Calculation of thermal conductivity coefficient of a binary mixture

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Abstract. A correct consideration of the thermal factors during the design phase of civil engineering structures in the permafrost area determines their reliable and safe exploitation. Among the important indicators in selection of design solutions is the thermal conductivity coefficient of the construction materials used. The thermal conductivity coefficient is usually chosen from reference tables, but when using mixtures, the thermal conductivity coefficient is determined through a calculation. The aim of the present research was to compare the calculated values of thermal conductivity coefficient of binary mixtures (a mixture of a binding material and a filler) obtained using Lichtenecker and Schwerdtfeger formulas. The comparison was conducted in the range of properties of materials used for thermal accumulation and thermal insulation mixtures. It was determined that for thermal accumilation binary mixtures the calculation results are quite similar within a wide range of initial values. For thermal insulation binary mixtures, the calculation results are significantly different. The divergencies are by hundreds of percents. At the current stage of research it is impossible to make a conclusion about suitability of either calculation method to determine the thermal conductivity of a thermal insulation binary mixture.

1 Introduction

The impact of thermal conditions on reliability and safety of exploitation of civil engineering structures has been widely researched [1-3] and is in the focus of attention [4- 6]. It was noted that to increase the reliability and safety of linear structures, such as roads, in the permafrost area, it is necessary to consider the influence of the thermal factor on the exploitation characteristics of the road surfaces and foundations during the design and construction stages [7-9]. To decrease the negative impacts of cryogenic processes it is proposed to use special thermal insulation structures [10-12] and mixtures of materials with the required thermal insulation and thermal accumulation properties [13-15]. When choosing the design solutions, the accuracy of the forecast of the thermal regime of automobile roads in the permafrost area largely depends on the accurate determination of thermal physical properties of construction materials of the road and the road foundation soil [16-19]. Artificial dispersed rocks (sand-based mixtures) are widely used in road construction [20-22]. Usually, such mixtures are

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binary, composed of two components, a binding material (binder) and a filling material (filler). In principle, even regular wet sand can be considered a binary mixture [13, 23].

The aim of the present work was to compare the results of thermal conductivity calculations for binary mixtures obtained using Lichtenecker [24, 25] and Schwerdtfeger [26] formulas.

2 Methods

The Schwerdtfeger formula has the form:

$$\lambda_S = \lambda_b \frac{q-m(1-g)}{q+\frac{m(1-g)}{2}},\ q = 1+\frac{g}{2} \tag{1}$$

where $g = \frac{\lambda_f}{\lambda_b}$, $m = \frac{V_f}{V_t}$. λ_f, λ_b are the thermal conductivity coefficients of the filler and the binder respectively in W/mK.$V_f, V_t$are the volume of the filler and the total volume of the binder and filler combined, m^3. If the filler only takes up the pores within the binder, the parameter *m* will be the equal to the porosity of the binder. Otherwise, for a binary mixture, the parameter *m* denotes the concentration of one of the mixture components.

The Lichtenecker formula has a simpler form [24, 25]:

$$_L = \lambda_b^m \cdot \lambda_f^{1-m} \tag{2}$$

Considering that $g = \frac{\lambda_f}{\lambda_b}$ it can be written

$$\lambda_L = g^{1-m} \cdot \lambda_b \tag{3}$$

A parameter of diverenge of the calculation results using different formulas is introduced:

$$\beta = \lambda_S/\lambda_L \tag{4}$$

The percentage difference of the results is determined using the equation:

$$e = abs(1-\beta) \cdot 100\% \tag{5}$$

Considering the formulas (1) and (3), the parameter β can be found using the formula:

$$\beta = \left(\frac{1+\frac{g}{2}-m(1-g)}{1+\frac{g}{2}+\frac{m(1-g)}{2}}\right) g^{m-1} \ = (a) \cdot c \tag{6}$$

3 Results and discussion

Variant calculations were done using the obtained formulas. The results of the calculations are presented as charts in the figures. Two types of binary mixtures were considered: thermal insulation mixtures (thermal conductivity coefficient of the binder is greater than the thermal conductivity coefficient of the filler, $\lambda_b > \lambda_f$) and thermal accumulation mixtures (thermal conductivity coefficient of the binder is smaller than the thermal conductivity coefficient of the filler, $\lambda_b < \lambda_f$). For the first type of mixtures, the range of the ratios of the thermal conductivity coefficients of the filler and the binder was 0.05 to 0.3. For the second type, the range was 1.0 to 2.0.

Figures 1 and 2 present the charts characterizing the change in the values of the dimensionless parameter β depending on the parameter *g* which describes the ratio of the thermal conductivity coefficients of the filler and the binder at different concentrations of the binary mixture *m*.

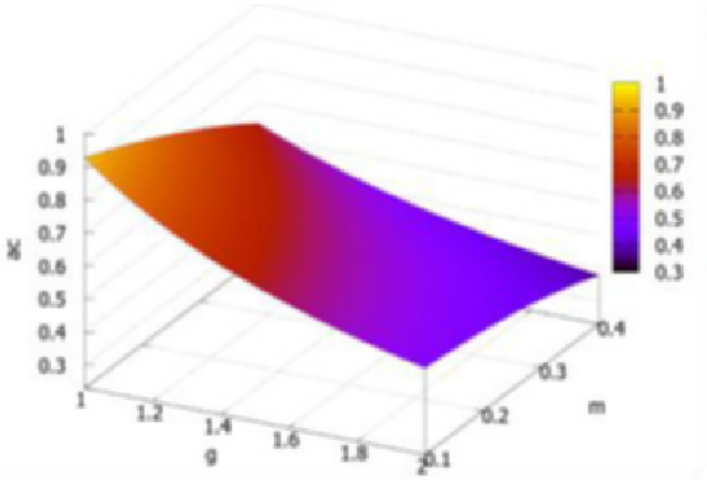

Fig. 1. Change in the parameter $\beta = a \cdot c$ for a thermal accumulation binary mixture depending on the concentration of the filler *m*.

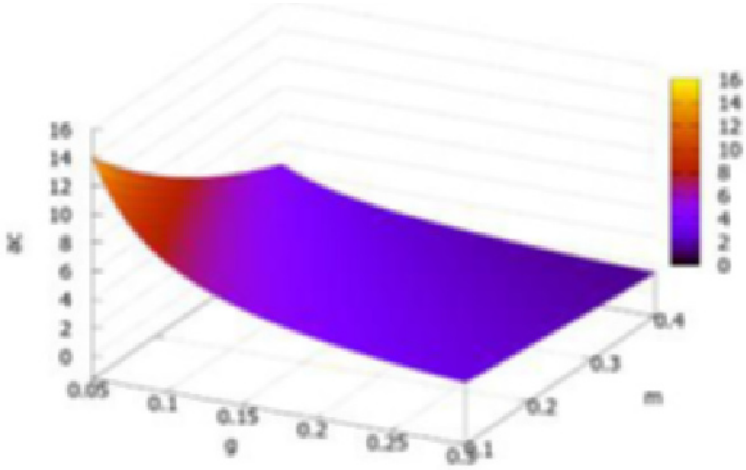

Fig. 2. Change in the parameter $\beta = a \cdot c$for the thermal insulation binary mixture depending on the concentration of the filler*m*.

The comparison of the charts in the figures shows a change in the parameter $\beta$by a factor during a transition from one type of a binary mixture to the other virtually along the entire range of the filler concentrations in the mixture. Figures 3 and 4 show the percent values of the divergence coefficient for values calculated using different formulas for the thermal accumulation and thermal insulation mixtures at different concentrations of the filler in the binary mixture.

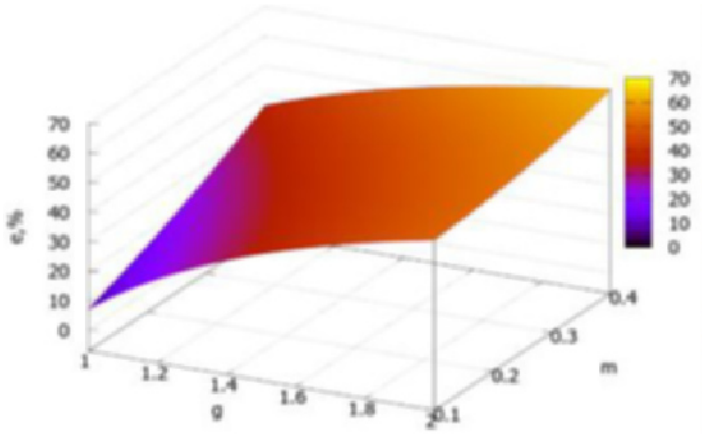

Fig. 3. Percent values of the divergence coefficient when calculating the thermal conductivity coefficient of a thermal accumulation binary mixture.

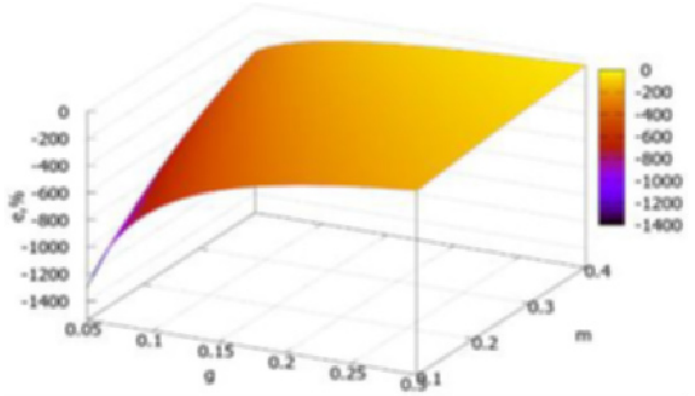

Fig. 4. Percent values of the divergence coefficient when calculating the thermal conductivity coefficient for thermal insulation binary mixture using diffent formulas.

As the charts show, for the thermal accumulation binary mixtures, both formulas return roughly similar results within a narrow range of initial values. For the thermal insulation mixtures, the results are widely differing along almost the entire considered range. For some ranges of the filler concentrations, the divergence is in hundreds and thousands of percents, which indicates a total divergence of the results. Thus, further research is required to determine which formula should be used for thermal insulation mixtures and in what ranges of initial data.

The obtained results and conclusions can also be applied in comparing the K. Lichteneker and Odelevsky [27] formulas, as the work [16] indicates. The work [16] conducted a comparison of Odelevsky and Schwerdtfeger formulas, which demonstrated that both formulas return roughly equivalent results along a wide range of initial values. In practical calculations, either one can be used. The difference in calculated thermal conductivity coefficient of a binary mixture does not exceed 10%, which is an error permissible in engineering practice.

The results presented previously show that the values obtained using the Lichtenecker formula significantly differ from the values obtained using the Schwerdtfeger formula across a wide range of initial values. For this reason, further surveys to determine the area of applicability of the Lichtenecker, Schwerdtfeger and Odelevsky formulas when determining

the thermal conductivity coefficient of binary mixtures is needed. Specifically, an area where the formulas produce similar results whereby the divergence in calculated values do not exceed 10% needs to be determined.

4 Conclusion

A comparison of thermal conductivity coefficients obtained using Lichtenecker and Schwerdtfeger formulas was made. The range of input values, the ratios of thermal conductivity coefficients of the filler to the binder varied from 0.05 to 0.3 for thermal insulation mixtures and from 1.0 to 2.0 for thermal accumulation mixtures. In both cases, the range of filler concentrations was from 0.1 to 0.4. It was demonstrated that for the thermal accumulation binary mixtures, the calculated values are quite close in a wide range of input values. For thermal insulation binary mixtures, the results differ significantly, by hundreds or thousands of percents. At this stage, it is not possible to make conclusions about applicability of either formula to determine the thermal conductivity coefficient of a thermal insulation binary mixture. The quantitative results and conclusions are valid for bothLichtenecker and Schwerdtfeger formulas, and also for the Odelevsky formula.

The article has both practical and methodological value and can be useful for both engineers designing structures in the permafrost area and for students of construction and permafrost engineering areas. Further research should be directed towards determining the area of appopriate use of the Lichtenecker, Schwerdtfeger and Odelevsky formulas when determining the thermal conductivity coefficient of binary mixtures, in particular thermal insulation binary mixtures. It is necessary to determine the range of input data for which all the formulas give similar results and the divergencies do not exceed the permitted error of 10%.

Additionally, the assessment of influence of accuracy of the determination of the thermal conductivity coefficient on specific design parameters is of interest. For example, in cases of determining the thawing depth of the road foundation or of the thermal resistance of the road structure. It would also be interesting to compare the analytical and experimental methods of determination of thermal conductivity coefficients of binary and multi-component mixtures of construction materials.

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