An improved multi-objective gray target model for project resource allocation selection Research

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Abstract: In response to the traditional method of using a single subjective or objective assignment, AHP (hierarchical analysis) subjective weights and CRITIC (hierarchical relevance of importance criteria method) objective weights are coupled with the Lagrange multiplier method to obtain combined weights, and the multi-objective gray target decision model is used to establish event set, countermeasure set, and situation set to obtain the resource ranking that affects the project resource deployment problem, which provides a basis for the project participants to make decisions and plan for the project. Taking the municipal road project as an example, a judgment matrix is established. The results show that the model constructed in this paper is relatively consistent with the actual situation.

Keywords: resource allocation; AHP; CRITIC; Lagrange multiplier method; gray target decision

1. Introduction

At present, the international community has formed a consensus on green transformation and taken practical actions. Efficient resource utilization has been a major issue that the construction of global ecological civilization has been striving to crack. Although the earth is rich in resources and the total amount of resources is relatively large, due to the rough development since the industrial revolution, resources such as land, energy, and water have formed a hard constraint and tight balance for development, and resource consumption has lit up the red light. In recent years, experts and scholars have devoted a lot of attention to the issue of resource deployment, and the areas involved include aviation[1], railroads[2], rail transportation[3], industrial workshops[4], and many other aspects.

Resource deployment is the rationalization of resource utilization, and the key to achieving optimization is to improve the influencing factors closely related to the desired project goals, such as project time management to make the project more efficient by optimizing the project schedule, cost management is to control the project budget by optimizing the consumption of resources required for the project, and quality management is to ensure the production quality by optimizing the production activities.[5] Quality management is the optimization of production activities to ensure the quality of production.

In engineering project management, the deployment of resources is often intricate and critical, and to analyze key resource elements such as labor, materials, and machinery, Cao et al.[6] To improve the assembly line management of assembly buildings, the defects of the existing production process are analyzed and an assembly line integrated supermarket is proposed to optimize the raw material scheduling method. Wu et al.[7] studied the tower crane planning problem of construction projects and proposed a Spatio-temporal planning model of tower cranes for construction projects based on simulated annealing, saving 31.10% of the total cost on average. For the engineering multi-resource balancing problem, Wang et al.[8] To solve the problem that the traditional multi-mode resource-constrained project scheduling problem (MRCPSP) model is difficult to characterize multiple relationships between process duration, cost, and resource requirements simultaneously, an MRCPSP model is proposed and the arithmetic case is solved using constrained planning (CP). Wang et al.[9] To efficiently and stably solve the multi-resource balancing problem in the process of construction project management, an optimization algorithm based on subset simulation is proposed, which has a large improvement in the stability of obtaining the optimal solution compared with the currently widely used genetic algorithm. To conduct a comprehensive resource scheduling analysis for project clusters, Feng et al.[10] borrowed the single project schedule optimization method under the condition of total resource constraint, constructed the project group schedule optimization model under the resource constraint, and determined the contract items that can be resource output and input to finally achieve the purpose of project group schedule optimization. Jiang et al.[11] conducted risk evaluation and prediction of resource conflict problem of hydropower project group from the perspective of construction enterprises, estimated the
weights of each evaluation index by entropy weight method, established homogeneous inverse risk evaluation and prediction model for static risk evaluation by using the five-element coefficient of association, and predicted risk development trend dynamically by using the partial coefficient of association.

From the above-related studies, scholars have made more research results on project resource deployment, but few of them have explored the overall deployment of project resources from a global perspective, and only a single consideration of qualitative or quantitative calculation has been made in calculating the weights of each index, thus resulting in the omission of some information. In this paper, through expert scoring and qualitative analysis of hierarchical analysis (AHP), weights are combined and calculated by the Lagrange multiplier method coupled with CRITIC (Criteria Importance Though Intercriteria Correlation), and then a multi-objective weighted gray target decision model is established to rank the importance of resource deployment problems of construction projects. Finally, the applicability and robustness of the model are verified by example projects. The findings are helpful for all parties involved in the project to identify the important and difficult points of the resource allocation problem before the project implementation and to plan the resources well in advance to ensure the smooth implementation of the project.

2. Conventional gray target decision model construction

Grey target decision making (GSDM) is proposed by Prof. Ju-Long Deng[12] It is roughly classified into rectangular, spherical, and ellipsoidal, and is mainly based on the set of evaluated sequences to construct grey targets. It is widely used to solve decision-making problems under multi-objective and uncertain conditions. It determines the bull's eye by unifying the objective data of each indicator, judging the benefit, cost, intermediate, and interval indicators, determining the bull's eye, and calculating the bull's eye distance of each solution by using the Euclidean space with the bull's eye as the reference point and ranking the bull's eye distance to determine the optimal solution. The model calculation steps are specified as follows.

STEP1: Determine the scope of the study event set, response set, and situation set. The set of events in a study area is the set of events in the study area, and is denoted as, where $A = \{a_1, a_2, \cdots, a_n\}$ is an individual event in the study area. The corresponding set of all possible responses is the response set, and is denoted as, where $B = \{b_1, b_2, \cdots, b_m\}$ is an individual response within the study area. The Cartesian product of the event set $A = \{a_1, a_2, \cdots, a_n\}$ and the response set $B = \{b_1, b_2, \cdots, b_m\}$ is the set of situations in the study area, denoted as $S_{ij}$.

$$u_j = (u_{ij}^{(1)}, u_{ij}^{(2)}, \cdots, u_{ij}^{(s)}) \in S^j$$ is the effect vector of the situation. $s_{ij} u_{ij}^{(k)} (k=1,2, \cdots, s)$ is the effective value of the situation $s_{ij}$ under the $k$ objective.

STEP2: Qualitative ranking of the situations under each indicator. Qualitative analysis of the effect vector

$$u_j = (u_{ij}^{(1)}, u_{ij}^{(2)}, \cdots, u_{ij}^{(s)}) \in S^j$$ of the situation $s_{ij}$ for each effect value $u_{ij}^{(k)}$ under the target $k (k=1,2, \cdots, s)$. The values are assigned according to the expert scoring method, marking the optimal effect value as 1 and the next best value as 2. The effect values are ranked according to this increasing value law.

STEP3: Determine the optimal effect vector and build the gray target model according to the optimal effect obtained from the ranking. For the solution decision problem with $s$ targets, the optimal effect vector is $r_0 = (r_0^{(1)}, r_0^{(2)}, \cdots, r_0^{(s)})$ after ranking, where: $r_0^{(k)}$ is the optimal effect value of the effect vector $t_{ij}$ under the target $k$. Where the spherical target space is formed based on each effect vector and the optimal effect vector $r_0$ is called $s$ dimensional spherical gray target which $R$ is

$$R=\left( r^{(1)}, \cdots, r^{(s)} \right) \left( r^{(1)} \cdots r^{(s)} \right)$$

(1)

Where: $r_0$ is the bull's-eye; $r^{(i)}$ is each effect vector. STEP4: Calculate the bull's-eye distance. For the program $r_i=(r_1^{(1)}, r_1^{(2)}, \cdots, r_1^{(s)}) \in R$, the bull's-eye distance is

$$|r_i-r_0| = \sqrt{(r_1-r_0)^2+(r_2-r_0)^2+\cdots+(r_s-r_0)^2}$$

(2)

STEP5: Sort the solutions according to the bullseye distance size. Calculate the bull's-eye distance for each situation effect vector based on the gray target and evaluate the decision options. For different situations $s_{ij}$, the different effect vectors corresponding to $s_{ij}$ are $u_{ij} = (u_{ij}^{(1)}, u_{ij}^{(2)}, \cdots, u_{ij}^{(s)})$ and $u_{ij} = (u_{ij}^{(1)}, u_{ij}^{(2)}, \cdots, u_{ij}^{(s)})$, and if the bull's-eye distance is $|u_{ij}-r_0| \geq |u_{ij}-r_0|$, then the situation $s_{ij}$ is said to be superior to $s_{ij}$, which is considered equivalent if the sign of the equation is valid.

3. AHP Method

Hierarchical analysis (AHP) is a systematic hierarchical modeling method that was first proposed by T.L. Saaty, a famous American operations researcher, in the 1970s.[13] It is a systematic hierarchical modeling method combining qualitative and quantitative approaches firstly proposed to establish a matrix by two-by-two comparison to determine the weights of each index, which has the advantages of low data volume requirement, the easy establishment of the evaluation system, low calculation workload and mature development of system tools. This method is often used in the study of multi-objective,
multi-criteria, multi-factor, multi-level unstructured complex decision problems, especially strategic decision problems. The specific steps of the hierarchical analysis method (AHP) are as follows.

**STEP1:** Build a recursive hierarchical model. Hierarchize the decision problem, construct a hierarchical structural model and decompose it into three levels: goal level, criterion level, and solution level.

**STEP2:** Construct the judgment matrix. Adopting the approach of factor two-by-two comparison to build the matrix, \( b_{ij} \) indicates the influence of the impact factor indicators \( B_i \) and on the target \( B_j \ A \), and similarly, \( 1 / b_{ij} \) indicates the influence of \( B_j \) and \( B_i \) on the target \( A \), to build the matrix \( B = (B_{ij})_{n \times n} \), and the determination of \( B_{ij} \) uses 1–9 and it’s reciprocal as the scale.

**STEP3:** Hierarchical single ranking and consistency test. The eigenvectors of the judgment matrix \( B \) corresponding to the largest eigenvalue \( \lambda_{max} \) are normalized to obtain the ranking weights of the relative importance of the corresponding factors at the same level as a factor at the previous level, which is the hierarchical single ranking. The consistency test of the judgment matrix needs to calculate the consistency index (CI) and consistency ratio (CR), and when \( CR < 0.1 \), the consistency of the judgment matrix is considered to be acceptable. CI The formulas for calculating RI and CR are

\[
\begin{align*}
CI &= \frac{\lambda_{max} - n}{n-1} \\
RI &= \frac{\lambda_{max} - n}{n-1} \\
CR &= CI / RI
\end{align*}
\]

Equation (3): \( \lambda_{max}' \) is the average of the maximum eigenvalues of 500 sample matrices constructed using a random method and randomly drawing numbers from 1 to 9 and their reciprocals to construct positive reciprocal inverse matrices.

**STEP4:** Hierarchical total ranking and consistency check. The weights are synthesized based on the results of the hierarchical single ranking to obtain the ranking weights of each factor on the total goal \( w_{AHP} \).

**4. CRITIC Method**

The conventional objective methods of weight calculation include the entropy method and the CRITIC method.[14] The CRITIC method is a more scientific and objective weighting method, compared with the entropy method, which only considers the information quantity of indicators and ignores the correlation between indicators, the CRITIC method determines the weights of indicators based on the comparison intensity and conflict between evaluation indicators. The larger the standard deviation of the value of the indicator, the greater the amount of information reflected, and the greater the weight; correlation refers to the correlation coefficient between two indicators, the smaller the correlation coefficient, the more similar the amount of information reflected, and the smaller the weight. The specific calculation steps are as follows.

**STEP1:** Standardized matrix.

\[
m_{ij} = \frac{x_{ij} - x_{i-min}}{x_{i-max} - x_{i-min}}
\]

**STEP2:** Calculate the standard deviation.

\[
\sigma_j = \frac{\sum_{i=1}^{n} (m_{ij} - \bar{m}_j)^2}{n-1}
\]

\[
\bar{m}_j = \frac{1}{n} \sum_{i=1}^{n} m_{ij}
\]

**STEP3:** Construct the correlation coefficient matrix.

\[
r_{ij} = \frac{\sum_{i=1}^{n} (m_i - \bar{m})(m_j - \bar{m})}{\sqrt{\sum_{i=1}^{n} (m_i - \bar{m})^2 \sum_{j=1}^{n} (m_j - \bar{m})^2}}
\]

**STEP5:** Normalize \( c_j \) to get the objective weight \( w_{CRITIC} \).

**5. Lagrange multiplier method**

Since the hierarchical analysis method is a subjective assignment method, which is subject to the interference of human factors, and the CRITIC method is an objective assignment method, which tends to ignore the subjective intention of the evaluator and the actual weight relationship among the evaluation indexes. Therefore, the two methods are coupled to form a combined weighting method to determine the comprehensive weight of each evaluation index by complementing the subjective and objective. The specific calculation process of combined weighting adopts the principle of minimum information entropy and the Lagrange multiplier method.[15] The specific calculation steps are as follows.
\[
\begin{align*}
\min F = & \sum_{j=1}^{m} \lambda_j \ln \lambda_j - \ln w_{AHP} + \sum_{j=1}^{m} \lambda_j \ln \lambda_j - \ln w_{CR}\nonumber \\
\text{s.t.} & \sum_{j=1}^{m} \lambda_j = 1, \quad \lambda_j > 0, \quad j = 1, 2, \ldots, m \nonumber 
\end{align*}
\]

The above optimization problem is solved according to the Lagrange multiplier method, calculated as

\[
\lambda_j = \frac{w_{AHP}w_{CR}}{\sum_{j=1}^{m} \sqrt{w_{AHP}w_{CR}}} \quad (12)
\]

6. Empirical analysis

6.1 Project basic introduction and decision index setting

YB Avenue Municipal Supporting Project is located in the northern part of Jiangxia District, Wuhan City, Hubei Province, China, and is an urban secondary road in Guanggu South Health Industry Park, which mainly undertakes the service functions of freight transportation and residents' traffic travel inside and outside the region, with a planned red line width of 30 meters. The main construction content includes road engineering, bridge engineering, drainage engineering, traffic engineering, lighting engineering, electric power telecommunications, and landscape greening engineering. The project construction period is 36 months, and the total investment is 472,517,800 yuan, including the engineering cost of 264,239,700 yuan. The project adopts a PPP (Public Private Partnership) mode of construction, the implementation agency is Jiangxia District Water and Construction Co. The project is a key project of the government investment infrastructure supporting the Guanggu South Health Industry Park, which is characterized by a tight construction cycle, high investment intensity, multiple cross-construction, and wide distribution of work surfaces. Therefore, the resource allocation of the project is a high concern for all parties involved in the construction. Based on the resources themselves, we believe that resource deployment will have an impact on five aspects: project financial evaluation, project extension cost, project cycle pressure, mission risk factor, and the number of related tasks. Indicators are detailed in Table 1, where \( X_1 \), \( X_2 \), \( X_3 \) is a property of the project itself and \( X_4 \), \( X_5 \) is a property of the tasks themselves.

<table>
<thead>
<tr>
<th>Indicator Name</th>
<th>Properties of the indicator</th>
<th>Marking of indicators</th>
<th>Literature on Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project Financial Evaluation</td>
<td>Beneficial</td>
<td>( X_1 )</td>
<td>[18]</td>
</tr>
<tr>
<td>Project Extension Costs</td>
<td>Cost Type</td>
<td>( X_2 )</td>
<td>[17]</td>
</tr>
<tr>
<td>Project Cycle Pressure</td>
<td>Mission Risk Factor</td>
<td>( X_3 )</td>
<td>[19]</td>
</tr>
<tr>
<td>Task Risk Factor</td>
<td>Number of related tasks</td>
<td>( X_4 )</td>
<td>[20]</td>
</tr>
<tr>
<td>Resources</td>
<td>Resources</td>
<td>( X_5 )</td>
<td>[37]</td>
</tr>
<tr>
<td>Boundary</td>
<td>Resources</td>
<td>( Y_1 )</td>
<td>[45]</td>
</tr>
<tr>
<td>Resources</td>
<td>Resources</td>
<td>( Y_2 )</td>
<td>[45]</td>
</tr>
<tr>
<td>Resources</td>
<td>Resources</td>
<td>( Y_3 )</td>
<td>[45]</td>
</tr>
<tr>
<td>Resources</td>
<td>Resources</td>
<td>( Y_4 )</td>
<td>[45]</td>
</tr>
<tr>
<td>Resources</td>
<td>Resources</td>
<td>( Y_5 )</td>
<td>[45]</td>
</tr>
<tr>
<td>Indicator</td>
<td>Variability Conflict Amount ( w_{AHP} ) indicator indicator information ( w_{CR} )</td>
<td>( \lambda_j )</td>
<td></td>
</tr>
<tr>
<td>Index</td>
<td>\text{AHP Method}</td>
<td>CRITIC Method</td>
<td>AHP-CRITIC</td>
</tr>
<tr>
<td>Indicator Name</td>
<td>Eigenvector</td>
<td>Variable Conflict Amount of ( w_{AHP} ) indicator indicator information of ( w_{CR} )</td>
<td>( \lambda_j )</td>
</tr>
<tr>
<td>Index</td>
<td>1.351</td>
<td>0.27</td>
<td>0.402</td>
</tr>
</tbody>
</table>

Combining the specific situation of this project, after reviewing the literature and consulting with project experts, it was concluded that the five resources \( Y_1 \), \( Y_2 \), \( Y_3 \), \( Y_4 \), and \( Y_5 \) best reflect the impact of their deployment issues on the project. Through a questionnaire survey of nine industry experts, index scoring boundaries were set, 0-1 scoring was completed, and the data were averaged to form the following scoring matrix Table 2.

### Table 2 Resource deployment impact indicators

<table>
<thead>
<tr>
<th>Resource Category</th>
<th>Project Financial Evaluation Costs</th>
<th>Project Extension Costs</th>
<th>Project Cycle Pressure</th>
<th>Mission Risk Factor</th>
<th>Number of Related Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resources</td>
<td>( X_1 )</td>
<td>( X_2 )</td>
<td>( X_3 )</td>
<td>( X_4 )</td>
<td>( X_5 )</td>
</tr>
<tr>
<td>( Y_1 )</td>
<td>0.045</td>
<td>0.233</td>
<td>0.175</td>
<td>0.134</td>
<td>0.138</td>
</tr>
<tr>
<td>( Y_2 )</td>
<td>0.064</td>
<td>0.342</td>
<td>0.286</td>
<td>0.152</td>
<td>0.156</td>
</tr>
<tr>
<td>( Y_3 )</td>
<td>0.086</td>
<td>0.462</td>
<td>0.312</td>
<td>0.171</td>
<td>0.347</td>
</tr>
<tr>
<td>( Y_4 )</td>
<td>0.061</td>
<td>0.382</td>
<td>0.254</td>
<td>0.202</td>
<td>0.191</td>
</tr>
<tr>
<td>( Y_5 )</td>
<td>0.047</td>
<td>0.217</td>
<td>0.183</td>
<td>0.336</td>
<td>0.198</td>
</tr>
<tr>
<td>Boundary</td>
<td>0.045</td>
<td>0.425</td>
<td>0.357</td>
<td>0.427</td>
<td>[0.120,0.45]</td>
</tr>
</tbody>
</table>

6.2 Calculation of project resource decision index weights

The AHP method and formula (3) are used to calculate weight index \( X_1 \sim X_2 \) \( w_{AHP} \), the CRITIC method and formula (4)-(10) are used to calculate \( X_1 \sim X_1 \) weight index \( w_{CR} \), and finally, the Lagrange multiplier method and formula (11)-(12) are used to calculate the comprehensive weight \( \lambda_j \), which forms the following judgment matrix Table 3.

### Table 3 Calculation of the weights of independent and coupled methods

<table>
<thead>
<tr>
<th>Indicator Name</th>
<th>Eigenvector</th>
<th>Variability Conflict Amount of ( w_{AHP} ) indicator indicator information of ( w_{CR} )</th>
<th>( \lambda_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index</td>
<td>1.351</td>
<td>0.27</td>
<td>0.402</td>
</tr>
<tr>
<td>Indicator Name</td>
<td></td>
<td>1.899</td>
<td>0.21</td>
</tr>
<tr>
<td>Index</td>
<td></td>
<td>4.727</td>
<td>0.252</td>
</tr>
</tbody>
</table>
According to the calculation results, the weight vector obtained by the AHP method alone is \( w_{\text{AHP}} = (0.270, 0.339, 0.100, 0.131, 0.160) \), \( X_2 \) has the largest weight value, and \( X_3 \) has the smallest weight value; the weight vector obtained by the CRITIC method alone is \( w_{\text{CRITIC}} = (0.218, 0.248, 0.257, 0.208, 0.069) \), \( X_3 \) has the largest weight value and \( X_5 \) has the smallest weight value. By coupling the two subjective and objective weight calculation methods through the Lagrange multiplier method, the combined weight of \( \lambda_j = (0.252, 0.301, 0.166, 0.171, 0.109) \), \( X_2 \) has the largest weight value and \( X_5 \) has the smallest weight value. Figure 3 shows that the objective and subjective biases can be better integrated through coupled calculations.

### 6.3 Project resource allocation decisions

According to Eqs. (1)-(2), the AHP-CRITIC combination weights are used to obtain the score of each resource based on the software, resource \( Y_1 \sim Y_5 = (0.627, 0.486, 0.457, 0.433, 0.577) \), formation Figure 2, i.e., \( Y_1 \sim Y_2 \sim Y_3 \sim Y_4 \sim Y_5 \), which results in the largest impact of the resource \( Y_1 \) ’s blending problem on the project, and the smallest impact of the resource \( Y_5 \) ’s blending problem on the project. By comparing AHP-Multi-objective Grey Target, CRITIC - Multi-objective Grey Target, and AHP-CRITIC-Multi-objective Grey Target, formation Table 4, the impact ranking of resource redeployment problem formed by multiple methods is similar, and AHP-CRITIC-Multi-objective Grey Target combines subjective judgment and objective evaluation to reflect the problem more comprehensively. Combined with the actual progress of this project example, its resource deployment problems do conform to the conclusion of this paper in order, that the resources \( Y_1 \) have a great impact on the progress of the project, and problems such as deployment difficulties often occur during construction, and have a certain impact on the financial measurement of the project; the resources \( Y_5 \) have the second largest impact, with a large amount of resource deployment and a high-risk factor, and are prone to various chain reactions due to their deployment problems; the impact gap between the resources \( Y_2, Y_3, Y_4 \) is smaller, but also in The project progress affects the project progress, there are certain mutual constraints, and it is easy to form resource cross-deployment, which causes a large coordination workload.

Table 4 Decision ranking of independent and coupled methods

<table>
<thead>
<tr>
<th>Resource Category</th>
<th>AHP</th>
<th>CRITIC</th>
<th>AHP-CRITIC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Score</td>
<td>Sort by Score</td>
<td>Score</td>
</tr>
<tr>
<td>Resources ( Y_1 )</td>
<td>0.561</td>
<td>2</td>
<td>0.701</td>
</tr>
<tr>
<td>Resources ( Y_2 )</td>
<td>0.457</td>
<td>3</td>
<td>0.511</td>
</tr>
<tr>
<td>Resources ( Y_3 )</td>
<td>0.449</td>
<td>4</td>
<td>0.462</td>
</tr>
<tr>
<td>Resources ( Y_4 )</td>
<td>0.401</td>
<td>5</td>
<td>0.471</td>
</tr>
<tr>
<td>Resources ( Y_5 )</td>
<td>0.564</td>
<td>1</td>
<td>0.602</td>
</tr>
</tbody>
</table>

Figure 1 Weight calculation and gray target decision score

Figure 2 Bull's-eye distance

### 7. Conclusion and improvement

(1) Considering the qualitative and quantitative analysis of index weights and calculating the comprehensive weights by coupling the Lagrange multiplier method, a multi-objective weighted gray target decision model based on AHP-CRITIC is constructed to study the project resource deployment selection problem.
(2) The multi-objective gray target weighted decision model is validated through the example calculation of the YB Avenue project, the Euclidean spatial bull's-eye distance is calculated, the bull's-eye distances of the options are compared, the ranking of the options is derived, and the key factors affecting the resource allocation problem of the project are identified.

(3) Through an in-depth analysis of the YB Avenue project, the reasonableness and feasibility of the conclusions in the example calculations were verified in light of the actual situation of project implementation. Meanwhile, this paper can further adapt the large-scale group decision algorithm to continuously improve the accuracy of the decision and form a decision database, which is easy to be used in various engineering projects.

References


