The Quantitative Research on Atmospheric Environmental Corrosion of Aluminum Alloy Products

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Abstract: In this paper, we conduct quantitative research on the atmospheric environmental corrosion of aluminum alloy based on the atmospheric environmental factors. we apply the elastic network regression method to construct a regression model for corrosion rates based on the identified damage factors, which allows us to calculate the dependent variable as long as we know the main environmental factors in a specific year. Finally, we introduce a measure of atmospheric environment corrosion based on the established regression which can characterize the severity of corrosion with different transformations.

Keywords: Aluminum alloy; Environmental adaptability; Random forest; Elastic network; Atmospheric environmental corrosion measure.

1. Introduction

According to GJB 4239 "General Requirements for Equipment Environmental Engineering"[1] and GJB 6117-2007 "Equipment Environmental Engineering Terms"[2], environmental adaptability of a product refers to the ability of a product to maintain normal function and performance in all predetermined environments, and the requirements for environmental adaptability[3] are the requirements of a product's tolerance to environmental stress.An extensive analysis of typical cases of products' environmental adaptability shows that the environmental adaptability of different products varies with the environment, thus the environmental adaptability level of a product should be considered systematically when choosing the materials[4]. In this paper, A regression model for the corrosion of aluminum alloy in the atmospheric environment is constructed. we define a new measure of atmospheric corrosion to reflect the severity of environmental conditions.

2. Variable Selection

This paper selects seven aluminum alloy materials, 2B06-T4, 2D12-T4, 3A21-H24, 5A05-O, 7A09-T6, 7B04-T6 and 5A90-T3S. The environmental corrosion of aluminum alloy materials during storage is quantified based on the factors influencing the average corrosion rate. We define the independent variables X_1 (temperature), X_2 (relative humidity), X_3 (Cl^- deposition rate), X_4 (pH of rainwater), etc., to represent the environmental factors that have effect on the corrosion of aluminum alloy material.

TABLE 1 gives the atmospheric average corrosion rates of the seven aluminum alloys in four typical environments (Jiangjin, Wanning, Mohe and Beijing). The dependent variable Y considered in this paper is the ratio of the average corrosion rate to the mean of each group. If the ratio is less than 1, it can be interpreted as "being averaged", while if the ratio is greater than 1, it indicates that the original average corrosion rate has made contributions to the final average. We construct a regression model of Y using the selected environmental factors as the independent variables. We finally construct an environmental corrosion measure H to characterize the severity of the environmental damage, which is a function of Y. The larger H, the more severe the environment will be, and thus the more severe corrosion will be if the aluminum alloy material is exposed to this environment.

Table 1. Average corrosion rates of seven aluminum alloys in different localities

Locality	Year	2B06-T4	2D12-T4	3A21-H24	5A05-O	7A09-T6	7B04-T6	5A90-T3S
Jiangjin	2011	0.508	0.967	0.522	0.479	2.115	0.295	0.099
	2012	1.284	0.197	0.135	0.359	0.643	0.179	0.163
	2013	0.389	0.416	0.646	0.414	0.389	0.176	0.112
	2014	0.22	0.208	0.453	0.223	0.508	0.073	0.142
Wanning	2011	2.075	2.737	2.251	0.861	15.417	3.208	0.865
	2012	1.31	1.999	0.969	0.509	6.734	2.324	1.25
	2013	1.125	1.62	1.491	0.696	5.034	——	0.846
	2014	1.449	1.137	——	——	6.327	1.97	1.03
	2015	——	1.347	1.561	0.75	5.667	1.509	0.67
Mohe	2011	0.001	0.141	——	0.441	0.126	0.09	0.001
	2012	0.018	0.021	0.026	0.244	0.062	0.014	0.031
	2013	0.061	0.029	0.041	0.181	0.045	0.025	0.064
	2014	0.084	0.105	0.304	0.088	——	0.05	0.044
Beijing	2011	0.34	0.573	0.17	0.271	0.612	0.333	0.181
	2012	0.165	——	——	——	——	0.126	——
	2013	0.109	——	——	——	——	0.114	——
	2014	——	0.326	0.051	0.169	0.332	——	0.086

3. Regression Model for Atmospheric Environmental Corrosion

For aluminum alloy products, this paper aims at establishing a regression model for the atmospheric environmental corrosion with independent variables X_i and response variable Y ,i.e.,

$$Y = Xw + \varepsilon \tag{1}$$

where the first component of the vector $X = (1, X_1, \cdots, X_n)$ represents the constant term, $w = (w_0, w_1, \cdots, w_n)$ are the regression coefficients, and $\varepsilon \sim N(0,1)$. The basic linear regression method uses the least square method to estimate the coefficients $w = (w_0, w_1, \cdots, w_n)$ as follows:

$$\min_w f(w) = \sum_{i=1}^{m} (y_i - x_i^T w)^2 = (y - Xw)^T (y - Xw) \tag{2}$$

where x_i , y_i are the i -th observation values of the independent variables and the response variable.

In this paper, we use 14 environmental variables excluding the chemical components as the independent variables for modeling, i.e., SO_2 deposition rate, temperature、relative humidity、pH of rainwater、solar radiation, Cl^- of rainwater, rain days, atmospheric pressure, NH_3, Cl^- deposition rate, snow days, average wind speed, fog days , and H_2S。

The variance inflation factors given in Table 2 suggest that there is serious multicollinearity between independent variables. Therefore, if simple linear regression is performed directly, the estimation results are not reliable and the variance of the parameter estimators will be too large.

Table 2. VIFs of independent variables

Independent variable	VIF	Independent variable	VIF
H_2S	36.91	Solar radiation	190.89
NH_3	234.72	Atmospheric pressure	1006.33
SO_2	217.31	Average wind speed	336.90
Temperature	2480.46	Rain days	211.35
Relative humidity	710.45	Fog days	161.28
pH of rainwater	521.87	Snow days	240.52
Cl^- of rainwater	84.85	Cl^- deposition rate	208.28

Elastic Network Regression is another regularized regression method[4], which combines the L_1 and L_2 penalties of the lasso and ridge methods. Its loss function is as follows,

$$f(w) = \sum_{i=1}^{m} (y_i - x_i^T w)^2 + \rho\alpha \sum_{i=1}^{n} |w_i| + \frac{1-\rho}{2} \alpha \sum_{i=1}^{n} w_i^2 \tag{3}$$

where $\rho\alpha$ is an important parameter to determine which regularization method dominates in the end. Elastic network regression is quite useful when there is a group of highly correlated variables. The value of λ obtained by cross-validation[9] is 0.140766, which gives us the following model,

$$\begin{aligned} Y = 2.813209 &+ 0.028445 x_{Temperature} + 0.000239 x_{Relative\ humidity} - 0.466436 x_{pH\ of\ rainwater} \\ &+ 0.101853 x_{Average\ wind\ speed} + 1.111378 x_{Cl^- deposition\ rate} \end{aligned} \tag{4}$$

For which, the mean square error is 0.16, and the goodness of fit is 97.80%.

Based on this model, the value of Y of a region can be obtained only based on the annual average values of temperature, relative humidity, pH of rainwater, average wind speed, and Cl^-deposition rate. The larger the value of Y, the more severe the corrosion of aluminum alloy products.

Based on the established model, we provide the values of Y in four different regions in Table 3 and plot the trend of each region in Figure 1.

Table 3. The value y of Y in four regions

Region	Year	y	Region	Year	y
Jiangjin	2011	1.010229	Mohe	2011	0.079678
	2012	0.952298		2012	0.118373
	2013	0.952537		2013	0.127745
	2014	0.914403		2014	0.206743
Wanning	2011	1.765198	Beijing	2011	0.384689
	2012	1.527571		2012	0.351415
	2013	1.494026		2013	0.295100
	2014	1.424120		2014	0.361261
	2015	1.386805			

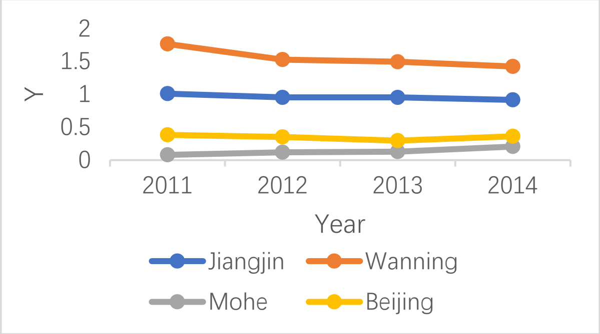

Fig. 1 Value trend chart of each region in the first four years

For each year, it can be seen that the relationship among the values of Y in the four regions is $Y_{B2} > Y_{B1} > Y_{B4} > Y_{B3}$. In other words, for aluminum alloy materials, the environment in Wanning is the worst, followed by that in Jiang, Beijing. In addition, except for Mohe, the values in other regions show a decreasing trend year by year. This may be because the strengthening of environmental protection measures in China in recent years has improved the atmospheric environment.

4. Environmental Corrosion Measurement

According to the corrosion regression model of aluminum alloy material in the atmospheric environment provided in Eq (4), we can calculate Y of a region in a certain year when the independent variables are known, which allows us to give a general judgment of the severity of the environment. The larger the value of Y, the harsher the environmental conditions for the aluminum alloy products are. However, to have a more uniform and intuitive understanding, this paper constructs the environmental corrosion measurement based on the dependent variable. We consider using the quadratic fraction transformation[5,6]:

$$h(y) = \frac{y^2}{1+y^2}, \tag{5}$$

or the logit transformation[10]：

$$h(y) = \frac{e^y}{1+e^y}, \tag{6}$$

to obtain a measurement of atmospheric environmental corrosion within [0,1], which allows us to better analyze the severity of the environment. A value of 0 indicates that the environment is the best for aluminum alloy products, and a value of 1 indicates that the environment is the harshest.

Moreover, we introduce two undetermined parameters α_0 and α_1 ($\alpha_1 > 0$) such that the transformed environmental corrosion measurement $H(Y,\alpha_0,\alpha_1)$ is a strictly increasing function of the dependent variable Y. Thus, a large $H(Y,\alpha_0,\alpha_1)$ also indicates a harsh environment. Specifically, the measure is given by,

$$H(Y,\alpha_0,\alpha_1) = \frac{(\alpha_0+\alpha_1 Y)^2}{1+(\alpha_0+\alpha_1 Y)^2} \tag{7}$$

or

$$H(Y,\alpha_0,\alpha_1) = \frac{e^{\alpha_0+\alpha_1 Y}}{1+e^{\alpha_0+\alpha_1 Y}}, \tag{8}$$

4.1 Quadratic fraction transformation

When applying the atmospheric environmental corrosion measurement, we hope that it can distinguish different typical environments as well as possible. Therefore, when determining the parameters α_0 and α_1, this paper tries to maximize the difference between the two regions with the smallest difference in the measurement of atmospheric environmental corrosion among the four regions. Let the environmental corrosion measure in Jiangjin, Wanning, Mohe, and Beijing be denoted by $H_1(Y_1,\alpha_0,\alpha_1)$, $H_2(Y_2,\alpha_0,\alpha_1)$, $H_3(Y_3,\alpha_0,\alpha_1)$, $H_4(Y_4,\alpha_0,\alpha_1)$, or abbreviated as H_1, H_2, H_3, H_4, there comes a constrained non-linear programming problem：

$$\begin{aligned} \max \quad & \min\left\{|H_1-H_2|^2,|H_2-H_3|^2,|H_3-H_4|^2,|H_4-H_1|^2\right\} \\ & +\lambda_1\sum_{i=1}^{4}|H_i|^2+\lambda_2\sum_{i=1}^{4}|1-H_i|^2 \\ s.t. \quad & \begin{cases} \alpha_1 > 0 \\ \alpha_0 \in \mathbb{R} \quad \text{(Field of real numbers)} \end{cases} \end{aligned} \tag{9}$$

Where $\lambda_1\sum_{i=1}^{4}|H_i|^2+\lambda_2\sum_{i=1}^{4}|1-H_i|^2$ denotes the penalty term, and λ_1, λ_2 are penalty coefficient satisfying $\lambda_1 > 0$ and $\lambda_2 > 0$.

Then the sequential quadratic programming method can be used to convert the problem into the following standard form:

$$\begin{aligned} \min \quad & -\min\left\{|H_1-H_2|^2,|H_2-H_3|^2,|H_3-H_4|^2,|H_4-H_1|^2\right\} \\ & -\lambda_1\sum_{i=1}^{4}|H_i|^2-\lambda_2\sum_{i=1}^{4}|1-H_i|^2 \\ s.t. \quad & \begin{cases} -\alpha_1 < 0 \\ \alpha_0 \in \mathbb{R} \end{cases} \end{aligned} \tag{10}$$

The values of H_1, H_2, H_3, H_4 are relevant to the specific year corresponding to environmental factors in the area, whereas Jiangjin and Wanning are different in the time dimension. Thus, we average Y_i of each region to get $\overline{y}_i$, and let $H_i(Y_i,\alpha_0,\alpha_1)=H_i(\overline{Y_i},\alpha_0,\alpha_1)$. The results are given in Table 4.

Table 4. Corrosion measurement expressions and $\overline{y}$ for four regions

Locality	$\overline{y}$	H
Jiangjin	0.957367	$H_1=\dfrac{(\alpha_0+0.957367\alpha_1)^2}{1+(\alpha_0+0.957367\alpha_1)^2}$
Wanning	1.519544	$H_2=\dfrac{(\alpha_0+1.519544\alpha_1)^2}{1+(\alpha_0+1.519544\alpha_1)^2}$
Mohe	0.133135	$H_3=\dfrac{(\alpha_0+0.133135\alpha_1)^2}{1+(\alpha_0+0.133135\alpha_1)^2}$
Beijing	0.348116	$H_4=\dfrac{(\alpha_0+0.348116\alpha_1)^2}{1+(\alpha_0+0.348116\alpha_1)^2}$

Let the penalty factors $\lambda_1=0.00001$, $\lambda_2=0.0001$, taking the values in the table into the nonlinear programming problem of Eq. (10) gives

$$\begin{cases}\alpha_0=-0.020926\\ \alpha_1=1.497601\end{cases} \tag{11}$$

Therefore, the measurement formula of atmospheric environmental corrosion under quadratic transformation is:

$$H(Y)=\frac{(1.497601Y-0.020926)^2}{1+(1.497601Y-0.020926)^2} \tag{12}$$

These four regions represent four typical atmospheric environments in China, namely, the humid and hot marine atmospheric environment, the sub-humid and hot industrial atmospheric environment, the cold rural atmospheric environment, and the warm and semi-rural atmospheric environment. The atmospheric environmental corrosion measurements for the four regions are given in Table 5.

Table 5. Environmental corrosion measurement in four regions after quadratic fraction transformation

Locality	Atmospheric environment	H
Jiangjin	Sub-humid and hot industrial atmospheric environment	0.6662
Wanning	Humid and hot marine atmospheric environment	0.8356
Mohe	Cold rural atmospheric environment	0.0309
Beijing	Warm and semi-rural atmospheric environment	0.2003

4.2 Logit transformation

Similarly, we establish the environmental corrosion measurement based on the Logit transformation[11,12]

$$H(Y,\alpha_0,\alpha_1)=\frac{e^{\alpha_0+\alpha_1}}{1+e^{\alpha_0+\alpha_1}},$$

To determine the parameters α_0 and α_1, we need to solve the following constrained non-linear programming problem.

$$\begin{aligned}\max\quad & \min\left\{|H_1-H_2|^2,|H_2-H_3|^2,|H_3-H_4|^2,|H_4-H_1|^2\right\}\\ & +\lambda_1\sum_{i=1}^{4}|H_i|^2+\lambda_2\sum_{i=1}^{4}|1-H_i|^2\\ s.t.\quad & \begin{cases}\alpha_1>0\\ \alpha_0\in\mathbb{R}\end{cases}\end{aligned} \tag{13}$$

Where $\lambda_1\sum_{i=1}^{4}|H_i|^2+\lambda_2\sum_{i=1}^{4}|1-H_i|^2$ denotes the penalty term, and λ_1, λ_2 are penalty coefficient satisfying $\lambda_1>0$ and $\lambda_2>0$. We can transform the problem into the following standard form:

$$\begin{aligned}\min\quad & -\min\left\{|H_1-H_2|^2,|H_2-H_3|^2,|H_3-H_4|^2,|H_4-H_1|^2\right\}\\ & -\lambda_1\sum_{i=1}^{4}|H_i|^2-\lambda_2\sum_{i=1}^{4}|1-H_i|^2\\ s.t.\quad & \begin{cases}-\alpha_1<0\\ \alpha_0\in\mathbb{R}\end{cases}\end{aligned} \tag{14}$$

In TABLE 6, we provide the average values $\overline{y}_i$ and the corresponding measurements $H_i(Y_i,\alpha_0,\alpha_1)=H_i(\overline{Y_i},\alpha_0,\alpha_1)$ for each region.

Table 6. Environmental corrosion measurement in four regions after logit transformation

Locality	$\overline{y}$	H
Jiangjin	0.957367	$H_1=\dfrac{e^{\alpha_0+0.957367\alpha_1}}{1+e^{\alpha_0+0.957367\alpha_1}}$,
Wanning	1.519544	$H_2=\dfrac{e^{\alpha_0+1.519544\alpha_1}}{1+e^{\alpha_0+1.519544\alpha_1}}$,
Mohe	0.133135	$H_3=\dfrac{e^{\alpha_0+0.133135\alpha_1}}{1+e^{\alpha_0+0.133135\alpha_1}}$,
Beijing	0.348116	$H_4=\dfrac{e^{\alpha_0+0.348116\alpha_1}}{1+e^{\alpha_0+0.348116\alpha_1}}$,

The sequential quadratic programming (SQP) algorithm[13,14] is used to solve the optimization problem (14), for which, we consider the following Lagrange function,

$$\begin{aligned}L(\alpha_0,\alpha_1,\lambda)=&-\min\left\{|H_1-H_2|^2,|H_2-H_3|^2,|H_3-H_4|^2,|H_4-H_1|^2\right\}\\ &-\lambda_1\sum_{i=1}^{4}|H_i|^2-\lambda_2\sum_{i=1}^{4}|1-H_i|^2+\lambda\cdot(-\alpha_1)\end{aligned} \tag{15}$$

Where λ is Lagrange multiplier. Based on this operation, this paper transforms an optimization problem with two variables and one constraint condition into an extreme value problem of a system of equations with three variables without any constraints. Next, we consider the quadratic approximation of the Lagrange function to improve the similarity of the quadratic programming subproblem, which makes the optimization problem with strong nonlinearity can also be calculated.

Let $\lambda_1=0.001$, $\lambda_2=0.0001$, we can get:

$$\begin{cases}\alpha_0=-1.944134\\ \alpha_1=3.647075\end{cases}$$

The environmental corrosion measurement with the transformation $h(y) = e^y / (1 + e^y)$ is given by

$$H(Y) = \frac{e^{-1.944134+3.647075 \cdot Y}}{1 + e^{-1.944134+3.647075 \cdot Y}}, \quad (16)$$

Table 7. Environmental corrosion measurement in four regions after logit transformation

Locality	Atmospheric environment	H
Jiangjin	Sub-humid and hot industrial atmospheric environment	0.8245
Wanning	Humid and hot marine atmospheric environment	0.9733
Mohe	Cold rural atmospheric environment	0.1887
Beijing	Warm and semi-rural atmospheric environment	0.3375

For this transformation, the environmental corrosion measures of all four regions also have the $H_2 > H_1 > H_4 > H_3$. The results show that the environment of Wanning is the worst among the four regions, followed by Jiangjin, which is consistent with the results in Section 4.1.

The related reference[15] demonstrates that Jiangjin is perennial and cloudy. Long-term exposure of metal materials to a humid atmosphere increases the thickness of the surface water film, impeding the passage of oxygen, thus reducing the corrosion rate of the metal materials.

While in Wanning where the measured value of the corrosion rate is the largest, the metal materials are exposed to the humid and hot marine atmospheric environment for a long time. The high temperature and humidity as well as the erosion of seawater will accelerate the corrosion of metal materials. Among the four regions, Mohe's atmospheric environmental corrosion measure is the smallest, where the metal materials are in the cold rural atmospheric environment. The low temperature and humidity will reduce the corrosion of metal materials. All these findings based on our model are in accordance with the objective law.

As shown in Section 4, the atmospheric environmental corrosion measures constructed based on the quadratic power transformation or logit transformation can all be used to evaluate the effects of typical atmospheric environments on the corrosion of aluminum alloy material in China. In this section, we explored the corrosion measures of seven typical atmospheric environments, i.e., humid and hot marine atmospheric environment, humid and hot rainforest atmospheric environment, sub-humid and hot industrial atmospheric environment, cool plateau atmospheric environment, warm and semi-rural atmospheric environment, dry and hot desert atmospheric environment, and cold rural atmospheric environment.

Referring to the atmospheric environment data of the seven atmospheric environments in 2016, this paper predicts the dependent variable values of Y in the corresponding environment based on the established model, as shown in TABLE 8.

Table 8. Typical environmental data and dependent variable values for 2016

environment	Temperature	Relative humidity	pH of rainwater	Average wind speed	Cl^- deposition rate	Y
Humid and hot marine atmospheric environment	25.625	84.50	5.57	2.642	0.841158	2.1674
Humid and hot rainforest atmospheric environment	21.467	77.58	6.15	0.850	0.00475	0.6656
Sub-humid and hot industrial atmospheric environment	19.607	77.50	5.40	1.050	0.001642	0.9802
Cold and warm plateau environment	8.617	40.18	6.32	2.408	0.003117	0.3704
Warm and semi-rural atmospheric environment	12.783	46.51	6.09	1.912	0.293433	0.8701
Dry and hot desert atmospheric environment	12.333	37.10	7.18	2.858	0.543875	0.7206
Cold rural atmospheric environment	2.442	66.17	5.83	2.533	0.003325	0.3043

The atmospheric corrosion measures for the seven typical atmospheric environments based on both the quadratic power and the logit transformations are given in TABLE 9 and TABLE 10, respectively.

Table 9. Measurements of Seven Typical Atmospheric Environments after Transformation

Type	Humid and hot marine atmospheric environment	Humid and hot rainforest atmospheric environment	Sub-humid and hot industrial atmospheric environment	Cold and warm plateau environment	Warm and semi-rural atmospheric environment	Dry and hot desert atmospheric environment	Cold rural atmospheric environment
$H(Y$	0.9123	0.4878	0.6768	0.2217	0.6218	0.5283	0.1590

Table 10. Measurements of seven typical atmospheric environments after logit transformation

Type	Humid and hot marine atmospheric environment	Humid and hot rainforest atmospheric environment	Sub-humid and hot industrial atmospheric environment	Cold and warm plateau environment	Warm and semi-rural atmospheric environment	Dry and hot desert atmospheric environment	Cold rural atmospheric environment
$H(Y$	0.9974	0.6186	0.8363	0.3558	0.7737	0.6646	0.3027

5. Summary

From the results in the above two tables, we can see that in 2016, the corrosion severity of the seven typical atmospheric environments has the following relationship: Humid and hot marine atmospheric environment > Sub-humid and hot industrial atmospheric environment > Warm and semi-rural atmospheric environment > Dry and hot desert atmospheric environment > Humid and hot rainforest atmospheric environment > Cold and warm plateau environment > Cold rural atmospheric environment. According to the long-term monitoring data of environmental factors and the field exposure test results of the seven environments[20], the order of environmental corrosion severity is: Marine atmospheric environment > Sub-humid and hot industrial environment > Dry and hot desert environment > Cold rural environment and Cold and warm plateau environment. So the results based on the atmospheric environment corrosion measurement in this paper are in line with the objective law.

References

1. Yaochang Zhu, Jianyong Sun. GJB 4239-"General Requirements for Material Environmental Engineering" and Its Application Analysis (I) (in Chinese) [J]. Aviation Standardization and Quality, 2006(05): 32-37.
2. GJB 6117-2007, Materiel environmental engineering term[S].
3. Zeqi Xu, Shiyan Zhang, Weifang Xuan. Environmental Worthiness Evaluation of Equipment (in Chinese) [J]. Equipment Environmental Engineering, 2012, 9(1):54-58.
4. Bowei Wu. Research on Sufficient Dimension Reduction Method Based on Regression Tree (in Chinese) [D]. East China Normal University, 2020.
5. Khaleghi S , Givehchi S , Karimi S . Multivariate correlation analysis and its application in environmental analysis[J]. World Applied Programming, 2013, 242(1):5-9.
6. Xiaoguang Wang and Junhui Fan. Variable selection for multivariate generalized linear models[J]. Journal of Applied Statistics, 2014, 41(2) : 393-406.
7. ISO 9223-2012, Corrosion of Metals and Alloys-Corrosivity of Atmospheres-Classification, Determination and Estimation[S].
8. Liyuan Wang, Xiutong Wang, Haofen Sun, Baorong Hou. Study of SO2 Influence on Metal Corrosion in Atmospheric Environment (in Chinese) [J]. Equipment Environmental Engineering, 2011, 8(02): 62-66.
9. Junxin Hu, Gongjie Zhang. K-Fold Cross-Validation Based Selected Ensemble Classification Algorithm [J]. Bulletin of Science and Technology (in Chinese), 2013, 29(12): 115-117.
10. Wenke Huang. The Diagnosis and Process Solutions in Multicollinearity of Multiple Regression Model (in Chinese) [D]. Harbin Institute of Technology, 2012.
11. Liu, Dacheng Tao. Review on Recent Method of Solving Lasso Problem (in Chinese) [J]. Journal of Data Acquisition & Processing, 2015, 30(01): 35-46.
12. Boya Wu. Research on New Elastic Network Algorithm for Cluster Analysis (in Chinese) [D]. Beijing University of Civil Engineering and Architecture, 2020.
13. Ying Li, Xuan Zhou, Huifang Meng, Haohua Wang. A Robust Logistic Regression with Exponential Squared Loss (in Chinese) [J]. Natural Science Journal of Hainan University, 2019, 37(04): 287-291.
14. Cong Ma, Zhe Liu, Xiaoxian Zhen. Combining penalty function with sequential quadratic programming method for norm minimization(in Chinese)[J]. Computer Engineering and Applications, 2013, 49(18): 212-216.
15. Laizheng Luo, Yong Xiao, Yan Su, Xiaofeng Li, Yong Zhong. Corrosion Behavior of 7050 High-strength Aluminum Alloy in Four Typical Atmospheric Environments in China (in Chinese) [J]. Equipment Environmental Engineering, 2015, 12(04): 49-53.