H_∞ control of a bilateral teleoperation system: application to a PV system

Fatima Zahra Darouiche[1,*], *El Houssaine* Tissir[1,**], and *Ghizlane* El Khaloufi[1,***]

[1]LISAC, Department of Physics, Faculty of Sciences Dhar El Mehraz, University of Sidi Mohamed Ben Abdellah, B.P. 1796, Fez-Atlas, Morocco

Abstract. This paper presents a novel approach for the H_∞ control of a bilateral teleoperation system by combining dilated Linear Matrix Inequalities (LMIs) and the Takagi-Sugeno (T-S) fuzzy approach. The proposed methodology aims to enhance the control performance and robustness of bilateral teleoperation systems by leveraging the advantages of both dilated LMIs and T-S fuzzy modeling. The proposed approach ensures robust stability by utilizing Lyapunov stability analysis. Robust H_∞ state feedback controllers are developed using both standard and dilated linear matrix inequalities to accommodate uncertainties inherent in the master/slave system. As an illustrative example, we enrich our approach by integrating a simulation model of a photovoltaic system. This extension demonstrates the versatility of our methodology by applying it to a distinct domain.

1 Introduction

Bilateral teleoperation systems have revolutionized the way humans interact with remote environments, enabling seamless control and manipulation tasks in diverse fields such as robotics, healthcare and hazardous environments[2–5]. These systems establish a connection between a human operator (master) and a remote robot or device (slave), enabling the operator to manipulate the slave and perceive haptic feedback.

One of the main challenges of two-way teleoperation is the modeling and control of uncertainty [6–8]. Fuzzy logic systems have emerged as a powerful tool for solving the difficulties associated with modeling and controlling such systems [9]. Fuzzy control allows the representation of imprecise and uncertain information through linguistic variables and rules, enabling adaptive and robust control in the face of uncertainty. The Takagi-Sugeno (T-S) fuzzy control framework, in particular, has been successfully applied to the modeling different processes such as finite frequency [10], active suspension systems [11, 14], vehicle lateral dynamics [12, 13], delayed systems [15] and 2D systems [16].

In recent years, there has been growing interest in integrating fuzzy control with bilateral teleoperation systems. This combination offers a flexible control approaches that can

*e-mail: fatimazahra.darouiche@usmba.ac.ma
**e-mail: elhoussaine.tissir@usmba.ac.ma
***e-mail: ghizlane.elkhaloufi@usmba.ac.ma

effectively handle uncertain dynamics. The T-S fuzzy control methodology provides a systematic framework for modeling and control design, allowing for the representation of teleoperation dynamics and the synthesis of robust control laws [17–19].

Moreover, the use of dilated Linear Matrix Inequalities (LMIs) has gained attention in the context of robust control design. Dilated LMIs extend the solution space of standard LMIs, offering less conservatism in controller synthesis. By incorporating dilated LMIs into the control design, the robustness and performance of bilateral teleoperation systems can be further enhanced [1].

This paper aims to contribute to the field of bilateral teleoperation by investigating the application of fuzzy control and dilated LMIs. The objective is to design a robust control strategy that ensures stable and accurate teleoperation, even in the presence of uncertainties. By leveraging the T-S fuzzy control approach and dilated LMIs, we aim to develop a control methodology that can address the challenges of uncertain dynamics and provide improved performance in bilateral teleoperation systems.
The key contributions of this work include the development of a T-S fuzzy model to capture the system dynamics, the design of a robust fuzzy controller using dilated LMIs to handle uncertainties, and the evaluation of the proposed approach through numerical simulations.
The remainder of this paper is organized as follows: The problem formulation and preliminaries are presented in Section 2. In Section 3, present Robust H_∞ controller with standard linear matrix inequalities while section 4 is devoted to the study of robust H_∞ controller with dilated linear matrix inequalities. In Section 5, an application example is given to evaluate the proposed approach. Finally, conclusion is presented in Section 6.

Nomenclature

The following notations will be used in this paper,

- $\mathbb{R}^i$ represents i-dimensional vector space.
- $\mathbb{R}^{p\times q}$ is a set of $p \times q$ dimensional real matrices.
- K^T stands for the transpose of the matrix K.
- The symmetric terms in a matrix are denoted by *.
- $He\{K\} = K + K^T$ stands for the transpose of the matrix K.
- $diag(.)$ denotes a diagonal matrix.
- I is an appropriately dimensioned identity matrix.
- $P > 0$ means that P is a positive-definite matrix.

2 Problem formulation and Preliminaries

Consider the bilateral teleoperation system shown in Figure 1[1]. The parameters of this model are defined in the following Table 1. The dynamic equations of the bilateral teleoperation system can be derived as follows [1];

$$\begin{cases} m_m\ddot{x}_m(t) + b_m\dot{x}_m(t) = u_m(t) + f_h(t), \\ m_s\ddot{x}_s(t) + b_s\dot{x}_s(t) = u_s(t) + f_e(t), \\ \\ f_h(t) = h_0(t) - k_h x_m(t) - c_h\dot{x}_m(t), \\ f_e(t) = k_e x_s(t) + b_e\dot{x}_s(t), \end{cases} \tag{1}$$

The tracking error between the reference signal and the master is;

$$e_{tm}(t) = f_h(t) - y_{tm}(t) \tag{2}$$

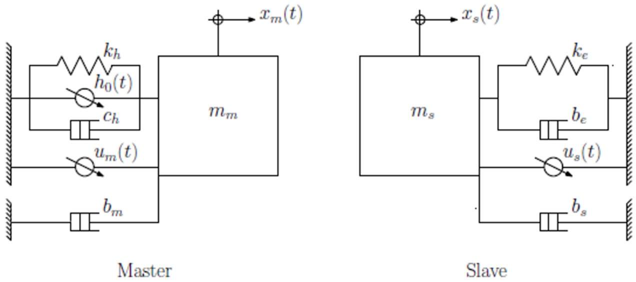

Figure 1. Bilateral teleoperation system.

Table 1. Bilateral Teleoperation System Parameters.

m_m	Mass of the master
m_s	Mass of the slave
$x_m(t)$	Horizontal displacement of the master
$x_s(t)$	Horizontal displacement of the slave
b_m	Coefficient of the master damping
b_s	Coefficient of the slave damping
c_h	Coefficient of damping and describe the dynamics of the human hand
k_h	Coefficient of stiffness and describe the dynamics of the human hand
$h_0(t)$	Reference input force
b_e	Coefficient of damping and describe the environmental-induced exogenous force while it is in-contact
k_e	Coefficient of stiffness and describe the environmental-induced exogenous force while it is in-contact
$u_m(t)$	Control force applied to the master
$u_s(t)$	Control force applied to the slave

The tracking error between the reference signal and the slave can be written as;

$$e_{ts}(t) = f_h(t) - y_{ts}(t) \tag{3}$$

Where;

$$\begin{cases} f_h(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \omega(t), \\ \\ y_{tm}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x_0(t), \\ y_{ts}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} x_0(t), \\ \\ x_0(t) = \begin{bmatrix} x_m(t) & \dot{x}_m(t) & x_s(t) & \dot{x}_s(t) \end{bmatrix}^T, \\ \omega(t) = \begin{bmatrix} f_h(t) & f_e(t) \end{bmatrix}^T, \\ u(t) = \begin{bmatrix} u_m(t) & u_s(t) \end{bmatrix}^T, \end{cases} \tag{4}$$

In the light of equation (1)-(4) and assuming that m_s and m_m are not exactly known, one can consider a state-space representation for the bilateral teleoperation system as;

$$\begin{cases} \dot{x}(t) = \tilde{M}x(t) + \tilde{N}u(t) + \tilde{D}\omega(t) \\ z(t) = Ex(t) + F\omega(t) \end{cases} \tag{5}$$

where $x(t) = \begin{bmatrix} x_m(t) & \dot{x}_m(t) & x_s(t) & \dot{x}_s(t) & \int e_{tm}(t) & \int e_{ts}(t) \end{bmatrix}^T$ is the state vector $\in \Re^n$, $u(t) \in \Re^m$ is the control input vector, $\omega(t) \in \Re^s$ is the exogenous input vector, $z(t) \in \Re^p$ is the set of controlled outputs that are selected as follows $z(t) = \begin{bmatrix} \frac{\int e_{tm}(t)dt}{|\int e_{tm}(t)dt|} & \frac{\int e_{ts}(t)dt}{|\int e_{ts}(t)dt|} & \frac{\int e_{\omega}(t)}{|\int e_{\omega}(t)|} \end{bmatrix}$, with $\int e_{tm}(t)dt$ is the integral tracking error between the reference signal and the master and $|\int e_{tm}(t)dt|$ denotes the estimated upper value of $\int e_{tm}(t)dt$. On the other hand, $\int e_{ts}(t)dt$ is an integral tracking error between the reference signal and the slave and $|\int e_{ts}(t)dt|$ is its estimated upper value. $e_{\omega}(t) = (f_h(t) - f_e(t))$ is the force error between the human operator and environmental-induced exogenous forces and $|\int e_{\omega}(t)|$ is the estimated upper value of $e_{\omega}(t)$, and;

$$\tilde{M} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{b_m}{m_m}(P_3 - 1) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{b_s}{m_s}(P_4 - 1) & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}, E = \begin{bmatrix} 0 & 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{N} = \begin{bmatrix} 0 & 0 \\ \frac{1}{m_m}(P_1 + 1) & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_s}(P_2 + 1) \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \tilde{D} = \begin{bmatrix} 0 & 0 \\ \frac{1}{m_m}(P_1 + 1) & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_s}(P_2 - 1) \\ 1 & 0 \\ 1 & 0 \end{bmatrix},$$

$F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \beta & -\beta \end{bmatrix}$ with P_1, P_2, P_3 and P_4 are maximum parameter uncertainty bounds of $\frac{1}{m_m}, \frac{1}{m_s}, \frac{b_m}{m_m}$ and $\frac{b_s}{m_s}$ respectively.

Using the approach in [24] consider $m_s(t)$ and $m_m(t)$, which are varying in given range: $m_s(t) \in [m_{s_{min}}, m_{s_{max}}]$ and $m_m(t) \in [m_{m_{min}}, m_{m_{max}}]$. This means that the uncertain mass $m_s(t)$ is delimited by its minimum value $m_{s_{min}}$ and its maximum value $m_{s_{max}}$. In addition, the uncertain mass $m_m(t)$ is delimited by its minimum value $m_{m_{min}}$ and its maximum value $m_{m_{max}}$. Then we get the values of $\frac{1}{m_s(t)}$ and $\frac{1}{m_m(t)}$ from $m_s(t) \in [m_{s_{min}}, m_{s_{max}}]$ and $m_m(t) \in [m_{m_{min}}, m_{m_{max}}]$ as follows;

$$max\frac{1}{m_s(t)} = \frac{1}{m_{s_{min}}} = \hat{m}_s \qquad min\frac{1}{m_s(t)} = \frac{1}{m_{s_{max}}} = \check{m}_s$$

$$max\frac{1}{m_m(t)} = \frac{1}{m_{m_{min}}} = \hat{m}_s \qquad min\frac{1}{m_m(t)} = \frac{1}{m_{m_{max}}} = \check{m}_m$$

We can represent the expressions $\frac{1}{m_s(t)}$ and $\frac{1}{m_m(t)}$ by

$$\frac{1}{m_s(t)} = \hat{P}_1(\phi_1(t))\hat{m}_s + \hat{P}_2(\phi_1(t))\check{m}_s;$$

$$\frac{1}{m_m(t)} = \hat{Q}_1(\phi_2(t))\hat{m}_m + \hat{Q}_2(\phi_2(t))\check{m}_m;$$

where $\phi_1(t) = \dfrac{1}{m_s(t)}$; $\phi_2(t) = \dfrac{1}{m_m(t)}$; $\hat{P}_1(\phi_1(t)) + \hat{P}_2(\phi_1(t)) = 1$ and $\hat{Q}_2(\phi_2(t)) + \hat{Q}_1(\phi_2(t)) = 1$.
The membership functions can be calculated as;

$$\hat{P}_1(\phi_1(t)) = \frac{\dfrac{1}{m_s(t)} - \check{m}_s}{\hat{m}_s - \check{m}_s};\ \hat{P}_2(\phi_1(t)) = \frac{\hat{m}_s - \dfrac{1}{m_s(t)}}{\hat{m}_s - \check{m}_s}$$

$$\hat{Q}_1(\phi_2(t)) = \frac{\dfrac{1}{m_m(t)} - \check{m}_m}{\hat{m}_m - \check{m}_m};\ \hat{Q}_2(\phi_2(t)) = \frac{\hat{m}_m - \dfrac{1}{m_m(t)}}{\hat{m}_m - \check{m}_m}$$

The membership functions shown in Fig. 2 are labelled by Heavy and Light. We can

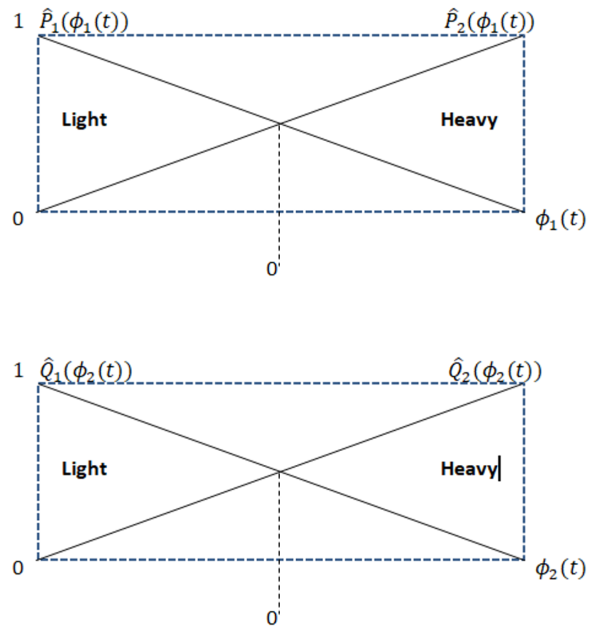

Figure 2. Membership functions $\hat{P}_1(\phi_1(t))$ and $\hat{P}_2(\phi_1(t))$ and membership functions $\hat{Q}_1(\phi_2(t))$ and $\hat{Q}_2(\phi_2(t))$ respectively.

represent the system (5) by the following fuzzy models;

Model Rule 1 : IF $\phi_1(t)$ is Heavy and $\phi_2(t)$ is Heavy, THEN

$$\begin{cases} \dot{x}(t) = \tilde{M}_1 x(t) + \tilde{N}_1 u(t) + \tilde{D}_1 \omega(t) \\ z(t) = Ex(t) + F\omega(t) \end{cases}$$

where the matrices $\tilde{M}_1, \tilde{N}_1$ and $\tilde{D}_1$ are obtained by replacing $\dfrac{1}{m_s(t)}$ and $\dfrac{1}{m_m(t)}$ in the matrices $\tilde{M}, \tilde{N}$ and $\tilde{D}$ with $\hat{m}_s$ and $\hat{m}_m$ respectively.

Model Rule 2 : IF $\phi_1(t)$ is Heavy and $\phi_2(t)$ is Light, THEN

$$\begin{cases} \dot{x}(t) = \tilde{M}_2 x(t) + \tilde{N}_2 u(t) + \tilde{D}_2 \omega(t) \\ z(t) = Ex(t) + F\omega(t) \end{cases}$$

where the matrices $\tilde{M}_2, \tilde{N}_2$ and $\tilde{D}_2$ are obtained by replacing $\frac{1}{m_s(t)}$ and $\frac{1}{m_m(t)}$ in the matrices $\tilde{M}, \tilde{N}$ and $\tilde{D}$ with $\hat{m}_s$ and $\check{m}_m$ respectively.

Model Rule 3 : IF $\phi_1(t)$ is Light and $\phi_2(t)$ is Heavy, THEN

$$\begin{cases} \dot{x}(t) = \tilde{M}_3 x(t) + \tilde{N}_3 u(t) + \tilde{D}_3 \omega(t) \\ z(t) = Cx(t) + D\omega(t) \end{cases}$$

where the matrices $\tilde{M}_3, \tilde{N}_3$ and $\tilde{D}_3$ are obtained by replacing $\frac{1}{m_s(t)}$ and $\frac{1}{m_m(t)}$ in the matrices $\tilde{M}, \tilde{N}$ and $\tilde{D}$ with $\check{m}_s$ and $\hat{m}_m$ respectively.

Model Rule 4 : IF $\phi_1(t)$ is Light and $\phi_2(t)$ is Light, THEN

$$\begin{cases} \dot{x}(t) = \tilde{M}_4 x(t) + \tilde{N}_4 u(t) + \tilde{D}_4 \omega(t) \\ z(t) = Ex(t) + F\omega(t) \end{cases}$$

where the matrices $\tilde{M}_4, \tilde{N}_4$ and $\tilde{D}_4$ are obtained by replacing $\frac{1}{m_s(t)}$ and $\frac{1}{m_m(t)}$ in the matrices $\tilde{M}, \tilde{N}$ and $\tilde{D}$ with $\check{m}_s$ and $\check{m}_m$ respectively.
Then, by fuzzy mixing, the global fuzzy model is deduced as follows;

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{4} h_i(\phi(t)) \left[\tilde{M}_i x(t) + \tilde{N}_i u(t) + \tilde{D}_i \omega(t)\right] \\ z(t) = Ex(t) + F\omega(t) \end{cases} \tag{6}$$

where $h_1(\phi(t)) = \hat{P}_1(\phi_1(t)) \times \hat{Q}_1(\phi_2(t)), h_2(\phi(t)) = \hat{P}_1(\phi_1(t)) \times \hat{Q}_2(\phi_2(t)),$

$h_3(\phi(t)) = \hat{P}_2(\phi_1(t)) \times \hat{Q}_1(\phi_2(t)), h_4(\phi(t)) = \hat{P}_2(\phi_1(t)) \times \hat{Q}_2(\phi_2(t)),$

Let the matrices;

$$\begin{bmatrix}\tilde{M} & \tilde{N} & \tilde{D}\end{bmatrix} = \sum_{i=1}^{4} h_i(\phi(t)) \begin{bmatrix}\tilde{M}_i & \tilde{N}_i & \tilde{D}_i\end{bmatrix}$$

It is obvious that the fuzzy weighting functions $h_i(\phi(t))$ satisfy $h_i(\phi(t)) \geq 0$ and $\sum_{i=1}^{4} h_i(\phi(t)) = 1$

The state feedback control is inferred as;

$$u(t) = Rx(t) \tag{7}$$

By substituting (7) into (6), the closed-loop fuzzy system can be represented as;

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{4} h_i(\phi(t)) \left[\left(\tilde{M}_i + \tilde{N}_i R\right) x(t) + \tilde{D}_i \omega(t)\right] \\ z(t) = Ex(t) + F\omega(t) \end{cases} \tag{8}$$

3 Robust H_∞ controller with standard linear matrix inequalities

In this Section, we will design the static output feedback (SOF) H_∞ control for the T-S fuzzy system.

Theorem 1 *Consider the closed-loop fuzzy system* (8)*, and a scalar* $\gamma > 0$*, the system is asymptotically stable with the* H_∞ *performance* $\gamma > 0$*, if there exist symmetric matrix* $Q = Q^T > 0$ *and a matrix* S *such that the following LMIs hold.*

$$\begin{bmatrix} Q\tilde{M}_i^T + \tilde{M}_i Q + S^T \tilde{N}_i^T + \tilde{N}_i S & \tilde{D}_i & QE^T \\ * & -\gamma I & F^T \\ * & * & -\gamma I \end{bmatrix} < 0, (i = 1, ..., 4) \tag{9}$$

The controller's gain is given by $R = SQ^{-1}$.

Proof 1 *A bounded real-lemma [22] can be written for the closed-loop fuzzy system* (8) *as follows ;*

$$\dot{V} + \frac{1}{\gamma} z^T z - \gamma \omega^T \omega < 0 \tag{10}$$

where $V(t) = x^T(t)Px(t)$*, if there exist matrice* $P = P^T > 0$*, we obtain;*

$$\begin{bmatrix} (\tilde{M}_i + \tilde{N}_i R)^T P + P(\tilde{M}_i + \tilde{N}_i R) + \gamma^{-1} E^T E & P\tilde{D}_i + \gamma^{-1} E^T F \\ * & \gamma^{-1} F^T F - \gamma I \end{bmatrix} < 0 \tag{11}$$

the H_∞ *norm of* (8) *from its input* ω *to output* z *is less than* γ.
Applying Schur complement on inequality (11) *and defining* $Q = P^{-1}$, $S = RQ$*, and pre- and post- multiplying* (11) *by* $diag\{Q, I, I\}$ *and its transpose gives inequality* (9)*, which ends the proof.*

4 Robust H_∞ controller with dilated linear matrix inequalities

The following lemma is essential for the development of controllers based on dilated LMIs.

Lemma 1 *([21])*
Given a symmetric matrix T *and two matrices* H *and* V *of appropriate dimensions whose range spaces are independent, there exists an unstructured matrix* U *that satisfies;*

$$He\left\{H^T U V\right\} + T < 0 \tag{12}$$

if and only if the following projection inequalities with respect to U *are satisfied;*

$$N_H^T T N_H < 0 \tag{13}$$

$$N_v^T T N_v < 0 \tag{14}$$

where N_H *and* N_v *are arbitrary matrices whose columns form a basis of the nullspaces of* H *and* V *respectively.*

Theorem 2 *The closed-loop fuzzy system* (8)*, and a scalar* $\gamma > 0$*, the system is asymptotically stable with the* H_∞ *performance* $\gamma > 0$*, if there exist scalar* $\lambda > 0$*, symmetric matrix* $K = K^T > 0$ *and matrices* Y*,* L *such that the following LMIs hold;*

$$\begin{bmatrix} \Pi & -Y^T \tilde{M}_i^T - L^T \tilde{N}_i^T + \lambda Y + K & -\lambda \tilde{D}_i & Y^T E^T \\ * & Y^T + Y & -\tilde{D}_i & 0 \\ * & * & -\gamma^2 I & F^T \\ * & * & * & -I \end{bmatrix} < 0, (i = 1, ..., 4) \tag{15}$$

where $\Pi = -\lambda Y^T \tilde{M}_i^T - \lambda \tilde{M}_i Y - \lambda L^T \tilde{N}_i^T - \lambda \tilde{N}_i L$ *and the controller's gain is given by* $R = LY^{-1}$.

Proof 2 *for all $t \geq 0$, it must be satisfied that,*

$$J(t) = \dot{V} + z^T z - \gamma^2 \omega^T \omega < 0, \tag{16}$$

Choosing the Lyapunov-Krasvskii functional as;

$$V(t) = x^T(t) P x(t),$$

where $P = P^T > 0$. using (8) *and* (16)*, we have;*

$$\begin{aligned} J(t) &= \sum_{i=1}^{4} h_i(\phi(t)) \left\{ x^T(t) \left[P\tilde{M}_i + \tilde{M}_i^T P + P\tilde{N}_i R + R^T \tilde{N}_i^T P \right] x(t) \right. \\ &\quad \left. + 2x^T(t) P \tilde{D}_i \omega(t) \right\} + \left[Ex(t) + F\omega(t) \right]^T \left[Ex(t) + F\omega(t) \right] - \gamma^2 \omega^T(t)\omega(t) \\ &= \sum_{i=1}^{4} h_i \Phi^T \xi^T \psi \xi \Phi < 0 \end{aligned} \tag{17}$$

where $\Phi = \begin{bmatrix} x^T(t) & \omega(t) \end{bmatrix}$; $\xi = \begin{bmatrix} I & 0 \\ \tilde{M}_i + \tilde{N}_i R & \tilde{D}_i \\ 0 & I \\ E & F \end{bmatrix}$; $\psi = \begin{bmatrix} 0 & P & 0 & 0 \\ P & 0 & 0 & 0 \\ 0 & 0 & -\gamma^2 I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$ *and* $h_i = h_i(\phi(t))$.

we can rewrite equation (17) *as follows;*

$$\sum_{i=1}^{4} h_i \Phi^T \begin{bmatrix} I & 0 \\ \tilde{M}_i + \tilde{N}_i R & \tilde{D}_i \\ 0 & I \end{bmatrix}^T \begin{bmatrix} E^T E & P & E^T F \\ P & 0 & 0 \\ F^T E & 0 & F^T F - \gamma^2 I \end{bmatrix} \begin{bmatrix} I & 0 \\ \tilde{M}_i + \tilde{N}_i R & \tilde{D}_i \\ 0 & I \end{bmatrix} \Phi < 0$$

Define

$$N_H = \begin{bmatrix} I & 0 \\ \tilde{M}_i + \tilde{N}_i R & \tilde{D}_i \\ 0 & I \end{bmatrix}; \; H = \begin{bmatrix} -\tilde{M}_i - \tilde{N}_i R & I & -\tilde{D}_i \end{bmatrix}; \; T = \begin{bmatrix} E^T E & P & E^T F \\ P & 0 & 0 \\ F^T E & 0 & F^T F - \gamma^2 I \end{bmatrix}.$$

This implies that the closed-loop fuzzy system is asymptotically stable.

By choosing $N_v = \begin{bmatrix} I & -\lambda I & 0 \\ 0 & 0 & I \end{bmatrix}^T$ *and* $V = \begin{bmatrix} \lambda I & I & 0 \end{bmatrix}$*, which verifies condition* (14) *of lemma 1, we can deduce that;*

$$He \left\{ \begin{bmatrix} \lambda I \\ I \\ 0 \end{bmatrix} U \begin{bmatrix} -\tilde{M}_i - \tilde{N}_i R & I & -\tilde{D}_i \end{bmatrix} \right\} + \begin{bmatrix} E^T E & P & E^T F \\ P & 0 & 0 \\ F^T E & 0 & F^T F - \gamma^2 I \end{bmatrix} < 0 \tag{18}$$

Defining $Y = U^{-1}$, $K = Y^T P Y$, pre- and post- multiplying (18) *by* $diag\left\{ Y^T, Y^T, I \right\}$ *and its transpose, using the definition $L = RY$ and by applying Schur Complement, we can obtain the inequality* (15) *with Y invertible matrix. This concludes the proof.*

Remark 1 *According to Theorem 2, the controller design incorporates the use of an auxiliary variable instead of a symmetric Lyapunov matrix. This approach expands the solution space of Linear Matrix Inequalities (LMIs), leading to less conservative controller synthesis compared to standard techniques. By utilizing dilated LMIs, we can achieve a broader range of feasible solutions and reduce conservatism in the controller design process.*

Remark 2 *It is important to highlight that the proposed control strategy represents a novel approach to controlling bilateral teleoperation systems using dilated LMIs. By applying an optimization procedure that minimizes γ, we can obtain the performance measure of H_∞ norm. This allows us to assess and optimize the robustness and disturbance rejection capabilities of the system. The utilization of dilated LMIs in this context provides a promising avenue for achieving improved control performance in bilateral teleoperation.*

5 Application to a PV system

In this example, we will attempt to demonstrate the effectiveness of our study by adopting a real photovoltaic system model, whose specifications are listed in Table 2.

Table 2. PV panel module: 1Soltech 1STH-FRL-4H-250-M60- BLK.

Specifications	Values
Cells per module (Ncell)	60
Series parallel	1
Maximum power	248.977 W
Voltage at maximum power point	30.7 V
Current at maximum power point	8.11 A
Open circuit voltage	38.4 V
Short-circuit current	8.85 A

Consider the following T-S PV model developed in [25] and described as;

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{4} h_i(\phi(t))\left[Ax(t) + B_i u(t) + D\omega(t)\right] \\ \\ y(t) = Ex(t) \end{cases} \tag{19}$$

where $A \in \Re^{n_a \times n_a}$, $B_i \in \Re^{n_a \times m_b}$, $D \in \Re^{n_a \times \omega_d}$ and $E \in \Re_{n_y \times n_a}$ are constant matrices with

$$A = \begin{bmatrix} \frac{-R_L}{L} & -\frac{1}{L} & \frac{1}{L} \\ \frac{1}{C_2} & -\frac{1}{R_{C2}} & 0 \\ \frac{-1}{C_1} & 0 & 0 \end{bmatrix},\ D = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{C_1} \end{bmatrix},\ B_1 = \begin{bmatrix} \frac{V_{C2min}}{L} \\ -\frac{I_{Lmin}}{C_2} \\ 0 \end{bmatrix},\ B_2 = \begin{bmatrix} \frac{V_{C2min}}{L} \\ -\frac{I_{Lmax}}{C_2} \\ 0 \end{bmatrix},\ B_3 = \begin{bmatrix} \frac{V_{C2max}}{L} \\ -\frac{I_{Lmin}}{C_2} \\ 0 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} \frac{V_{C2max}}{L} \\ -\frac{I_{Lmax}}{C_2} \\ 0 \end{bmatrix},\ E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } x(t) = \begin{bmatrix} I_L(t) \\ V_{C2}(t) \\ V_{PV}(t) \end{bmatrix},\ \omega(t) = I_{PV}(t)$$

we choose the state feedback control as $u(t) = Rx(t)$.

Considering the parameter mentioned in [25]; $L = 10mH$, $R_L = 0.01\Omega$, $C_1 = 500\mu F$, $C_2 = 100\mu F$ and $R = 20\Omega$ and by using our Theorem 2 we find

$$R = \begin{bmatrix} -0.1447 & 0.0028 & 0.0021 \end{bmatrix}$$

for the same parameters, we obtain the following figures.
Figure 3 shows the PV power response and figure 4 the PV voltage response.

Figures 3 and 4 show that both the PV power response and the PV voltage response

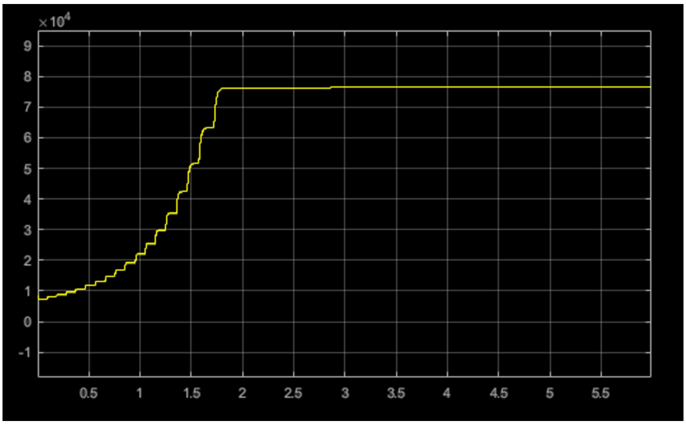

Figure 3. The power of PV.

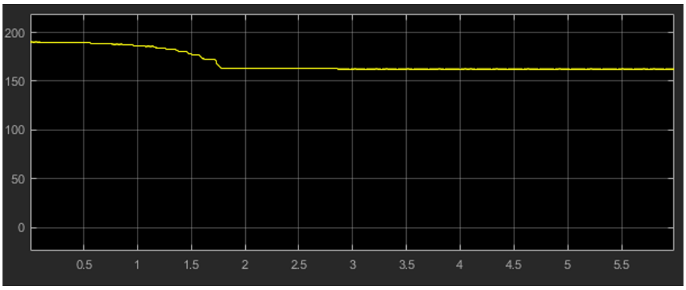

Figure 4. The voltage of PV.

converge precisely to the desired value. This result validates the effectiveness of the proposed approach.

6 Conclusion

In this study, we have proposed a novel approach for the robust H_∞ control of bilateral teleoperation systems, combining dilated linear matrix inequalities (LMIs) and the Takagi-Sugeno (T-S) fuzzy control approach. Our aim was to achieve improved control performance and robustness in the presence of uncertainties.
By representing the bilateral teleoperation system using the T-S fuzzy control model and employing Lyapunov stability analysis, we ensured asymptotic stability. The design of robust H_∞ state feedback controllers based on both standard and dilated LMIs effectively handled uncertainties in the master/slave system.

In a practical example, we extended our approach by adding a simulation model of a photovoltaic system. This model enabled us to demonstrate how our approach can be adapted to other fields. By integrating this photovoltaic model, we illustrated the flexibility of our methodology and its ability to be applied to a variety of complex systems.

References

[1] B. Gormus and H. Yazici and İ. B. Küçükdemiral, Robust H_∞ control of an uncertain bilateral teleoperation system using dilated LMIs, *Transactions of the Institute of Measurement and Control* **44**(6), 1275-1287 (2022).

[2] Y. Sung Min, K. Won Jae and L. Min Cheol, Design of bilateral control for force feedback in surgical robot, *International Journal of Control, Automation and Systems* **13**(4), 916-925 (2015).

[3] C. Passenberg and A. Peer and M. Buss, A survey of environment-, operator-, and task-adapted controllers for teleoperation systems, *Mechatronics* **20**(7), 787-801 (2010).

[4] H. Shuang, H. Lingyan and L. Peter X, Sliding mode control for a surgical teleoperation system via a disturbance observer, *IEEE Access* **7**, 43383-43393 (2019).

[5] P. Yong, C. Chengjun, L. Dongnian and Z. Zhengxu and H. Jun, Augmented reality-based robot teleoperation system using RGB-D imaging and attitude teaching device, *Robotics and Computer-Integrated Manufacturing* **71**, 102167 (2021).

[6] C. Ho and B. Wayne J, Internet-based bilateral teleoperation based on wave variable with adaptive predictor and direct drift control, *Robotics and Computer-Integrated Manufacturing* 86-93 (2006).

[7] A. Alireza, B. Ali, Y. Mahdi and T. Heidar Ali, Design and implementation of robust-fixed structure controller for telerobotic systems, *Journal of Intelligent & Robotic Systems* **83**, 253-269 (2016).

[8] A. Alireza, K. Alireza and L. Ali, Swarm-based structure-specified controller design for bilateral transparent teleoperation systems via μ synthesis, *IMA Journal of Mathematical control and Information* **31**(1), 111-136 (2014).

[9] M. Sugeno and T. Takagi, Fuzzy identification of systems and its applications to modelling and control, *Readings in Fuzzy Sets for Intelligent Systems* **15**(1), 387-403 (1993).

[10] I. Er-Rachid, R. Chaibi, E.H. Tissir and A. Hmamed, Robust finite frequency static output feedback control for uncertain fuzzy systems via a sum-of-squares approach, *Optimal Control Applications and Methods* **43**(1), 158-177 (2022).

[11] J. Mrazgua, R. Chaibi, E.H. Tissir and M. Ouahi, Static output feedback stabilization of T-S fuzzy active suspension systems, *Journal of Terramechanics* **97**, 19-27 (2021).

[12] R. Alika, E.M. Mellouli, and E.H. Tissir, Adaptive higher-order sliding mode control based fuzzy logic T-S for lateral dynamics of autonomous vehicles, *12th International Conference on Information and Communication Systems (ICICS)* 358-363 (2021).

[13] N. El Youssfi and R. El Bachtiri and R. Chaibi and E.H Tissir, Static output-feedback H_∞ control for T-S fuzzy vehicle lateral dynamics, *SN Applied Sciences* **2**(1), 101 (2020).

[14] J. Mrazgua, E.H. Tissir and M. Ouahi, Fault-tolerant H_∞ control approach, application to active half-vehicle suspension systems with actuators failure accounts, *8th International Conference on Systems and Control (ICSC)* 271-276 (2019).

[15] S. Idrissi, Y. El Fezazi, N. El Fezazi and E.H. Tissir, Further Improvements on Robust Stability Analysis for Takagi-Sugeno Fuzzy Systems with Time-Varying Delay, *Int. J. Automation and Control* **1**, 1-10 (2023).

[16] A. El Hajjaji, B. Bensalem, A. El Amrani and I. Boumhidi, H_∞ Model Reduction of 2D Discrete. time TS Fuzzy Systems, *International journal of autonomous and adaptive communications systems* **16**(5), 1 (2023).

[17] Y. Bouteraa and K.A. Alattas and T. Peng and A. Fekih and R. Rahmani and S. Mobayen, Design of robust adaptive fuzzy control for uncertain bilateral teleoperation systems based on backstepping approach, *IET Control Theory & Applications* **17**(7), 800-813 (2023).

[18] S. Da, L. Qianfang and R. Hongliang, Type-2 fuzzy modeling and control for bilateral teleoperation system with dynamic uncertainties and time-varying delays, *IEEE Transactions on Industrial Electronics* **65**(1), 447-459 (2017).

[19] P. M. Kebria, A. Khosravi, S. Nahavandi, D. Wu and F. Bello, Adaptive type-2 fuzzy neural-network control for teleoperation systems with delay and uncertainties, *IEEE Transactions on Fuzzy Systems* **28**(10), 2543-2554 (2019).

[20] A. Aktas, H. Yazici and M. Sever, LMI-based design of an I-PD+PD type LPV state feedback controller for a gantry crane, *Transactions of the Institute of Measurement and Control* **41**(6), 1640-1655 (2019).

[21] P. Gahinet and P. Apkarian, A linear matrix inequality approach to H_∞ control, *International journal of robust and nonlinear control* **4**(4), 421-448 (1994).

[22] S. Boyd and L. El Ghaoui and E. Feron and V. Balakrishnan, Linear matrix inequalities in system and control theory *Adaptive Signal Processing* (1994).

[23] G. Pipeleers, B. Demeulenaere, J. Swevers and L. Vandenberghe, Extended LMI characterizations for stability and performance of linear systems, *Systems & Control Letters* **58**(7), 510-518 (2009).

[24] J. Mrazgua and E.H Tissir and M. Ouahi, Fuzzy fault-tolerant H_∞ control approach for nonlinear active suspension systems with actuator failure, *Procedia Computer Science* **148**, 465-474 (2019).

[25] H. El Aiss, K.A. Barbosa, H. Chavez and C. Rodriguez, Takagi–Sugeno State Delayed Feedback and Integral Control for PV Systems: Modeling, Simulation, and Control, *International Journal of Photoenergy* 1-14 (2021).