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Research on time-varying economic dispatch of smart grid based on Lagrangian pairing

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Abstract. To design a class of time-varying economic dispatching algorithms for smart grids based on the Lagrangian dual idea for the time-varying load problems that arise in practical applications in smart grids, and to demonstrate the convergence of the algorithms.

1. Introduction

Currently, smart grid is a hot issue in power system research[1,2]. Smart grid, called "Grid 2.0", is an intelligent grid that combines the traditional grid with advanced information and communication facilities to achieve two-way information transfer between the grid and end users. It isolates faulty components of the grid and automatically restores the system to its normal state to ensure system stability; it facilitates user participation in managing the operation of the power system; it provides a fast solution for restoring power when the grid is attacked and fails; and it allows all types of power generation systems to be easily connected to the system[3]. All countries attach great importance to the construction and development of smart grids. For example, the U.S. is building a smart grid in three directions: 1) upgrading the grid infrastructure to improve the reliability of power supply; 2) applying advanced information, communication and computer technologies to the power system; and 3) enhancing the information transfer between customers and power companies through advanced meter improvements. The main focus of the European smart grid is to improve the utilization of new energy sources to improve environmental, climate and energy issues. In addition, the main focus of China's smart grid is to try to meet the energy supply needed for the country's economic development. The development of smart grid is accompanied by a series of new technologies, new equipment updates and breakthroughs, and will encounter many new problems[4]. Among them, the issue of economic dispatch is widely The main focus is on the power distribution of the power generation, which aims to save the cost of power generation and ensure the balance of the grid..

The existing economic scheduling studies of smart grids mainly focus on the case of constant load, which is too strict an assumption[5]. A new class of economic dispatching algorithms is designed considering the time-varying load case in practical applications, and the convergence of the algorithms is rigorously demonstrated.

2. Symbols and problem descriptions

2.1 Symbols

Let $x_i \in R^{ni}$, $i = 1,2,\dots,m$, $\mathrm{col}(x_1,\dots,x_m) = [x_1^T,\dots,x_m^T]^T$, where x_i^T is the transpose of x_i. I_n is an $n \times n$ unit matrix. $\nabla_x f(x,t)$ and $\nabla_{xx} f(x,t)$ denote the first order as well as the second order partial derivatives of the function $f(x,t)$ with respect to the vector x. $\nabla_{xt} f(x,t)$ denotes the function $\nabla_x f(x,t)$ with respect to t for the first-order partial derivative.

2.2 Problem Description

Suppose there is a grid with N buses and each bus line contains a generator and a local load, and denote by $P_i(t) \in \mathbb{R}$ and $D_i(t) \in \mathbb{R}$ denote the active power generated by the generators on the bus and the active power required by the load on the bus, respectively, and assume that each bus has a local load. the active power generated by the generator on the bus and the active power required by the load on the bus, respectively, and assuming that each each bus is assigned a local cost function.

$$f_i(P_i) = a_i P_i^2 + b_i P_i + c_i \quad (1)$$

where $a_i > 0$, $b_i > 0$, $c_i > 0$. Next consider the following economic scheduling problem:

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$$P_i^* = argmin \sum_{i=1}^{n} f_i(P_i) \quad (2)$$

$$s.t. \sum_{i=1}^{N} P_i = \sum_{i=1}^{N} D_i(t) \quad (3)$$

$$0 \le P_i \le P_i^{max} \quad (4)$$

where f_i is known only for the bus. The goal of this paper is to design an algorithm for the bus is to design an algorithm such that the system

$$P_i^* = u_i \quad (5)$$

realizes $P_i \to P_i^*(t)$, $t \to \infty$, where $P_i^*(t)$ is the above optimal solution of the optimization problem.

Assume 1. The first-order and second-order derivatives of the load D_t exist and are bounded.

Note 1. From $a_i > 0$ it can be inferred that the overall cost function $\sum f_i(P_i)$ is convex , so the optimal solution of problem (2) exists. Assumption 1 can be satisfied in many cases because the actual load demand always varies slowly.

3. Algorithm derivation

Theorem 1. Let $a_i^* = a_i + \frac{c(t)}{2(P_i - P_i^{max})^2}$,$c(t) = e^{-2t}$,$b_i^* = b_i + 2a_i P_i^{max}$, $\theta^* = \sum_{i=1}^{N} \frac{1}{2a_i^*}$. Consider the following The following economic scheduling algorithm is

$$\dot{P}_i = -P_i + P_i^{max} - \frac{b_i^*}{2a_i^*} + \frac{1}{2a_i^* \theta^*} \sum_{i=1}^{N} \left(D_i + \dot{D}_i - P_i^{max} + \frac{b_i^*}{2a_i^*} \right) \quad (6)$$

$$\dot{\lambda} = -\lambda + \frac{1}{\theta^*} \sum_{i=1}^{N} \left(D_i + \dot{D}_i - P_i^{max} + \frac{b_i^*}{2a_i^*} \right) \quad (7)$$

The algorithm can realize the time-varying economic dispatch problem of smart grid.

Proof: The economic scheduling problem is essentially a constrained optimization problem. problem, and according to the design ideas of typical optimization algorithms, the optimization algorithm can be designed by constructing Lagrangian function, based on the idea of duality to design the optimization algorithm. Specifically, the following Lagrangian function is defined that,

$$L_2 = \sum_{i=1}^{n} f_i(P_i) + \lambda \left(\sum_{i=1}^{N} D_i(t) - \sum_{i=1}^{N} P_i \right) - c(t) \sum_{i=1}^{N} log(P_i^{max} - P_i) \quad (8)$$

Consider the original optimization variables and the dyadic variables in combination that is, let $z = col(P_1, \dots, P_N, \lambda)$. Thus the above economic scheduling algorithm can be written in a more concise form, by means of the Lagrangian function The expression can be obtained whose first-order gradient has the following form:

$$\nabla_z L_2 = \begin{pmatrix} 2a_1 P_1 + b_1 - \lambda - \frac{c(t)}{(P_1 - P_1^{max})} \\ \vdots \\ 2a_N P_N + b_N - \lambda - \frac{c(t)}{(P_N - P_N^{max})} \\ \sum_{i=1}^{N} D_i - \sum_{i=1}^{N} P_i \end{pmatrix} \quad (9)$$

$$\nabla_{zt} L_2 = \begin{pmatrix} (P_1 - P_1^{max}) \\ \vdots \\ -\frac{c(t)}{(P_N - P_N^{max})} \\ \sum_{i=1}^{N} D_i \end{pmatrix} \quad (10)$$

In addition, the Lagrangian function expression gives its second order gradient degree has the following form.

$$\nabla_{zz} L_2 = \begin{bmatrix} & \Delta_{11} & 0 & \cdots & 0 & -1 \\ & 0 & \Delta_{22} & \cdots & 0 & -1 \\ -1 & \vdots & \vdots & \ddots & \vdots & \vdots \\ & 0 & 0 & \cdots & \Delta_{NN} & -1 \\ & -1 & -1 & \cdots & -1 & 0 \end{bmatrix} \quad (11)$$

where $\Delta_{ii} = 2a_i + \frac{c(t)}{(P_t - P_t^{max})^2}$, $i = 1, \dots, N$. To further investigate the convergence properties of the algorithm, by simplifying the calculation of the Lagrangian Langeland function first-order gradient and its partial derivative with respect to time has a special symmetric form. Specifically, it is not difficult to compute

$$\nabla_{zz} L_2 = \begin{bmatrix} & 2a_1^* & 0 & \cdots & 0 & -1 \\ & 0 & 2a_2^* & \cdots & 0 & -1 \\ -1 & \vdots & \vdots & \ddots & \vdots & \vdots \\ & 0 & 0 & \cdots & 2a_N^* & -1 \\ & -1 & -1 & \cdots & -1 & 0 \end{bmatrix} \quad (12)$$

$$\nabla_z L_2 + \nabla_{zt} L_2 = \begin{pmatrix} 2a_1^*(P_1 - P_1^{max}) + b_1^* - \lambda \\ \vdots \\ 2a_N^*(P_N - P_N^{max}) + b_N^* - \lambda \\ \sum_{i=1}^{N} D_i - \sum_{i=1}^{N} P_i + \sum_{i=1}^{N} \dot{D}_i \end{pmatrix} \quad (13)$$

Further, the inverse matrix of the Hessian matrix can be obtained as the following expressions are obtained:

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$$\nabla_{zz}^{-1}L_2 = \begin{bmatrix} \Lambda_{11} & -\frac{1}{4a_1^*a_2^*\theta^*} & \cdots & -\frac{1}{4a_1^*a_N^*\theta^*} & -\frac{1}{2a_1^*\theta^*} \\ -\frac{1}{4a_2^*a_1^*\theta^*} & \Lambda_{22} & \cdots & -\frac{1}{4a_2^*a_N^*\theta^*} & -\frac{1}{2a_2^*\theta^*} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{1}{4a_N^*a_1^*\theta^*} & -\frac{1}{4a_N^*a_2^*\theta^*} & \cdots & \Lambda_{NN} & -\frac{1}{2a_N^*\theta^*} \\ -\frac{1}{2a_1^*\theta^*} & -\frac{1}{2a_2^*\theta^*} & \cdots & -\frac{1}{2a_N^*\theta^*} & -\frac{1}{\theta^*} \end{bmatrix} \quad (14)$$

where $\Lambda_{ii} = \frac{1}{2a_i^*} - \frac{1}{4(a_i^*)^2\theta^*}$, $i = 1, \dots, N$. The symmetry of the form as above is carefully studied, and the trajectory of the original optimal variables and the pairwise variables over time can be investigated. Based on the symmetric form of the first-order gradient of the Lagrangian function and its partial derivatives with respect to time, the following relationship exists between the original optimal variables and the trajectory of the dyadic variables with respect to time:

$$\dot{z} = -\nabla_{zz}^{-1}L_2(S\nabla_z L_2 + \nabla_{zt}L_2) \quad (15)$$

where $S = I$. It is further obtained that

$$\dot{\nabla_z L_2} = -\nabla_z L_2 \quad (16)$$

The equation further states that there exists $\|z - z^*\| \le ce^{-t}\|\nabla_z L_1(z(0), 0)\|_2$, that is, the variable z will converge to the optimal solution. In addition, it is known from the above equation that the convergence of z has an exponential rate of convergence, so it is robust to external perturbations, especially to perturbation signals of small amplitude.

4. Simulation Example

Considering the specific economic scheduling problem, the corresponding parameters are shown in Table 1 shown in Table 1, the load-time variable track line can be characterized as

$$D_i = 70 + 10isin(0.5t) \quad (17)$$

Table 1. IEEE-14 bus parameters.

Bus	a_i	b_i	c_i
1	**0.04**	**2.0**	**561**
2	**0.03**	**3.0**	**310**
3	**0.035**	**4.0**	**4.0**
6	**0.03**	**4.0**	**4.0**
8	**0.04**	**2.5**	**78**

Based on the algorithm in Theorem 1, the numerical simulation leads to the specific of the bus load trajectory, as shown in Figures 1 and 2. It can be seen that the algorithm can achieve the economic scheduling goal and the convergence rate has exponential characteristics.

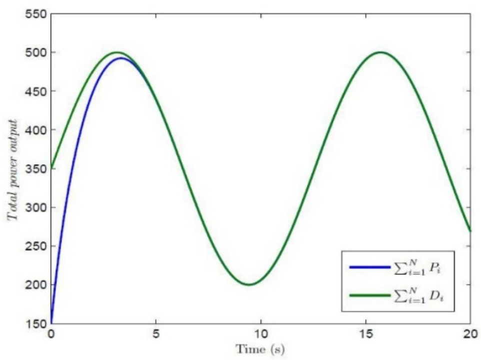

Figure 1. Time-varying load and power generation curves.

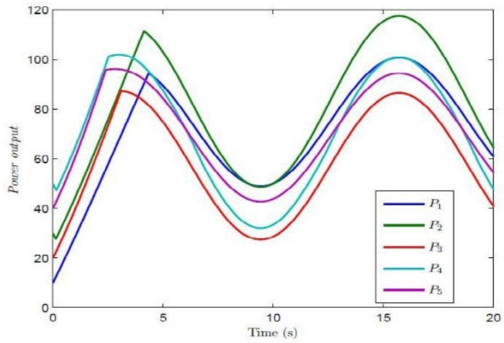

Figure 2. Different bus generation power rail lines.

5. Conclusion

The time-varying economic dispatching problem of smart grid is studied, and the corresponding algorithm is designed based on the Lagrangian pair idea, and the effectiveness of the algorithm is demonstrated. The designed scheduling algorithm is simple in form, low in complexity, and easy to calculate, which is well adapted to the actual grid system operation requirements; in addition, the algorithm has exponential convergence speed and is robust to external disturbance signals, so it is applicable to smart grids of different scales. Future research can turn to the economic dispatching problem of communication-constrained smart grid, the economic dispatching problem of information transmission with time delay and the economic dispatching problem of finite time convergence, etc.

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