Modeling of natural convection by lattice Boltzmann in a partially heated cavity

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Abstract. This study introduces the lattice Boltzmann method for modeling natural convection in a partially heated square cavity filled with air for varying Rayleigh numbers. A temperature gradient affects the vertical walls, whereas the other walls are treated as adiabatic. The velocity and temperature fields are determined using the double population model of Lattice Boltzmann Method (LBM), which uses two different distribution functions one for the velocity field and another for the temperature field. The results obtained are showcased through streamlines, isotherms, velocity, and temperature profiles, these results align well with existing literature.

1 Introduction

Heat transfer in enclosed cavities by natural convection is a subject that arouses considerable interest in the industry, due to its many applications such as thermal design in the field of buildings, furnaces, nuclear reactors, solar energy collectors, etc. Natural convection in cavities, which can take various forms subjected to different boundary conditions, has been extensively studied. Numerous researchers have investigated and modeled natural convection, both turbulent and laminar, employing numerical techniques including the Lattice Boltzmann Method (LBM), Finite Volume Method (FVM), and Finite Difference Method (FDM). The LBM was introduced in the late 1980s by Mac Namara and Zanetti [1]. Its origins are inspired by lattice gas cellular automata (LGCA) and the kinetic theory of gases. This method tends to describe the behavior of fluid molecules at the mesoscopic scale using the distribution function of the fluid particles. Macroscopic properties, density, temperature, and velocity vector can be calculated from the distribution functions means [2-4]. According to researchers who have utilized this, we find Mohamed [5] who compared results obtained by the LBM and the FDM methods. Similarly, convection flow in a square cavity with high Rayleigh numbers was simulated using LBM by Dixit et al. [6]. They conducted a thorough comparison and validation of both laminar and turbulent convection types, using an interpolation method to supplement the Lattice Boltzmann approach (ISLB). Additionally, Abouricha et al. [7] utilized LBM to model turbulent natural convection in a square cavity that was partially heated and cooled, across a range of Rayleigh number values spanning from 5x106 to 108.

The goal of our contribution is to investigate natural convection within an enclosed space filled with air and partially heated. To achieve this goal, we have employed the double population Lattice Boltzmann Method (LBM), which employs two distribution functions, to calculate density/velocity and temperature fields. We have to evaluate the Rayleigh (Ra) number's influence on streamlines, temperature, as well as the velocity and temperature profiles along the cavity's horizontal midplane.

2 Problem statement

In this study, the numerical exploration of natural convection inside a two-dimensional, partially heated square cavity filled with air (which is considered Newtonian, incompressible fluid with a Prandtl number value Pr = 0.71) is carried out. Whereas the other walls and chips are thought to be adiabatic, a thermal gradient is applied to the centered portions of the vertical walls.

Boundary conditions associated with the problem under investigation are as follows:

- Dynamic boundary conditions

The bounce-back boundary condition is utilized to ensure that a zero-velocity vector is achieved on all walls within the cavity: $U = V = 0$

- Thermal boundary conditions

In the hot part: Tc = 1
In the cold part: Tf = 0
On the other parts and walls, adiabatic conditions are applied: $\frac{\partial \theta}{\partial n} = 0$, where 'n' represents the direction perpendicular to the adiabatic parts and walls considered.

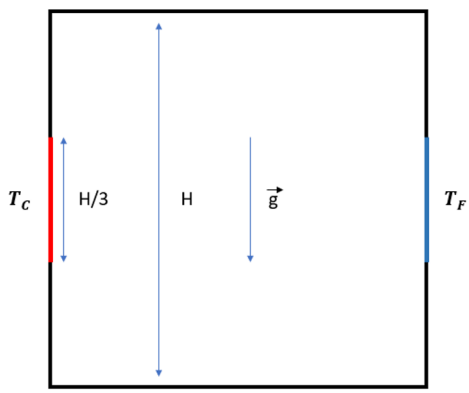

Fig. 1. Problem configuration.

2.1. LBM method

In this problem, we employed two distribution functions, f(x, t) and g(x, t): the first one for simulating the moving field, and the second one for simulating the thermal field. To achieve

this objective, we adopted a model of the lattice Boltzmann Method in two dimensions with nine discrete velocities, known as the D2Q9 model, utilizing a square grid with a consistent geometric increment of Δx = Δy = 1 [8].

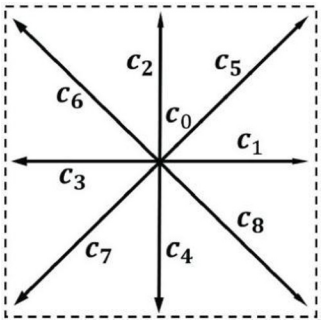

Fig. 2. Model D2Q9.

Fluid particles travel in the direction of k from one grid node to the next, moving at discrete velocities determined by:

$$c_k = \begin{cases} (0\,,\ 0\,), & k = 0 \\ (\pm 1\,, 0\,)\,;\ (0\,, \pm 1\,), & k = 1 - 4 \\ (\pm 1\,, \pm 1\,), & k = 5 - 8 \end{cases} \tag{1}$$

The equations governing the dual population according to the BGK model in the lattice Boltzmann method are as follows:

$$f_k(\mathbf{x} + \mathbf{c}_k \Delta t\,, t + \Delta t) = f_k(\mathbf{x}\,, t)(1 - w_m) + w_m f_k^{eq}(\mathbf{x}\,, t) + \Delta t\, \mathbf{F}_k, \tag{2}$$

$$g_k(\mathbf{x} + c_k \Delta t\,, t + \Delta t) = g_k(\mathbf{x}\,, t)(1 - w_s) + w_s g_k^{eq}(\mathbf{x}\,, t), \tag{3}$$

with $\mathbf{w_s} = \frac{\Delta t}{\tau_s} = \frac{1}{3\alpha + 0.5}$, $\mathbf{w_m} = \frac{\Delta t}{\tau_m} = \frac{1}{3\vartheta + 0.5}$, where $\boldsymbol{\tau_s}$ and $\boldsymbol{\tau_m}$ are respectively the relaxation times for the thermal and dynamic fields; α and ϑ are thermal diffusivity and kinematic viscosity, respectively.

Each of the expressions takes into account the collision and particle propagation steps as well as the equilibrium distribution functions, where:

$$f_k^{eq}(\mathbf{x}, t) = \omega_k\, \rho(\mathbf{x}, t) \left(1 + 3\frac{\mathbf{c}_k \cdot \mathbf{u}}{c^2} + \frac{9}{2}\frac{(\mathbf{c}_k \cdot \mathbf{u})^2}{c^4} - \frac{3}{2}\frac{u^2}{c^2}\right), \tag{4}$$

$$g_k^{eq} = \omega_k T(\mathbf{x}, t) \left[1 + 3\frac{\mathbf{c}_k \cdot u}{c^2}\right], \tag{5}$$

where the lattice velocity magnitude $c = \frac{\Delta x}{\Delta t} = \frac{\Delta y}{\Delta t}$ and $\Delta t = 1$ is the time step.
$\mathbf{F}_k$ corresponds to an external forces field provided by:

$$\mathbf{F}_k = 3\, \omega_k \frac{\rho\, \beta\, g_r\, \Delta T\, \mathbf{c}_k}{c^2} \tag{6}$$

The parameters involved include density (ρ), temperature differential (ΔT), coefficient of thermal expansion (β), and gravitational field strength (gr). While ω_k is called the nodal weights given by:

$$\omega_0 = \frac{4}{9}, \omega_{1-4} = \frac{1}{9}, and\ \omega_{5-8} = \frac{1}{36}.$$

To calculate the macroscopic quantities: temperature T, velocity vector **u** (U, V), and mass density ρ we utilize the following summations:

$$\rho(\mathbf{x},t) = \sum_0^8 f_k(\mathbf{x},t) \quad (7)$$

$$\rho\, u(\mathbf{x},t) = \sum_0^8 \mathbf{c}_k\, f_k(\mathbf{x},t) \quad (8)$$

$$T(\mathbf{x},t) = \sum_0^8 g_k(\mathbf{x},t) \quad (9)$$

2.2. Nusselt number

The amount of heat exchange within the enclosure, whether by convection or conduction, is determined using the Nusselt number, which is calculated using the following expression [9]:

$$\overline{Nu} = \frac{1}{H}\int_0^H \left.\frac{\partial T}{\partial X}\right|_{y=0} dY$$

3 Validation

The model was confirmed in the case of a differentially heated square cavity filled with air. The table offers a comparison of the average Nusselt number values calculated through our method (LBM) with those found in the literature. These results illustrate a good agreement between the values, where the relative error of the average Nusselt number is minimal [10, 11].

Table 1. Values of $\overline{\boldsymbol{Nu}}$ for Ra = 105 and 106.

	Ra =10^5	Ra = 10^6
Present work	4.564	8.684
De Vahl Davis [10]	4.519	8.800
Error	0.995 %	1.310 %
Kuznik et al. [11]	4.518	8.803
Error	1.018 %	1.350 %

For the physical validation, we observed that our method successfully reproduced the flow effects (streamlines and isotherms) described in the literature. Consequently, the interpretation of the physical phenomenon is consistent with the description provided in the literature [11].

4 Results and discussion

The results presented come from simulations carried out with variations in the Rayleigh number, ranging from 10^3 to 10^6.

4.1. Stream lines

The contours of streamlines shown in the following figures represent how the fluid circulates in the square cavity:

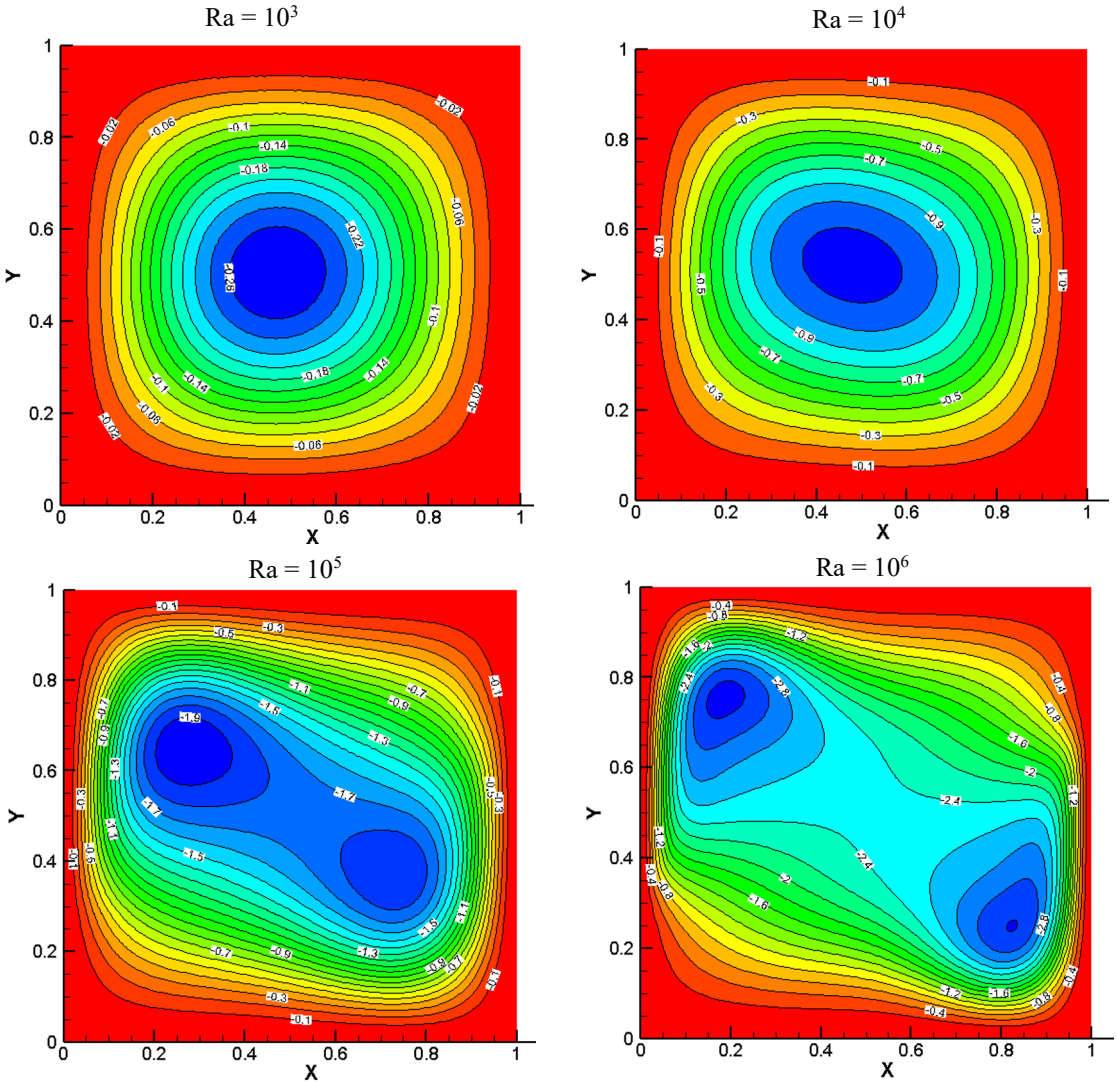

Fig. 3. Stream functions for various Rayleigh numbers.

With a Rayleigh number equal to Ra = 10^3, a stagnant spherical area appears in the middle of the cavity, indicating relatively weak convection.
For Ra = 10^4, the previously stagnant circular area in the center of the cavity evolves into an oval-shaped reflow zone, indicating the establishment of convection. When the number of Rayleigh reaches high values, around Ra = 10^5, we observe the presence of two partial cells. More active convection is evident at the corners of the cavity, with one extending towards the upper left corner and the other towards the lower right corner.

With an even higher Rayleigh number, Ra = 10^6, the streamlines weaken in the center of the cavity and become significantly more intense around the active zones, due to the thermal boundary conditions.

4.2. Isotherm lines

The representation of the thermal field's appearance is illustrated by the temperature contours, as seen in Figure 4, for a range of Rayleigh numbers between 10^3 and 10^6.

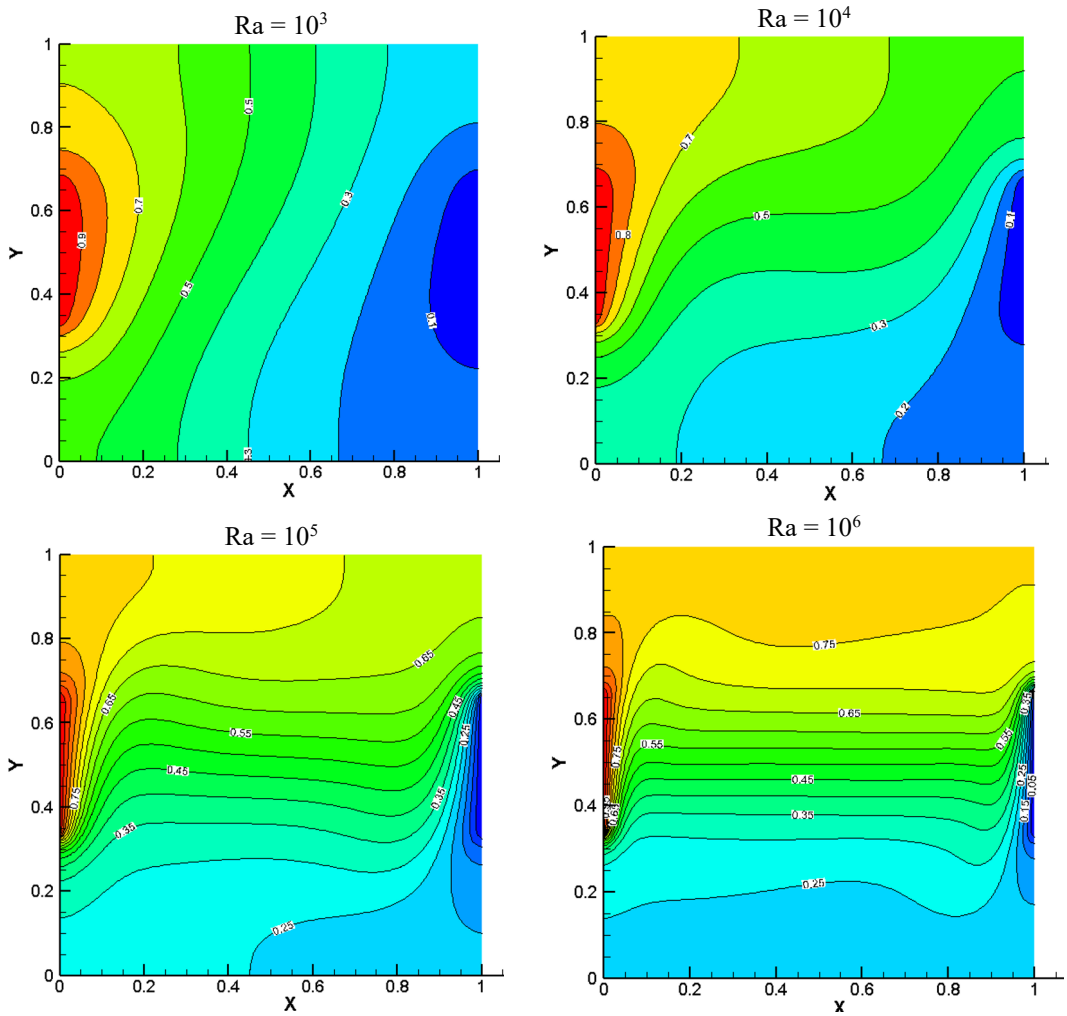

Fig. 4. Isotherms for various Rayleigh numbers.

For Ra =10^3, the isotherms appear to be parallel to the vertical walls and perpendicular to the horizontal walls. This suggests that conduction is the dominant mode of heat transfer within the cavity; heat from the hot wall is conducted to the fluid, resulting in an increase in temperature in the fluid layers adjacent to the left wall and so on.

For Ra = 10^4, The heat transfer intensifies and becomes evident through a sudden alteration in the configuration of the isotherms. These isotherms converge and concentrate near the

active regions. It is evident that the isotherms take on an almost horizontal appearance at a Rayleigh number of 10^5 in the center of the cavity, while they become narrower near the vertical walls. Convection becomes the prevailing mode of heat transfer in this case.
With a higher Rayleigh number of 10^6, the isotherms exhibit a more pronounced horizontal alignment at the cavity's center and progressively narrow as they approach the vertical walls. This shift suggests that convection is increasingly becoming the dominant heat transfer mechanism.

4.3. Temperature profiles

To obtain more results concerning temperature distribution, the figure below presents the temperature profiles along the horizontal median plane for the same values of the Rayleigh number that have been discussed previously.
With a Rayleigh number of Ra $=10^3$, the temperature decreases linearly. This reflects the dominance of the transfer mode of heat by conduction.
From the number Ra $= 10^4$, a region of temperature stability becomes evident, extending from X=0.32 to X=0.63. This area of stability widens as the Rayleigh number increases, finally extending over the interval X ∈ [0.1; 0.9] when the Rayleigh number value reaches Ra $=10^6$.

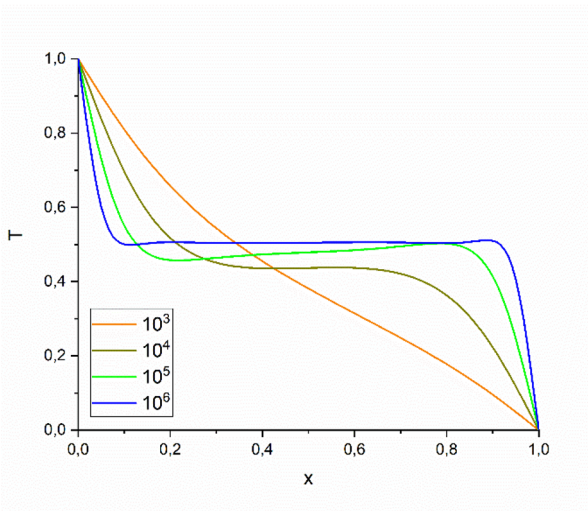

Fig. 5. Temperature profiles along the horizontal midplane.

4.4. profiles of Horizontal velocity

Figure 6 displays the profiles of horizontal velocity (U) along the vertical midplane for a range of Rayleigh numbers from 10^3 to 10^6.

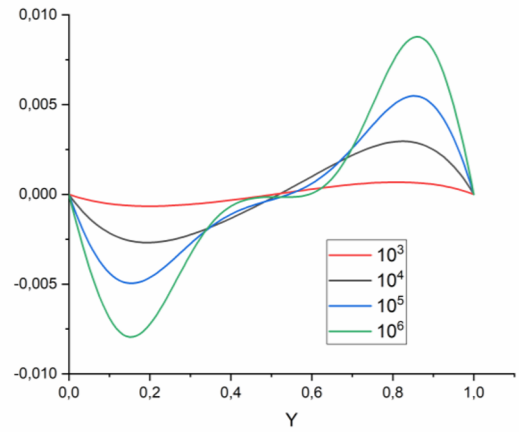

Fig. 6. Horizontal velocity profiles long the vertical midplane.

The shape of the curve representing the horizontal velocity component remains anti-symmetrical with respect to the line passing through Y=0.5. The velocity is zero at the wall Y=0, then gradually decreases until it reaches a minimum. Afterward, it starts increasing until it becomes zero at the position corresponding to the center of the cavity, and the same pattern repeats in the opposite direction. It's noteworthy that at Rayleigh numbers $Ra=10^5$ and $Ra=10^6$, a region where the velocity is zero emerges, extending between Y=0.44 and Y=0.58, and between Y=0.41 and Y=0.60, respectively. The results obtained confirm that as the Rayleigh number increases, there is an increase in the amplitude of fluid velocity.

Conclusion

This paper focuses on the investigation of natural convection of air within a square cavity that is partially heated. The numerical method employed is the lattice Boltzmann approach with a dual-population thermal model. The numerical study showed that the rate of heat transfer is markedly affected by the Rayleigh number. An increase in this number generates significantly more active fluid flow along the vertical walls, which favors considerably improved heat transfer.

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