Study and Design of Bandstop Filters Based on a Multiple Co-Directional Complementary Split Ring Resonator (CSRR)

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Abstract. The objective of this study is to enhance the performance of metamaterial filters. Indeed, the distinctive electromagnetic characteristics of metamaterials, along with their capacity to guide and manipulate electromagnetic waves in ways unattainable by natural materials, have rendered metamaterials progressively appealing in recent years. One type of metamaterial that has been studied extensively is the Split Ring Resonator (SRR) and its complementary counterpart (CSRR). Both of these structures possess dimensions significantly smaller than the wavelength. They demonstrate bandstop characteristics and exhibit negative permeability or negative permittivity in a narrow frequency range centered around their resonant frequencies. These frequency bands can be precisely and easily controlled in terms of frequency selectivity and rejection level by adjusting various parameters. As a result, these resonators are designed and optimized for use in the design of new filters. The obtained results confirm that the proposed metamaterial resonators significantly enhance the performance of the targeted applications.

1 Introduction

Metal inclusions in the host media, such as the substrate, are used to create metamaterials, which are synthetic materials with strange, unnatural properties. The manner in which a metamaterial interacts with an electromagnetic field depends on the form, size, alignment, arrangement, and electromagnetic properties of the host medium. Instead of receiving such qualities directly from their constituents, these materials obtain their useful features via their structures. These materials have an anti-parallel relationship between the phase velocity and the group velocity. Application areas for metamaterials in microwave engineering encompass waveguides, antennas, filters, phase shifters, delay lines, and more. Victor Veselago introduced the concept of metamaterials in 1968 when he analyzed the propagation of incident uniform plane waves in a medium, assuming that both permittivity and permeability were negative [1]. Left-handed metamaterials (LHM) or double negative metamaterials (DNG) are the names he gave to these substances. If only permeability ($\mu$) or...
permittivity ($\varepsilon$) are negative, they are referred to as single negative (SNG) metamaterials and are referred to as Mu Negative (MNG) and Epsilon (ENG) metamaterials, respectively. Pendry suggested using SRR in order to realize negative permeability and thin wire in order to realize negative permittivity, as well as combining the two in order to create LHM [4]. Later, in 2000, Smith and his coworkers established the simultaneous display of negative permittivity and permeability in metamaterials, and they conducted microwave experiments to examine the material's peculiar features [1-2].

Smith et al. conducted an experimental demonstration of negative refraction in 2001 using copper strips and metamaterials with repetitive Split Ring Resonator (SRR) unit cells [3]. In 2002, Marques et al. explored the bianisotropy of the SRR unit cell structure. In an effort to mitigate bianisotropy, they proposed a modified SRR form known as broadband coupling (BC-SRR) in the same publication. By placing printed metallic rings of the BC-SRR on both sides of the dielectric substrate with their splits oriented 180 degrees apart, they provided a comparative analysis of traditional (or edge-coupled) SRR and BC-SRR configurations [4].

In 2007, Bilotti and colleagues introduced and investigated unit cells of Split Ring Resonators (SRRs) featuring multiple rings. They presented a model incorporating Multiple Split Ring Resonators (MSRRs) and Spiral Rings (SRs) to enhance miniaturization within the same structure. As the number of turns in SRs and rings in MSRRs increased, a saturation effect on the resonant frequency was observed [1].

In 2010, Joshi et al. introduced a microstrip patch antenna incorporating Split Ring Resonators (SRRs) and demonstrated that by loading the patch antenna with SRRs, the resonant frequency could be shifted to a lower frequency range. In 2011, Pattnaik et al. also observed that altering the distance between the patch antenna and SRRs resulted in a shift and reduction in the resonant frequency, with the frequency decreasing as the distance increased [5].

In 2012, Joshi et al. conducted research on a rectangular slotted microstrip patch antenna with a ground plane partially loaded with metamaterial Multiple-Split Ring Resonators (MSRRs). They showed that in the unloaded condition, the resonant frequency of the rectangular patch antenna was higher, but it decreased when loaded with MSRRs [6].

In 2014, Radovan Bojanic et al. presented an improved equivalent circuit approach for modeling the magnetic/electric interaction of individual Split Ring Resonators (SRRs) with printed lines. They extracted various parameters related to microstrip lines with parallel and perpendicular gaps to the line [1].

In this article, bandstop filters are designed by incorporating Complementary Split Ring Resonators (CSRRs) into planar structures. These bandstop filters are based on a multi CSRR co-directional. Each filter is constructed using a ground plane composed of both circular and square CSRRs, with an FR4 epoxy substrate on one side and a Rogers 3010 substrate on the other. A microstrip is placed on top of the substrate. The filtered frequency bands are those that experience interference with other communication systems, such as the Worldwide Interoperability for Microwave Access (WiMAX, 3.3-3.7 GHz) and local area networks (WLAN, 2.4-2.48 GHz, 5.15-5.35 GHz, and 5.725-5.825 GHz).

2 Design of transmission lines based on metamaterial structures

The design of transmission lines based on metamaterial structures can be achieved either by positioning SRR (Split Ring Resonator) cells in close proximity or by engraving CSRR (Complementary Split Ring Resonator) cells into the ground plane. A microstrip line loaded with a CSRR, which is more suitable for planar structures as previously described, is further investigated, the substrate employed is made of FR4-epoxy with a dielectric
constant $\varepsilon_r=4.4$ and a loss tangent of 0.02. This structure and its equivalent circuit model are depicted in the figure 1.

![Image of transmission line loaded with the CSRR and its equivalent circuit model](image)

**Fig. 1.** Transmission line loaded with the CSRR and its equivalent circuit model.

From the equivalent circuit, the transmission resonance frequency ($F_Z$) of the CSRR-loaded transmission line is defined as follows [7]:

$$F_Z = \frac{1}{2\pi} \sqrt{L_C (C_C + C)}$$

Where $C$ represents the coupling capacitance between the transmission line and the CSRR. The latter is represented as a parallel LC (inductance $L_C$ - capacitance $C_C$) circuit.

To induce the bandstop effect of the CSRR at the frequency ($f_Z$), the entire length of the CSRR, $L_{\text{total}}$, is usually adjusted to be half the wavelength at that specific frequency:

$$L_{\text{total}} \approx \frac{\lambda}{2} = \frac{c}{2F_Z \sqrt{\varepsilon_{\text{eff}}}}$$

$$\varepsilon_{\text{eff}} = \frac{(\varepsilon_r+1)}{2} + \frac{(\varepsilon_r-1)}{2\sqrt{1+12\frac{h}{w}}}$$

In this equation, where $c$ represents the speed of light in a vacuum, $w$ is the width of the transmission line, and $h$ and $\varepsilon_r$ denote the height and the dielectric constant of the substrate, respectively.

In the case of a circular resonator, the radius is calculated from $L_{\text{total}}$, which represents the circumference of the circle in this case, using the formula:

$$L_{\text{total}} \approx 2\pi r$$

Therefore

$$r = \frac{L_{\text{total}}}{2\pi}$$

This approximate length, $L_{\text{total}}$, is used for the design. This has led us to design multi-band filters.

### 3 Design of a bandstop filter based on a multiple co-directional CSRR
3.1 Circular multiple co-directional CSRR filter

The design of metamaterial filters and the simulation of results are carried out using the High-Frequency Structure Simulator (HFSS) software. Figure 2 illustrates the structure of the proposed filter. It consists of a microstrip line with a width of \( w = 3 \text{ mm} \), providing a characteristic impedance of \( 50 \Omega \), and a ground plane where the complementary split ring resonators are etched. The substrate used is FR4-epoxy with dimensions of \( 30 \times 15 \times 1.6 \text{ mm}^3 \), featuring a dielectric constant \( \varepsilon_r = 4.4 \) and a loss tangent of \( 0.02 \).

![Figure 2. Circular multiple co-directional CSRR filter](image)

The dimensions of the complementary split ring resonators are chosen to suppress the signal in a specific frequency band. By adjusting various geometric parameters, the center operating frequencies of the filter (CSRR resonance frequencies) are obtained while considering the rejection level. Table 1 displays the total length and optimized dimensions of the CSRR.

<table>
<thead>
<tr>
<th>( F_z ) (GHz)</th>
<th>( L_{\text{Total}} ) (mm)</th>
<th>( r ) (mm)</th>
<th>( c ) (mm)</th>
<th>( g ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>38.1</td>
<td>5.6</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>3.2</td>
<td>26.1</td>
<td>4.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>4.4</td>
<td>16.6</td>
<td>3</td>
<td>0.6</td>
<td>1</td>
</tr>
</tbody>
</table>

The reflection and transmission coefficients of the filter are depicted in figure 3

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Table 1. The total length and optimized dimensions of the CSRRs are as follows.
It can be observed that the transmission coefficient of the filter is -18.17 dB, -17.63 dB, -19.8 dB at frequencies of 2.4 GHz, 3.2 GHz, and 4.4 GHz, respectively. Consequently, signal rejection within these frequency bands is achieved.

To further enhance the performance of this filter in terms of rejection level and bandwidth, a second-order filter is recommended. For this purpose, two multiple co-directional CSRR cells are utilized, while keeping the substrate dimensions identical to those of the first-order filter (figure 4).
Figure 5 depicts the reflection and transmission coefficients of the second-order filter.

![Graph showing reflection and transmission coefficients](image)

**Fig. 5.** The reflection and transmission coefficients of the second-order filter

The transmission coefficient of the second-order filter achieves, in HFSS, -31.08 dB, -30.72 dB, -37.76 dB at frequencies of 2.4 GHz, 3.2 GHz, and 4.4 GHz, respectively. It's worth noting that the rejection level was, in HFSS, at -18.17 dB, -17.63 dB, -19.8 dB at frequencies of 2.4 GHz, 3.2 GHz, and 4.4 GHz, respectively, for the first-order filter. Indeed, an improvement in filter performance is observed in terms of filtered bandwidth and rejection level, all while maintaining the same substrate dimensions.

### 3.2 Rectangular multiple co-directional CSRR filter

To assess the impact of the CSRR type on the filter's performance, multiple co-directional rectangular CSRRs are integrated into the structure to serve the filtering function. The same substrate dimensions as those of the circular CSRR filter are applied. Figure 6 depicts the structure of the second-order filter.

![Diagram of second-order filter structure](image)

**Fig. 6.** First-order rectangular multiple co-directional CSRR filter
The total length and optimized dimensions of the rectangular CSRRs are compiled in table 2.

**Table 2.** The total length and optimized dimensions of the CSRRs are as follows.

<table>
<thead>
<tr>
<th>Fz (GHz)</th>
<th>L_total (mm)</th>
<th>a (mm)</th>
<th>b (mm)</th>
<th>c (mm)</th>
<th>g (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>38.1</td>
<td>9.2</td>
<td>10</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>3.5</td>
<td>26.1</td>
<td>8</td>
<td>6.2</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>5.5</td>
<td>16.6</td>
<td>6.2</td>
<td>4</td>
<td>0.6</td>
<td>1.4</td>
</tr>
</tbody>
</table>

The figure 7 illustrates the reflection and transmission coefficients of the first-order filter.

**Fig. 7.** The reflection and transmission coefficients of the first-order filter

It can be observed that the transmission coefficient of the filter is -14.69 dB, -16.90 dB, -18.30 dB at frequencies of 2.4 GHz, 3.4 GHz, and 5.7 GHz, respectively. Consequently, signal rejection within these frequency bands is achieved.

To further enhance the performance of this filter, two multiple co-directional CSRR cells will be employed, all while maintaining the substrate dimensions identical to those of the first-order filter.
Fig. 8. Second-order circular multiple co-directional CSRR filter

Figure 9 illustrates the reflection and transmission coefficients of the second-order filter.

Fig. 9. The reflection coefficient S11 and transmission coefficient S21 of the second-order filter.

It can be observed that the attenuation of the transmission coefficient is approximately -36.29 dB, -33.97 dB, -30.11 dB at frequencies of 2.3 GHz, 3.4 GHz, and 5.6 GHz, respectively. Therefore, the filtering property is achieved with broader rejection bands, notably in the third band from 5-6 GHz. It is also evident that the rejection level is increased in this second-order filter compared to the first-order filter (Figure 6) studied previously.
### 3.3 Filter based on RO3010

A substrate of the Rogers 3010 type, with dimensions of 25x15x0.835mm³, a dielectric constant $\varepsilon_r = 10.2$, and a loss tangent of 0.0035, is employed to observe its impact on filter performance. The structure comprises a microstrip line with a width of $w=0.6\,\text{mm}$ to achieve a characteristic impedance of 50 $\Omega$, and a ground plane where complementary split-ring resonators are etched. Using both circular and rectangular resonators, two configurations are presented.

![Diagram](image)

**Fig. 10.** Filter based on multiple co-directional CSRR: (a) Circular. (b) Rectangular

Table 3 displays the total length and optimized dimensions of the CSRRs for both configurations.
Table 3. The total length and optimized dimensions of the circular and rectangular CSRRs

<table>
<thead>
<tr>
<th>Fz (GHz)</th>
<th>L_Total (mm)</th>
<th>a (mm)</th>
<th>b (mm)</th>
<th>c (mm)</th>
<th>g (mm)</th>
<th>r (mm)</th>
<th>c (mm)</th>
<th>g (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>26.3</td>
<td>7.5</td>
<td>6.5</td>
<td>0.2</td>
<td>0.4</td>
<td>4.5</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>3.5</td>
<td>18.2</td>
<td>6.5</td>
<td>4</td>
<td>0.2</td>
<td>0.6</td>
<td>3.1</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>5.5</td>
<td>11.5</td>
<td>4.8</td>
<td>3.1</td>
<td>0.6</td>
<td>1.4</td>
<td>2.2</td>
<td>0.4</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Figure 11 depicts the reflection and transmission coefficients of the filter for configuration (a).

Fig. 11. The frequency response of the transmission and reflection coefficients corresponding to configuration (a).
Figure 12 depicts the reflection and transmission coefficients of the filter for configuration (b).

![Graph showing reflection and transmission coefficients](image)

**Fig. 12.** The frequency response of the transmission and reflection coefficients corresponding to configuration (b).

It is noticeable that the transmission coefficients of filters with an FR4-epoxy or Rogers 3010 substrate, whether with rectangular or circular CSRRs, are nearly identical. The use of the Rogers 3010 substrate has allowed for size reduction of the structure and improved rejection levels while preserving the desired filtering properties.

### 4 Comparison of performance

The proposed and designed filters are first-order and second-order filters that enable the creation of three rejection bands while maintaining smaller, simpler, and easier-to-implement structures. Additionally, the filtered bands exhibit an acceptable rejection level and can be easily controlled by adjusting the geometric parameters of the CSRRs.
Table 4. Comparison of the performance of the designed bandstop filters

<table>
<thead>
<tr>
<th>bandstop filters</th>
<th>filter order</th>
<th>rejection level</th>
<th>Frequency bands filtered at -10 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular multiple co-directional CSRR filter (FR4-epoxy)</td>
<td>1</td>
<td>-18.17 dB</td>
<td>2.28GHz-2.47GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-17.63 dB</td>
<td>3.13GHz-3.29GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-19.8 dB</td>
<td>4.30GHz-4.57GHz</td>
</tr>
<tr>
<td>Second-order circular multiple co-directional CSRR filter (FR4-epoxy)</td>
<td>2</td>
<td>-31.08 dB</td>
<td>2.17GHz-2.63GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-30.72 dB</td>
<td>2.97GHz-3.35GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-37.76 dB</td>
<td>4.15GHz-4.66GHz</td>
</tr>
<tr>
<td>Rectangular multiple co-directional CSRR filter (FR4-epoxy)</td>
<td>1</td>
<td>-14.69 dB</td>
<td>2.28GHz-2.44GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-16.90 dB</td>
<td>3.27GHz-3.48GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-18.30 dB</td>
<td>5.50GHz-5.86GHz</td>
</tr>
<tr>
<td>Second-order circular multiple co-directional CSRR filter (FR4-epoxy)</td>
<td>2</td>
<td>-36.29 dB</td>
<td>2.16GHz-2.48GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-33.97 dB</td>
<td>3.08GHz-3.54GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-30.11 dB</td>
<td>5.25GHz-5.80GHz</td>
</tr>
<tr>
<td>Filter with co-directional circular RAFC (Rogers)</td>
<td>2</td>
<td>-27.42 dB</td>
<td>2.11GHz-2.45GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-37.78 dB</td>
<td>2.92GHz-3.30GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-35.06 dB</td>
<td>4.08GHz-5.01GHz</td>
</tr>
<tr>
<td>Filter with co-directional rectangular RAFC (Rogers)</td>
<td>2</td>
<td>-40.40 dB</td>
<td>2.24GHz-2.51GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-36.56 dB</td>
<td>3.10GHz-3.49GHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-53.70 dB</td>
<td>5.11GHz-6.57GHz</td>
</tr>
</tbody>
</table>

5 Conclusion

The objective of this article is to develop new devices by harnessing the properties of metamaterial resonators, including split-ring resonators (SRRs) and their complementary counterpart, the complementary split-ring resonator (CSRR). These resonators have been applied to microwave circuits, particularly for implementing filtering functions. Indeed, bandstop filters have been studied and analyzed using multiple co-directional CSRRs to achieve a multitude of filtered bands. A Rogers substrate has been employed to miniaturize these filters, and the filter order plays a crucial role in controlling the desired frequency bands, whether for signal rejection or transmission. This control pertains to frequency selectivity, rejection level, or transmission level. In addition to their satisfactory performance, these designs show great promise in terms of compactness and ease of implementation.
References


