Set Cover Model with Path-Segment Concept to Determine the Optimum Rest Areas on Toll Road as the Location of Electric Vehicle Battery Charging Stations

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Abstract. The rapid adoption of electric vehicles (EVs), especially battery-based electric vehicles (BEVs), requires battery recharging facilities. Battery charging station (BCS) is an important component in EVs ecosystem. EVs requires BCS infrastructure for battery recharging. Limited battery range in one full charge is a concern for EVs users which is often called range anxiety so the number and placement of BCS must be adequate to prevent EVs from running out of battery in the middle of the trip. This study uses the real data of Trans Java toll road network, specifically the Semarang-Ngawi section for running numerical experiment to test the performance of the proposed model. Rest areas on toll roads are considered as candidate points for BCS locations. The proposed model is a set cover model formulated into binary integer programming. The Simplex algorithm in Microsoft Excel Solver is used to find the optimum solution on numerical experiment. Four locations are found as the optimum locations of BCS, i.e rest area r3, r6, r9, and rest area r12. The novelty of this study is combining the path-based approach with node-based approach to get a more compact set cover model.

1 Introduction

Carbon emissions caused by the use of fossil energy are very detrimental to life on earth such as causing climate change, extreme temperatures, and other negative impacts. One solution around the world to achieve carbon emission targets is the adoption of battery-based electric vehicles in the transportation sector replacing conventional fossil fuel vehicles. Electric vehicles (EVs) are capable of mitigating climate change [1], reducing environmental problems [2, 3], and providing long-term economic benefits to consumers compared to fossil fuel vehicles [4, 5].

The Government of Indonesia is committed to contribute to overcoming the problem of carbon emissions by adopting battery-based electric vehicles (BEVs) by the enactment of a number of laws and regulations to ensure the implementation of these commitments. Among these regulations are presidential regulation number 55 Year 2019 about acceleration of battery-based electric vehicle for roads transportation [6], presidential regulation number 22 Year 2017 about general plan of national energy [7], executive order number 7 Year 2022 about use of BEVs as official vehicle for central and local government [8], ministerial regulation ministry of energy and mineral resources number 13 Year 2020 about charging infrastructure development for BEVs [9], and ministerial regulation ministry of industrialisation number 27 Year 2020 about the targeted usage of battery electric vehicles (motorcycle and car) at the end of 2030 [10].

Around the world EVs has grown rapidly over the past decade. Apart from the growing interest in environmentally friendly renewable energy, EV's growth is also due to the improvement of EV's technology and concerns about oil prices [11]. EV's global sales in 2021 were 6.6 million units, doubling the sales in 2020. While in 2012 EVs were sold only 120,000 units. In 2021, about 10% of global car sales were electric cars, equivalent to quadrupling the electric car market share in 2019. Global sales of electric cars continued to increase rapidly into 2022 with 2 million units sold in the first quarter, increasing 75% from the same period in 2021 [12].

EVs requires BCS infrastructure for battery recharging. Limited battery range in one full charge is a concern for EVs users which is often called range anxiety [1315] so the number and placement of BCS must be adequate to prevent EVs from running out of battery on the trip [11]. Potential consumers tend not to be interested in buying an electric car unless there is an adequate battery charging stations [16]. On the other hand, investors are not interested for investing on BCSs unless the population of EVs is quite large, so it is often called a chicken-and-egg problem [17-20].

BCSs placement optimization interests many researchers. A flow capture location model in which a single facility anywhere along the path can meet the

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demand along the path is proposed in [21]. Electric vehicles may need to charge batteries on multiple BCSs while traveling long distances so a single facility can not meet the demand. Considering the limited range of electric vehicles, Kuby and Lim (2005) [22] introduced a flow refueling location model (FRLM) that allows vehicles to be charged multiple times before completing a trip.

Mirhassani and Ebrazii (2013) [23] reformulated the model of [22] by introducing a dummy source node and a dummy sink node for each path, simplifying the mathematical formulation and reducing computation time compared to model of [22]. Yildiz et al. (2016) [24] introduced the concept of path segments, corresponding to the earlier concept of expanded networks proposed by [23]. FRLM in [25-27] also adopted the concept of path segments when developing models.

A round trip of an intercity transport network passing through each city once was considered on [25]. The system characterization is analogous to the traveling salesman problem, an example of the Hamilton cycle in graph theory. Capar and Kuby (2012) [28] proposed a location model for an undirected network where EVs traverse the same road on go trip and return trip so one BCS serves EVs from two different directions. The same network structure as in [28] was considered by [23, 29-31].

This paper considers a different network structure compared to [22-31]. The proposed model use a toll road network operated by a closed toll system. A closed toll system is an example of Eulerian graph. An Eulerian graph has even number of arcs on each node [32]. A toll plaza on a toll road consists of entrance gate and exit gate (two arcs with different direction). Toll roads are growing in popularity among transportation policymakers for relieving congestion on highways and providing funding for the construction of new lanes on congested city corridors [33].

Each vehicle is charged a fixed rate at the toll station according to its origin and destination (O-D trip) [34]. A trip is a round trips via different roads (different toll lanes). An origin or destination node is represented by a toll gate that is considered an entrance/exit to a city. On a round trip, the origin/destination node is traversed twice, one outbound and one inbound. The proposed model minimizes the total number of BCSs while satisfying all charging demand of EVs from all paths so EVs never run out its battery along the trip.

2 System Description

2.1 Toll Road Operations

The Trans Java toll road is a closed system toll road. On closed system toll roads, there are a number of toll plaza or toll gate as entrance and exit point of toll roads. Vehicles can not turn around to reverse direction. Turning around can only be done at the toll plaza. In addition, on closed system toll roads it is not possible to change lanes to the opposite direction, for example, to take a rest at the rest area across the road is not possible. This led to the rest areas being built on both opposite sections of the toll road. The same applies to the construction of BCS which is intended to serve EVs. BCS must be built on both lanes of opposite directions. This is in contrast to free roads (not a toll road) where vehicles are allowed to cross to opposite lanes through certain crossing points making it possible to use one BCS facility to serve EVs from both directions. Fig. 1 is an illustration of the system on toll roads. Exit toll or toll plaza is modeled as city A and city B which also represents the origin point and destination point of a trip (O-D path). A trip is assumed a round trip. Lanes on toll roads are modeled with blue lines with arrows indicating one-way roads. The rest area that is a candidate for BCS location is modeled with a small circle with the letter r1-r6.

2.2 Minimum Driving Range

The distance between the two rest areas is known and the minimum driving range (MDR) is also known. MDR is the maximum distance between two consecutive BCSs [25] which describes the minimum range requirement that an EV must meet on a single recharge if it travels on a toll road network. An EV can not complete a trip on a toll road if it start a trip with battery range less than MDR. When starting the trip from the point of origin, the EV’s battery is assumed to be charged at least to MDR level. When arriving at the destination point, EV does not recharge the battery in the destination city but in the rest area on the way back to the origin city. On the toll road, it is not possible to drive backwards or drive to reverse direction except through the toll plaza. Suppose the MDR is 100 Km, then the distance between two consecutive BCSs should not exceed 100 Km in order to EV’s battery can be recharged before it runs out.

![Fig. 1. Closed toll system](image)

### Table 1. The length of path-segments (Km) (MDR=100Km)

<table>
<thead>
<tr>
<th></th>
<th>r1</th>
<th>r2</th>
<th>r3</th>
<th>r4</th>
<th>r5</th>
<th>r6</th>
</tr>
</thead>
<tbody>
<tr>
<td>r1</td>
<td>&gt;MDR</td>
<td></td>
<td>37</td>
<td>45</td>
<td>82</td>
<td>&gt;MDR</td>
</tr>
<tr>
<td>r2</td>
<td>&gt;MDR</td>
<td>&gt;MDR</td>
<td>8</td>
<td>45</td>
<td>64</td>
<td>&gt;MDR</td>
</tr>
<tr>
<td>r3</td>
<td>93</td>
<td>&gt;MDR</td>
<td>&gt;MDR</td>
<td>37</td>
<td>56</td>
<td>&gt;MDR</td>
</tr>
<tr>
<td>r4</td>
<td>82</td>
<td>&gt;MDR</td>
<td>&gt;MDR</td>
<td>&gt;MDR</td>
<td>&gt;MDR</td>
<td>&gt;MDR</td>
</tr>
<tr>
<td>r5</td>
<td>45</td>
<td>64</td>
<td>&gt;MDR</td>
<td>&gt;MDR</td>
<td>&gt;MDR</td>
<td>&gt;MDR</td>
</tr>
<tr>
<td>r6</td>
<td>26</td>
<td>45</td>
<td>82</td>
<td>90</td>
<td>&gt;MDR</td>
<td>&gt;MDR</td>
</tr>
</tbody>
</table>

2.3 Battery Charging Station Locations

If a rest area is selected as the location of a BCS, then the next BCS must be located on a rest area with the same direction and the distance between the two BCSs does not exceed MDR, or the next BCS can be located on a rest area of the opposite direction passing through a toll plaza (city) with a distance not exceeding MDR. Table 1 provides examples of distance data between two toll plazas.
rest areas. If two rest areas are connected, a path-segment is formed that connects the two nodes. By using the concept of path-segment and concept of coverage status, it can be seen from Table 1 that BCS at point r1 can serve node r2, r3, and r4. Not only nodes are served, but also the line or path-segment (road segment) connecting r1 to r2 to r4 are served. Node r1 is prohibited to serve itself because path-segment (r1,r1) is not exist. Another example, if the BCS is built on the rest area r6, then the BCS can serve nodes r1, r2, r3, and r4. City A and city B representing toll plazas are neglected in Table 1 because those cities are located on the path-segment (r6,r1) and (r3,r4) respectively. If both path-segments are served, then automatically both cities are also served. In the next section, we will develop the binary integer programming formulation to represent the system described above.

3 Model Formulation

The model developed aims to determine the best rest areas as the optimal locations of BCS. The model objective is to minimize the number of BCSs but EVs are served on all paths without running out of battery along the trip, assuming EVs always recharge the battery at least to MDR level when passing through a BCS. The cost of building a BCS is considered the same at every rest area node so that the model objective is the minimization of the number of BCSs which is equivalent to the minimization of the total cost to build BCSs. The proposed model is referred as the set cover model because city A city B r1 r2 r3 r6 r5 r4 it requires all paths to be served, i.e. EVs never runs out of battery on the way on any route or path. Rest areas are modeled as demand nodes as well as server nodes.

3.1 Basic Set Cover Model

Toregas et al (1971) [35] proposed the first set cover model without considering the path or network traveled by demand which is termed the node-based approach. The set cover formulation of [35] is as follows:

\[
Min Z = \sum_{i \in I} x_i
\]

s.t.

\[
\sum_{j \in I} x_i \geq 1, \forall j \in J
\]

\[
x_j \in \{0,1\}, \forall j \in J
\]

Xi = binary decision variables

Xi = 1 if a facility is built at node i; 0 otherwise

Ni = set of facility (nodes i) that can cover or serve demand node j (based on distance)

I = set of facility nodes with index i

J = set of demand nodes with index j

Z = number of facilities (objective function)

Basic set cover model represented by equations (1) to (3) aims to minimize the number of facilities (eq. (1)) in which a number of selected facilities are able to serve all demand points (eq. (2)). One facility is only able to serve a few nearby demand nodes based on coverage requirement (distance or time) so more than one facility is needed to serve all demand nodes. Each demand node must be served at least by one facility (eq.( 2)).

3.2 Node-Based Set Cover Model Considering Paths

The proposed model in this study is based on the model of [35] by considering different system characterization. The system that will be modeled in this study is the toll road network represented by toll plazas, rest areas, and pathsegments (road segment). The toll road network consists of many travel routes (paths) representing different points of origin and destination that is not considered in [35]. The length of path-segment will be converted to a coverage status parameter to enable node-based set cover formulation. In addition, the proposed model considers many different paths that exist on the toll roads.

The mathematical notation on the proposed model are as follows:

Sets

I = set of servers (all rest areas on the network as BCS candidate locations)

J = set of demand nodes (all rest area on the network as demand nodes)

Q = set of paths (O-D round trip) on the network to be covered

Indices

q = index of path

i,j = index of node (rest area)

Parameters

t_{ij}^q = 1 for i=j and path-segment (i,j) on path q is feasible (the length < MDR) (node i can cover node j and path-segment (i,j) completely)

= 0 for i=j or for i≠j and path-segment (i,j) on q is not feasible (the length > MDR) (node i can not cover node j and path-segment (i,j) completely)

Decision variables

yi = 1 if a BCS is located at node i; 0 otherwise

The set cover model is formulated as follows:

\[
Min Z = \sum_{i \in I} y_i
\]

\[
\sum_{i \in I} t_{ij}^q y_i \geq 1, j \in J, q \in Q
\]

\[
y_i \in \{0,1\}, \forall j \in J
\]

The objective (4) minimizes the total number of BCSs. Constraints (5) represent the requirement that every demand node must be covered at least by one BCS. Constraints (6) represent requirement of binary decision variables. Example of binary parameters, t_{ij}^q, is shown on Table 2. For the path on Fig.1 with MDR 100 Km, the length of path-segments in kilometres as shown on Table 1 is converted to binary parameters, t_{ij}^q, namely coverage status (1=yes, 0=No) as shown on Table 2. If
the length is more than MDR then \( t_{ij} = 0 \) for \( i \neq j \). In addition, \( t_{ii} = 0 \) for \( i = j \) because there is no pathsegment from \( i \) to \( i \) or from \( j \) to \( j \). If the length is less than or the same as MDR then \( t_{ij} = 1 \) for \( i \neq j \). On the first row of Table 2, BCS on rest area \( r_1 \) can serve demand nodes \( r_2, r_3, r_4, r_5 \) and all the path-segments \((r_1,r_2), (r_1,r_3), (r_1,r_4), (r_2,r_3),(r_2,r_4), \) and \((r_3,r_4)\).

### Table 2. Example of binary parameters, \( t_{ij} \) (coverage status)

<table>
<thead>
<tr>
<th></th>
<th>( r_1 )</th>
<th>( r_2 )</th>
<th>( r_3 )</th>
<th>( r_4 )</th>
<th>( r_5 )</th>
<th>( r_6 )</th>
</tr>
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<tbody>
<tr>
<td>( r_1 )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<td>1</td>
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<tr>
<td>( r_2 )</td>
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<td>1</td>
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</tr>
<tr>
<td>( r_3 )</td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( r_5 )</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( r_6 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### 4 Result of Numerical Experiment

Real data are used to illustrate the applicability of the proposed model. The Semarang-Ngawi toll road section consisting of three paths (round trips), i.e. M-O, M-G and OG, is shown in Fig.2. Semarang (M) is represented by toll plaza at KM420 (kilometers 420), Solo (O) by toll plaza at KM504, and Ngawi (G) by toll plaza at KM579. All 12 rest areas in the network are shown in Table 3 and Fig.2. There are more than 10 exit tolls on the Semarang-Ngawi toll road section, but in this example only 3 are represented in the model. The distance between two rest areas can be easily \( (1) \), \( (2) \), \( (3) \), \( (4) \), \( (5) \), \( (6) \) calculated using the data in Table 3, considering the locations of the three cities. To calculate the length of path-segment, we need to know the distance from one rest area to another rest area. In this example, the MDR is assumed 100 km. The length of path-segments on each path of the three paths are converted to coverage status (parameters \( t_{ij}^d \)) as shown on Table 4 to Table 6. The cost to build BCS on a rest area is assumed the same on all rest areas.

### Table 3. Semarang-Ngawi toll road section: location of rest areas

<table>
<thead>
<tr>
<th>Toll lane</th>
<th>km429 (r1)</th>
<th>km458 (r2)</th>
<th>km487 (r3)</th>
<th>km519 (r4)</th>
<th>km538 (r5)</th>
<th>km575 (r6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-G</td>
<td>km429 (r12)</td>
<td>km456 (r11)</td>
<td>km487 (r10)</td>
<td>km519 (r9)</td>
<td>km538 (r8)</td>
<td>km575 (r7)</td>
</tr>
</tbody>
</table>

**Fig. 2.** Twelve rest areas (r1-r12) in three path (MO, M-G, and O-G)

Microsoft excel solver with simplex LP algorithm is used to solve the numerical example. To get optimum decisions, the solver solves the model in less than one second. From the solver output is known that the minimum number of BCS to serve all three paths is four. The optimal locations of BCSs are found on rest area \( r_3, r_6, r_9, \) and \( r_{12} \) as shown on Fig.3. For the path M-G, the length of path-segment \((r_3,r_6)\) is 88 Km, path-segment \((r_6,r_9)\) is 64 Km, path-segment \((r_9,r_{12})\) is 90 Km, and path-segment \((r_{12},r_3)\) is 76 Km. The path M-G needs four BCSs. For the path M-O, the length of path-segment \((r_3,r_{12})\) is 69 Km, and path-segment \((r_{12},r_3)\) is 76Km. The path M-O needs two BCSs. For the path O-G, the length of and path-segment \((r_6,r_9)\) is 64 Km, and pathsegment \((r_9,r_6)\) is 82 Km. The length of a path-segment can be interpreted as the distance between two consecutive BCSs optimized by solver on the proposed model. Four selected rest areas can serve all three paths. BCS on rest area 3 and rest area 12 are used by path M-G and path M-O, and BCS on rest area 6 and rest area 9 are used by path M-G and path O-G.

### Table 4. Coverage status, \( t_{ij}^d \), of path/round trip M-G (path 1)

<table>
<thead>
<tr>
<th></th>
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<th>( r_3 )</th>
<th>( r_4 )</th>
<th>( r_5 )</th>
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</table>

### Table 5. Coverage status, \( t_{ij}^d \), of path/round trip M-G (path 2)

<table>
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### Table 6. Coverage status, \( t_{ij}^d \), of path/round trip M-G (path 3)

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</table>

**Fig. 3.** Four optimal locations of BCSs: r3, r6, r9, r12
5 Conclusion

The set cover model developed in this study adopts the concept of path-segment which is the term used in the pathbased location model approach, especially FRLM models. The length of path-segment is converted to a coverage status parameter with a value of 1 if the length is less than or the same as MDR and a value of 0 if the length is more than MDR. Coverage status is a commonly used concept in the nodebased location model approach. The novelty of this study is combining the path-based approach with node-based approach to get a more compact set cover model with fewer constraints and fewer variables compared to FRLM models proposed in [22, 23], and [28-31]. The set cover model proposed in this study was tested on a real system of Trans Jawa toll road network specifically Semarang-Ngawi toll road section which consists of three paths (three round trips), namely path of Semarang-Ngawi, path of Semarang-Solo, and path of Solo-Ngawi. The model aims to determine the best rest areas as the optimal locations of BCSs while minimizing the total number of BCSs. In other words, the model is used to find the minimum number of BCSs to serve all three paths, i.e. EVs will never run out of battery on the toll road if the driver always stops at BCS anywhere along the toll road to charge the battery at least up to MDR level. However, the proposed model assumed uncapacitated BCSs and a static single period planning horizon. Those assumptions may be less realistic for some cases. Therefore, a new model can be developed to relax those assumptions on further research.

References


