cos[\alpha(x)] - \varepsilon\sigma\left[T\left(x,\,\frac{h}{2};\,t\right)^4 - T_c^4\right], \\ 0 \le x \le a,\ z = \dfrac{h}{2},\ t > 0; \\ \lambda \dfrac{\partial T\left(x,\,-\frac{h}{2};\,t\right)}{\partial z} = -\varepsilon\sigma\left[T\left(x,\,-\frac{h}{2};\,t\right)^4 - T_c^4\right], \\ 0 \le x \le a,\ z = -\dfrac{h}{2},\ t > 0; \\ T(x,\,z,\,0) = T_0,\ 0 \le x \le a,\ -\dfrac{h}{2} \le z \le \dfrac{h}{2},\ t = 0. \end{cases} \tag{2}$$

In formulation (2), internal heat fluxes propagate along the x and z axes (Figure 3). The temperature field of each layer of the elastic element is not homogeneous, but the temperature does not depend on the y coordinate.

Let us further assume that the initial deflection of the elastic element is such that the segments parallel to the y axis cannot be considered rectilinear (Fig. 3 case 3). In this case, the initial deflection is a function of the x and y coordinates; $w_0=w_0(x, y)$. In this case, the heating of the surface layer becomes even more uneven. A two-dimensional formulation of the problem is not enough for this case. The three-dimensional problem of thermal conductivity should be considered. The following system is valid for it:

$$\begin{cases} \dfrac{\partial T(x,\,y,\,z;\,t)}{\partial t} = a^2\left[\dfrac{\partial^2 T(x,\,y,\,z;\,t)}{\partial x^2} + \dfrac{\partial^2 T(x,\,y,\,z;\,t)}{\partial y^2} + \dfrac{\partial^2 T(x,\,y,\,z;\,t)}{\partial z^2}\right], \\ 0 \le x \le a,\ -\dfrac{b}{2} \le y \le \dfrac{b}{2},\ -\dfrac{h}{2} \le z \le \dfrac{h}{2},\ t > 0; \\ \lambda \dfrac{\partial T\left(x,\,y,\,\frac{h}{2};\,t\right)}{\partial z} = Q\cos[\alpha(x)]\cos[\beta(y)] - \varepsilon\sigma\left[T\left(x,\,y,\,\frac{h}{2};\,t\right)^4 - T_c^4\right], \\ 0 \le x \le a,\ -\dfrac{b}{2} \le y \le \dfrac{b}{2},\ z = \dfrac{h}{2},\ t > 0; \\ \lambda \dfrac{\partial T\left(x,\,y,\,-\frac{h}{2};\,t\right)}{\partial z} = -\varepsilon\sigma\left[T\left(x,\,y,\,-\frac{h}{2};\,t\right)^4 - T_c^4\right], \\ 0 \le x \le a,\ -\dfrac{b}{2} \le y \le \dfrac{b}{2},\ z = -\dfrac{h}{2},\ t > 0; \\ T(x,\,y,\,z,\,0) = T_0,\ 0 \le x \le a,\ -\dfrac{b}{2} \le y \le \dfrac{b}{2},\ -\dfrac{h}{2} \le z \le \dfrac{h}{2},\ t = 0. \end{cases} \tag{3}$$

In formulation (3), internal heat fluxes propagate along all coordinate axes, and the temperature field is substantially inhomogeneous.

3 Main results of the work

With a flat position of elastic elements at the moment of the temperature shock, a one-dimensional formulation of the thermal conductivity problem is sufficient, assuming that the initial temperature field is homogeneous. In this case, for a homogeneous and stationary heat flux (solar radiation) normally incident on the surface of the elastic element, the surface layer warms up as quickly as possible. At the same time, the effect of the temperature shock on the small spacecraft rotational motion is maximal.

Since large elastic elements of the small spacecraft make natural oscillations also due to the temperature shock, a one-dimensional formulation may not be enough for a correct assessment of the real, and not the maximum effect of the temperature shock. Assuming that the temperature shock is a more transient phenomenon than the elastic elements oscillations, it is possible to take into account only the initial deflection, and not its dynamic behavior during the impact period. At the same time, if the width of the elastic element is significantly less than its length, then the deflection of the plate points can be considered a function only of its longitudinal coordinate. This situation leads to a two-dimensional formulation of the thermal conductivity problem. It, as well as the one-dimensional one, is a static problem, because it takes into account only the deflection of the elastic element at the time of the temperature shock, which causes the inhomogeneity of the surface layer heating along the longitudinal coordinate. In comparison with the one-dimensional formulation, the heating is less intense here, since not all parts of the elastic element are located normally to the incident heat flux.

However, in some cases, a two-dimensional statement may also not be enough. This was clearly demonstrated by experiments with promising ROSA solar panels on the International Space Station [17]. Then we should abandon the assumption that the deflection depends only on the longitudinal coordinate. In this case, the problem of thermal conductivity will have to be solved in the most general three-dimensional formulation. The heating of the surface layer will be even less intense due to the more complex shape of the elastic element at the time of the temperature shock. Figure 4 shows the results of modeling the average temperature of the surface layer for one-dimensional, two-dimensional and three-dimensional thermal conductivity problems for the elastic element having dimensions of 1x0.5 m.

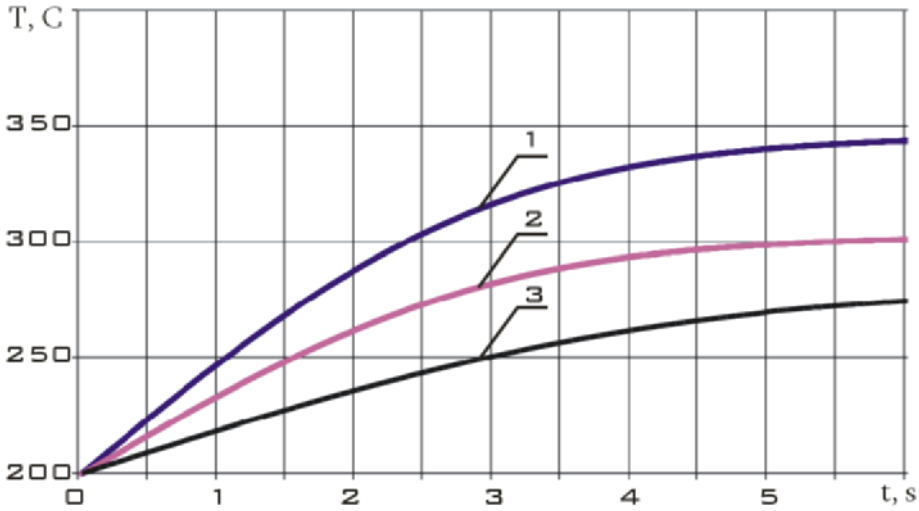

Fig. 4. Dynamics of the average temperature of the surface layer of the elastic element during the temperature shock for 1 - one-dimensional; 2 - two-dimensional; 3 - three-dimensional thermal conductivity problems.

These results were obtained in the ANSYS environment and indicate a significant effect of the initial deflection of the elastic element at the time of the temperature shock on the temperature field.

4 Conclusions

The paper presents a comparative analysis of the average temperature dynamics of the elastic element surface layer during the temperature shock. It is shown that the one-dimensional formulation gives the maximum estimation of warming up. For a three-dimensional model, heating is the least intense of the three formulations due to the fact that most of the elastic element is not located normally to the incident heat flux. The results of the work can be used in the study of the small spacecraft rotational motion dynamics, taking into account the temperature shock, to effectively solve the problems of remote sensing of the Earth, the implementation of gravity-sensitive processes and the transportation of space debris.

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