Creation of an electromechanical model of an exoskeleton link in the form of Lagrange-Maxwell equations for agricultural mechanization

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Abstract. The electromechanical model of exoskeleton link has been developed for agricultural mechanization. First, a differential equation of link motion was compiled without taking into account electric drives in the form of the Lagrange equation of the second kind. For it, the inverse and direct problems of dynamics are solved. Then, this equation takes into account the influence of the rotating rotor and the gear ratio of the gearbox on the dynamics of the link. For this model, the direct problem of dynamics is solved. A significant influence on the results of the numerical solution for taking into account the rotating rotor of the electric drive has been established. Then the system of differential equations, describing dynamics of the controlled motion of the model, in the form of Lagrange-Maxwell equations has been composed. The local system of coordinates has been applied in the model, since the electric drive changes the angles between the links. Direct and inverse dynamics problems have been solved. The comparative analysis of the models with electric drive and without it has been made. It has been established that taking into account the electrical drive system allows achieving good results in modeling accuracy.

1 Introduction

Since agricultural workers are constantly faced with increased loads on the musculoskeletal system, it becomes urgent to create active exoskeletons to make their daily work easier. This article develops an electromechanical model of the link, from which it will be possible to assemble a full-fledged active exoskeleton. The increase in degrees of freedom of the mechanisms, due to the growth of computing power and control algorithm development, is the current trend in the evolution of exoskeletons with variable geometry and imposed constraints. These factors allow creating mechanisms with greater capabilities than the currently available devices, which provides the relevance of this study. The results of the development of exoskeletons and their individual components are presented in [1-9].

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2 Materials and methods

The proposed exoskeleton model differs from the previously created ones by Borisov A.V., Chigarev A.V. [9] by applying the angles between the links. These angles correspond to the operation of electric drives with reduction gears that change the relative angles between the links. The predefined exoskeleton motion is performed by changing the configuration of its links, applying internal efforts, and imposing external constraints. Consider a mechanism model with one link that moves in the vertical plane (Figure 1).

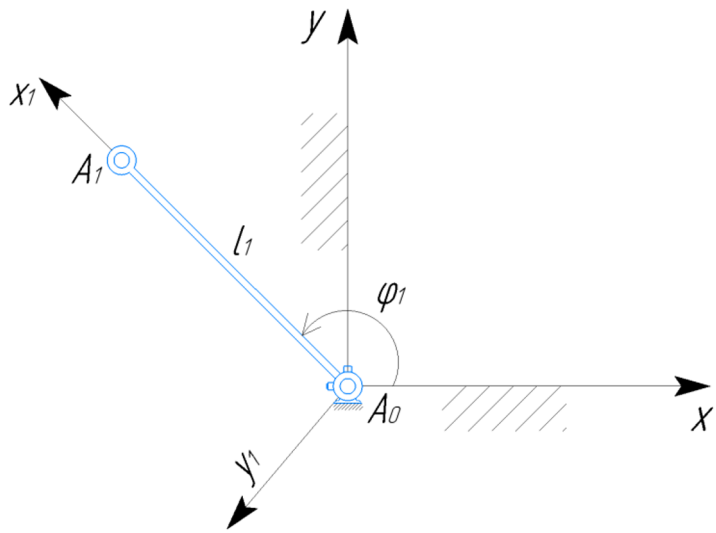

Fig. 1. The exoskeleton model for agricultural mechanization with one mobile link that moves in the vertical plane and the introduced local system of coordinates

The moment of inertia for the link A_0A_1, relative to the axis perpendicular to the motion plane $x_1A_0y_1$ and passing through its beginning – the point A_0, is designated I_1. Consider the electromechanical model of the link drive. It includes the electric motor with reduction gear at the fixed hinge A_0. The axis of the link rotation and the rotor of electric motor are assumed to be along the same line. Consider the impact of motor inertia on the differential equation of motion (2). Since the electric motor with the reduction gear is firmly fixed and immobile, their masses do not impact kinetic and potential energy of the link. The rotor of the electric drive that performs rotational motion relative to the immobile axis passing through the point A_0 is mobile. Its contribution into the kinetic energy of the electromechanical system is as follows:

$$2T_R = \left(I_1 + I_R k_R^2\right)\dot{\varphi}_1^2 . \tag{1}$$

Here I_R – the moment of inertia of the electric motor rotor relative to its rotation axis, k_R – the transmission ratio of the reduction gear.

The contribution of the reduction gear rotating elements into the kinetic energy of the system is neglected. The friction in the bearings of all rotating elements across the mechanism is also neglected. The differential equation of motion is as follows:

$$\left(I_1 + I_R k_R^2\right)\ddot{\varphi}_1 + (1/2)m_1 l_1 g \cos\varphi_1 = M_1 . \tag{2}$$

Here M_1 – the torque developed by the drive of the mechanism.

Let's introduce the electrical part of the electric motor control. The motion equations with the subsystem describing the electrical part are written in the form of Lagrange-Maxwell equations:

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\varphi}_1}\right) - \dfrac{\partial L}{\partial \varphi_1} + \dfrac{\partial \Phi}{\partial \dot{\varphi}_1} = 0, \\ \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial i_1}\right) + \dfrac{\partial \Phi}{\partial i_1} = U_1. \end{cases} \tag{3}$$

The Lagrange and Rayleigh functions for the considered model are as follows:

$$2L = \left(I_1 + I_R k_R^2\right)\dot{\varphi}_1^2 + L_1 i_1^2 + c_1 k_R \varphi_1 i_1 - m_1 l_1 g \sin\varphi_1, \tag{4}$$

$$2\Phi = R_1 i_1^2. \tag{5}$$

Now, the system of motion equations for the proposed electromechanical model can be written as follows:

$$\begin{cases} \left[\left(I_1 + I_R k_R^2\right)\ddot{\varphi}_1 + (1/2) m_1 l_1 g \cos\varphi_1\right] / \left(c_1 k_R\right) = i_1, & (6) \\ L_1 \dot{i}_1 + c_1 k_R \dot{\varphi}_1 + R_1 i_1 = U_1, & (7) \end{cases}$$

here R_1, L_1 – the generalized resistance and inductance of the electric motor rotor windings, c_1 – the coefficient of electromechanical interaction, $i_1 = i_1(t)$ – the electric current in the external circuit of the rotor, $U_1 = U_1(t)$ – the motor voltage, I_R – the moment of inertia of the electric motor rotor relative to its rotation axis, k_R – the transmission ratio of the gear.

The system of equations (6)-(7) can be transformed by substituting the expression for the electrical current i_1 from the equation (6) and its derivative into the equation (7) thus reducing it just to one differential equation of exoskeleton link motion:

$$\begin{aligned} &\frac{1}{c_1 k_R}\Big[\left(I_1 + I_R k_R^2\right) L_1 \dddot{\varphi}_1 + \left(I_1 + I_R k_R^2\right) R_1 \ddot{\varphi}_1 + \\ &+ \left(c_1^2 k_R^2 - (1/2) L_1 l_1 m_1 g \sin\varphi_1\right)\dot{\varphi}_1 + R_1 (1/2) m_1 l_1 g \cos\varphi_1\Big] = U_1. \end{aligned} \tag{8}$$

The voltage is the controlling parameter in the system. The link rotation angle φ_1 is the unknown function that should be found. As an example, let's select a suitable electric motor and reduction gear from the nomenclature of currently manufactured devices based on the peak values of load torque and velocity. As a result, we select the permanent magnet synchronous motor TECNOTION QTR-A-78 [10] and reduction gear Harmonic Drive CPL-2A [11]. The moment of inertia of the selected electric motor rotor is $I_R = 1.3 \cdot 10^{-5}$ kg·m^2, the transmission ratio of the reduction gear is $k_R = 100$, the rated voltage is $U_1 = 48$ V, the resistance of the motor armature is $R_1 = 0.857$ Ohm, the inductance of the armature is $L_1 = 0.00135$ H. The coefficient of electromechanical interaction is included into the

equation (6) as a product of transmission ratio of the reduction gear, therefore, it can be defined together with: $c_1 k_R = M_{1P}/U_1$. The starting torque on the output shaft of the reduction gear can be evaluated based on the following formula: $M_{1P} \approx g m_1 l_1/2$. Hence, we get the following values: $c_1 k_R = g m_1 l_1/2U_1 \approx 0.115$ N·m/V, $c_1 \approx 0.00115$ N·m/V. The initial conditions are as follows: $\varphi_1(0) = 1.82$ rad, $\dot{\varphi}_1(0) = 0$ rad/s, $\ddot{\varphi}_1(0) = 0$ rad/s^2.

Thus, an electromechanical model of the exoskeleton link was created. An exoskeleton can be assembled from such links to mechanize the work of agricultural workers.

3 Results

Let it simulate the motion of the human supporting leg shin. In this case, the trajectory of its motion can be specified by the following function:

$$\varphi_1(t) = \pi/2 + j_1 \sin[f_1 - (1 - \cos[2\pi t/T])\pi/2], \tag{9}$$

here T – the period of the walk, j_1 and f_1 – the parameters of the walk.

Let's select the values for the mechanism properties that correspond to the human shin. This information can be found in the monograph [9]. The length of the link is $l_1 = 0.385$ m, the mass of the link is $m_1 = 2.91$ kg, the moment of inertia of the link is calculated as that for the rod relative to the axis passing perpendicularly through its end $I_1 = 0.144$ kg·m^2. The acceleration due to gravity is $g = 9.81$ m/s^2. The period of single-support step phase, i.e. $^1/_2$ of walk period is $t_k = 0.36$ s. the walk parameters are $j_1 = 0.25$, $f_1 = \pi/2$.

Let's apply the programmed control of the model motion specified by the formula (9), solving the direct dynamics problem for the system of motion equations (7)-(8), we can find the amperage and the voltage as functions of time that are plotted in the Figure 2.

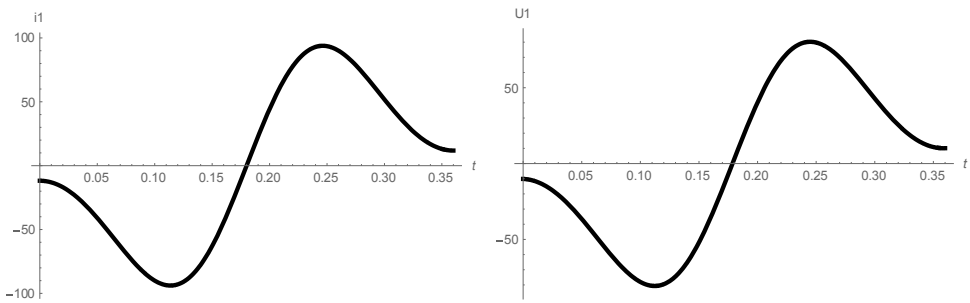

Fig. 2. The curves for the current i_1 (A) and voltage U_1 (V) as functions of time t (s).

Next, the obtained current and voltage as functions of time can be approximated with step functions (Figure 3).

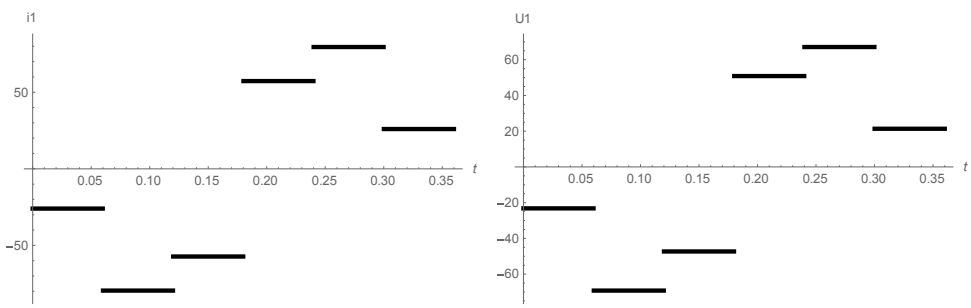

Fig. 3. The plots of current i_1 (A) and voltage U_1 (V) approximated with step function of time t (s).

The link rotation angle, angular velocity, and angular acceleration as functions of time have been obtained as a result of the Cauchy problem solution for the equation (8) with the control voltage in the form of step function presented in the Figure 8. The obtained functions are shown in the Figure 4.

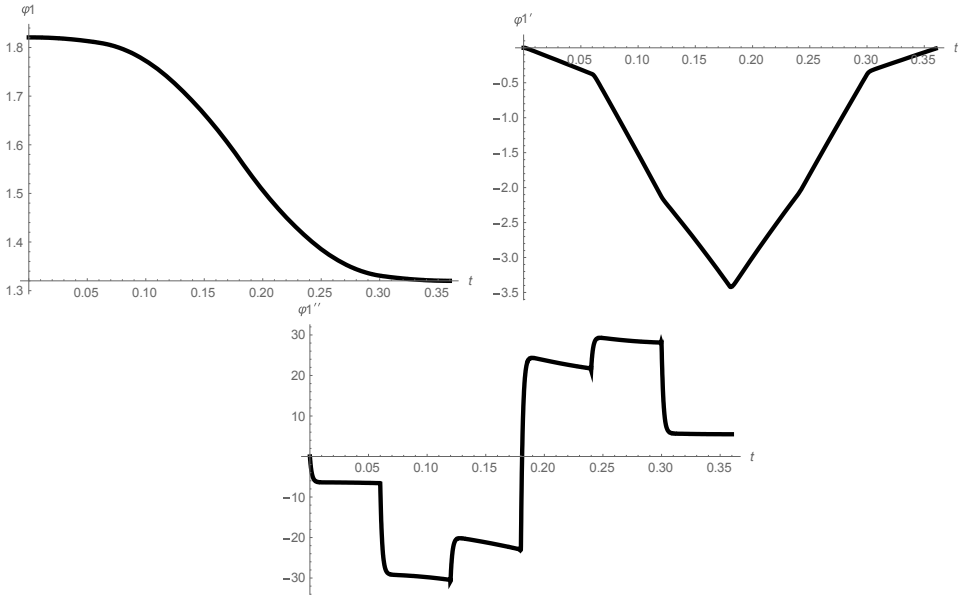

Fig. 4. The Cauchy problem solution: rotation angle φ_1 (rad), angular velocity $\dot{\varphi}_1$ (rad/s), angular acceleration $\ddot{\varphi}_1$ (rad/s^2) of the link as functions of time *t* (s).

The obtained results of the Cauchy problem solution (Figure 4) are in good agreement with the results obtained for the model with ideal drives, i.e. neglecting their masses. It suggests that the application of a more precise electromechanical model, taking into account the inertia properties of the drive, yields good results that are very close to the initial mechanism motion. This result indicates the need for building more precise models compared to the previously constructed ones [9].

4 Discussion

A mathematical model of the operation of the exoskeleton link has been created. The method of specifying the programmed motion for an exoskeleton, as well as solving direct and inverse dynamics problems has been developed in the study. This confirms the functionality of this model.

5 Conclusion

The created electromechanical model of the exoskeleton is relevant due to its numerous applications. It can be used in the practical creation of exoskeletons for the mechanism of labor of agricultural workers, since in their daily work they regularly face significant and prolonged physical stress. Such an exoskeleton will reduce the load on the musculoskeletal system, increase endurance, and protect against injuries and occupational diseases.

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