Filtration velocity under roller squeezing

Sh. Khurramov¹*, F. Khalturaev¹, and F. Kurbanova¹

¹Tashkent Institute of Architecture and Civil Engineering, 13, Navoi, Tashkent, Uzbekistan

Abstract. Analytical dependencies are determined that describe the patterns of change in filtration rates in the squeezing area. It was established that the fluid filtration rate along the abscissa axis at the boundaries of the compression zone is zero, and inside the zone, it takes negative values. It was found that the fluid filtration rates along the ordinate axis are zero at the beginning of the compression zone; they increase to a maximum at a point lying on the line of centers, and in the zone of strain restoration, they depend on the angle that determines the position of the point where the fluid changes direction. Keywords: roll squeezing, moisture filtration, filtration speed, pressing area.

1 Introduction

The theoretical description of the roller pressing process is one of the most difficult tasks in modern mechanics. The problem lies in the need to jointly solve the problem of contact in a two-roll module (a contact problem) and moisture filtration in a deformable non-homogeneous porous medium (a hydraulic problem). In this case, one or both rollers are coated with a material having viscoelastic properties. The situation is also aggravated by the large strains in the processed materials and the nonlinearity of the equations describing the process.

In [1-6], the main contact problems of roller pressing were solved. The main hydraulic problems of roller pressing are the analytical description of the distribution of hydraulic pressure and mathematical modeling of the residual moisture content in the squeezed material. To solve these problems, it is necessary to know the patterns of change in filtration rates in the squeezing area.

An analysis of the publications devoted to the study of hydraulic problems of roller pressing [7–14] showed that the existing patterns of change in filtration rates in the pressing area were obtained with the introduction of models of roll devices and materials that do not correspond to the real physical phenomena of roller squeezing. Therefore, they do not provide a possibility to solve problems that allow the revealing of the hydraulic phenomenon of roller squeezing.

In [15], analytical dependencies were determined that describe the patterns of changes in filtration rates in the pressing area for a symmetrical two-roll module. However, many roller-squeezing machines are asymmetrical. Several types of asymmetry are often realized simultaneously, for example, two types of geometric asymmetry - different diameters and inclination of the material layer relative to the horizontal line. In this regard, a generalized

* Corresponding author: shavkat-xurrnov59@mail.ru
two-roll module of a squeezing machine was developed in order to systematize the studies of roller pressing [9].

Fig. 1. Scheme of a two-roll module of squeezing machine.

To further develop theoretical concepts given in [1,2], the object of study is a generalized two-roll module, in which the rolls are located relative to the vertical line with an inclination to the right at an angle of, have unequal diameters with elastic coatings, a layer of wet (processed) material of a uniform thickness is fed tilted downward with respect to the line of centers at an angle of (Fig. 1).

2 Materials and methods

The lower roll contact curve (curve \( A_1A_2 \)) consists of two zones \( A_1A_3 \) and \( A_3A_2 \). In zone \( A_1A_3 \), the material being processed is compressed, so the fluid flows from it into the roll coating at a polar angle [14].

In the process of filtration in the wringing zone, the continuity equation is fulfilled [13]:

\[
\frac{\partial (\varepsilon_{11} v_m + u_{11x})}{\partial x_{11}} + \frac{\partial (u_{11y})}{\partial y_{11}} = 0, \tag{1}
\]

where \( v_{11x}, v_{11y} \) — absolute and relative velocities of fluid in zone \( A_1A_3 \) along the axis \( Ox \); \( v_{11a}, v_{11y} \) — absolute and relative velocities of fluid in zone \( A_1A_3 \) along the axis \( Oy \); \( u_{11x}, u_{11y} \) — is the fluid filtration rate in zone \( A_1A_3 \); \( \varepsilon_{11} \) — is the relative strain of the material in zone \( A_1A_3 \); \( v_m \) — squeezing material speed.

From equation (1) it follows

\[
v_m \frac{\partial \varepsilon_{11}}{\partial x_{11}} + \frac{\partial u_{11x}}{\partial x_{11}} + \frac{\partial u_{11y}}{\partial y_{11}} = 0. \tag{2}
\]
Let us proceed to differentiation in one variable $\theta_1 + \gamma$:

$$v_m \frac{d\varepsilon_{11}}{d(\theta_1 + \gamma)} \frac{d(\theta_1 + \gamma)}{dx_{11}} + \frac{du_{11x}}{d(\theta_1 + \gamma)} \frac{d(\theta_1 + \gamma)}{dx_{11}} + \frac{du_{11y}}{d(\theta_1 + \gamma)} \frac{d(\theta_1 + \gamma)}{dy_{11}} = 0.$$  

After transformation we obtain

$$\frac{du_{11y}}{d(\theta_1 + \gamma)} = -v_m \frac{d\varepsilon_{11}}{d(\theta_1 + \gamma)} \frac{dy_{11}}{dx_{11}} \frac{du_{11x}}{d(\theta_1 + \gamma)} \frac{dy_{11}}{dx_{11}} = 0.$$ (3)

Note, that

$$u_{11y} = u_{11x} \tan(\theta_1 + \gamma).$$ (4)

Let us differentiate

$$\frac{du_{11y}}{d(\theta_1 + \gamma)} = \frac{du_{11x}}{d(\theta_1 + \gamma)} \tan(\theta_1 + \gamma) - u_{11x} \frac{1}{\sin^2(\theta_1 + \gamma)}.$$ (5)

Considering this, from equality (3) we obtain

$$\frac{dx_{11}}{d(\theta_1 + \gamma)} \cos(\theta_1 + \gamma) + \frac{dy_{11}}{d(\theta_1 + \gamma)} \sin(\theta_1 + \gamma) \frac{du_{11x}}{d(\theta_1 + \gamma)} - \frac{dx_{11}}{d(\theta_1 + \gamma)} \sin(\theta_1 + \gamma) \frac{dy_{11}}{d(\theta_1 + \gamma)} - \frac{1}{\sin^2(\theta_1 + \gamma)} u_{11x} = -v_m \frac{d\varepsilon_{11}}{d(\theta_1 + \gamma)} \frac{dy_{11}}{dx_{11}}.$$ (6)

From figure 1, it follows that

$$x_{11} = r_1 \sin(\theta_1 + \lambda), \quad y_{11} = r_1 \cos(\theta_1 + \gamma), \quad \varepsilon_{11} = \frac{1}{h_{11}^0} \left( r_1 - R_1 \cos(\theta_1 + \gamma) \right).$$

where

$$h_{11}^0 = \frac{\sin(\varphi_2 - \gamma)}{\sin(\varphi_1 + \varphi_2)}.$$ 

Hence
\[
\frac{dx_{11}}{d(\theta_{11} + \gamma)} = r'_{11} \sin(\theta_{11} + \gamma) + r_{11} \cos(\theta_{11} + \gamma),
\]
\[
\frac{dy_{11}}{d(\theta_{11} + \gamma)} = r'_{11} \cos(\theta_{11} + \gamma) - r_{11} \sin(\theta_{11} + \gamma),
\]
\[
\frac{d\varepsilon_{11}}{d(\theta_{11} + \gamma)} = \frac{1}{h_{11}^{0}} \left( r'_{11} - R_{1} \frac{\cos(\theta_{11} + \gamma) \sin(\theta_{11} + \gamma)}{\cos^{2}(\theta_{11} + \gamma)} \right).
\]

Substituting these derivatives into equalities (6) and considering \( r'_{11} \sin^{2}(\theta_{11} + \gamma) \approx 0 \), after simple transformations we find
\[
= \frac{1}{\cos(\theta_{11} + \gamma) \sin(\theta_{11} + \gamma)} \cdot \frac{du_{11x}}{d(\theta_{11} + \gamma)} - \frac{u_{11x}}{\sin^{2}(\theta_{11} + \gamma)} = \nu \frac{r'_{11} \cos(\theta_{11} + \gamma) - r_{11} \sin(\theta_{11} + \gamma)}{r'_{11} \sin(\theta_{11} + \gamma) + r_{11} \cos(\theta_{11} + \gamma)} \cdot \frac{1}{h_{11}^{0}} \left( r'_{11} - R_{1} \frac{\cos(\theta_{11} + \gamma) \sin(\theta_{11} + \gamma)}{\cos^{2}(\theta_{11} + \gamma)} \right).
\]

For the considered two-roll module, the expressions \( r_{11} \) and \( r'_{11} \) have the form [2]:
\[
r_{11} = \frac{R_{1}}{1 + k_{11} \lambda_{11}} \left( 1 + k_{11} \lambda_{11} \frac{\cos(\phi_{11} + \gamma_{1})}{\cos(\theta_{11} + \gamma)} \right), \quad -\left(\phi_{11} + \gamma_{1}\right) \leq \theta_{11} + \gamma \leq 0,
\]
\[
r'_{11} = \frac{k_{11} \lambda_{11} R_{1}}{1 + k_{11} \lambda_{11}} \frac{\cos(\phi_{11} + \gamma_{1})}{\cos \phi(\theta_{11} + \gamma)} \tan(\theta_{11} + \gamma),
\]

where \( k_{11} = \frac{m_{11} H_{1} \sin(\phi_{11} + \varphi_{21})}{m_{11} \delta_{1} \sin(\varphi_{21} - \gamma_{1})} \), \( \lambda_{11} = \frac{A^{*}_{1} m_{11} (\Delta_{11})_{cp} - (A_{11}(1 - m_{11}) - A_{11}(1 - m_{11}^{*})) h_{11}^{0}}{A_{11} m_{11} (\Delta_{11})_{cp} + (A_{11}(1 - m_{11}) - A_{11}(1 - m_{11}^{*})) H_{1}} \),
\[
(\Delta_{11})_{cp} = R_{1} \left( 1 - \frac{\sin 2(\phi_{11} + \gamma_{1})}{2(\phi_{11} + \gamma_{1})} \right).
\]

here \( m_{11} \) – is the coefficient of strengthening of the points of elastic coating of the lower roll under compression; \( m_{11}^{*} \) – is the coefficient of strengthening of the points of the processed material under compression.

Using expressions (8) and (9), we obtain
\[
r'_{11} \sin(\theta_{11} + \gamma) + r_{11} \cos(\theta_{11} + \gamma) = \frac{R_{1}}{1 + k_{11} \lambda_{11}} \frac{k_{11} \lambda_{11} \cos(\phi_{11} + \gamma_{1}) + \cos^{3}(\theta_{11} + \gamma)}{\cos^{2}(\theta_{11} + \gamma)}.
\]
\[ r'_1 \cos(\theta_1 + \gamma) - r_1 \sin(\theta_1 + \gamma) = -\frac{R_1}{1 + k_{11} \lambda_{11}} \sin(\theta_1 + \gamma), \]

\[
\frac{d \xi_{11}}{d(\theta_1 + \gamma)} = -\frac{R_1}{h_{11}^0(1 + k_{11} \lambda_{11})} \sin(\theta_1 + \gamma) \cos(\theta_1 + \gamma). \quad (10)
\]

After substituting these expressions and some transformations, equation (7) takes the following form

\[
\frac{du_{11x}}{d(\theta_1 + \gamma)} = \cos(\theta_1 + \gamma) \sin(\theta_1 + \gamma) u_{11x} =
\]

\[
= v_m R_1 \cos(\varphi_{11} + \gamma_1) \cdot \frac{\sin^3(\theta_1 + \gamma) \cos(\theta_1 + \gamma)}{(k_{11} \lambda_{11} \cos(\varphi_{11} + \gamma_1) + \cos^3(\theta_1 + \gamma))}. \quad (11)
\]

The homogeneous part of the differential equation (11) has the solution

\[ u_{11x} = C_{11}(\theta_1 + \gamma) \sin(\theta_1 + \gamma). \quad (12) \]

hence

\[
\frac{du_{11x}}{d(\theta_1 + \gamma)} = \frac{dC_{11}}{d(\theta_1 + \gamma)} \sin(\theta_1 + \gamma) + C_{11} \cos(\theta_1 + \gamma).
\]

Substituting \( \frac{du_{11x}}{d(\theta_1 + \gamma)} \) and \( u_{11x} \) into equation (12) and applying the assumptions

\[ \sin(\theta_1 + \gamma) \approx \theta_1 + \gamma, \quad \cos(\theta_1 + \gamma) \approx 1 - \frac{(\theta_1 + \gamma)^2}{2}, \]

we have

\[
\frac{dC_{11}}{d(\theta_1 + \gamma)} = a_{11} \frac{(\theta_1 + \gamma)^2}{m_{11}^2 - (\theta_1 + \gamma)^2}, \quad (13)
\]

where

\[ a_{11} = \frac{2v_m R_1 \cos(\varphi_{11} + \gamma_1)}{3h_{11}^0(1 + k_{11} \lambda_{11})}, \quad m_{11}^2 = \frac{2}{3} (1 + k_{11} \lambda_{11} \cos(\varphi_{11} + \gamma_1)). \quad (14) \]

Integrating equality (21), we obtain

\[
C_{11} = a_{11} \left( -\left( \theta_1 + \varphi \right) + \frac{m_{11}^2}{2} \ln \left[ \frac{m_{11} + (\theta_1 + \gamma)}{m_{11} - (\theta_1 + \gamma)} \right] \right) + C_{11}^*. 
\]
Expanding the logarithmic function into a series and being limited to terms in the third power with respect to \((\theta_1 + \gamma)\), we have \(C_{11} = \frac{a_{11}}{3m_{11}^2} (\theta_1 + \gamma)^3 + C_{11}^*\).

Substitute \(C_{11}\) into equation (12)

\[
u_{1lx} = \left(\frac{a_{11}}{3m_{11}^2} (\theta_1 + \gamma)^3 + C_{11}^*\right) \sin(\theta_1 + \gamma).
\]

Taking into account the initial condition \(u_{1lx}(-\varphi_1 + \gamma_1) = 0\), we find

\[
u_{1lx} = b_1((\varphi_1 + \gamma_1)^3 + (\theta_1 + \gamma)^3) \sin(\theta_1 + \gamma), \quad -\varphi_1 + \gamma_1 \leq \theta_1 + \gamma \leq 0, \quad (15)
\]

where \(b_1 = \frac{v_m R_l \cos(\varphi_1 + \gamma_1)}{3h_1^0 (1 + k_1 A_1)(1 + k_1 A_1 \cos(\varphi_1 + \gamma_1))}\).

Then, taking into account equality (4), we have

\[
u_{1lx} = -b_1((\varphi_1 + \gamma_1)^3 + (\theta_1 + \gamma)^3) \cos(\theta_1 + \gamma), \quad -\varphi_1 + \gamma_1 \leq \theta_1 + \gamma \leq 0, \quad (16)
\]

\[
u_{1lx} = -b_1((\varphi_1 + \gamma_1)^3 + (\theta_1 + \gamma)^3), \quad -\varphi_1 + \gamma_1 \leq \theta_1 + \gamma \leq 0. \quad (17)
\]

The sign \((-)\) in formulas (16) and (17) means that the fluid flows from the fibrous material down into the coatings of the lower roll.

Formula (17) determines the change in the filtration rate of the liquid flowing from the material into the coatings of the lower roll in the compression zone.

In zone \(A_3 A_2\), the processed material, restoring the strain, can absorb fluid from the roll coating. In this case, the fluid, to the left of some point \(A_4\) of zone \(A_3 A_2\), flows from the material into the roll coating; to the right, it flows into the material being processed. Depending on the roll design of the squeezing machine and its coating, point \(A_4\) can coincide with any point of the strain restoration zone \([9, 10, 13]\).

Let point \(A_4\) be determined by the angle of \(\varphi_4 + \gamma_4 = \xi_1(\varphi_2 + \gamma_2), 0 < \xi_1 \leq 1\). Point \(A_4\) divides the strain restoration zones into two sections: the first section \(A_3 A_4\), where \(0 \leq \theta_2 + \gamma \leq \varphi_4 + \gamma_4\) and the second section \(A_4 A_2\), where \(\varphi_4 + \gamma_4 \leq \theta_2 + \gamma \leq \varphi_2 + \gamma_2\). At point \(A_4\) filtration rates are zero, that is, \(u_{12x}(\varphi_4 + \gamma_4) = 0\) and \(u_{12y}(\varphi_4 + \gamma_4) = 0\).

Similarly to formulas (15) - (17) for the section \(A_3 A_4\) we have:

\[
u_{12x} = b_1((\varphi_4 + \gamma_4)^3 - (\theta_2 + \gamma)^3) \sin(\theta_2 + \gamma), \quad 0 \leq \theta_2 + \gamma \leq \varphi_4 + \gamma_4, \quad (18)
\]

\[
u_{12y} = -b_1((\varphi_4 + \gamma_4)^3 - (\theta_2 + \gamma)^3) \cos(\theta_2 + \gamma), \quad 0 \leq \theta_2 + \gamma \leq \varphi_4 + \gamma_4, \quad (19)
\]

\[
u_{12\theta} = -b_1((\varphi_4 + \gamma_4)^3 - (\theta_2 + \gamma)^3), \quad 0 \leq \theta_2 + \gamma \leq \varphi_4 + \gamma_4, \quad (20)
\]
where \( b_{12} = \frac{\nu_m R_1 \cos(\varphi_{12} + \gamma_{12})}{3h_{12}^0(1 + k_1 \lambda_{12})(1 + k \lambda_{12} \cos(\varphi_{12} + \gamma_{12}))} \).

The authors of [9,13] believe that in the second section \( A_4A_2 \) of the strain restoration zone, the filtration rate along the \( Ox \) axis is zero, i.e., \( u_{12x}(\theta_{12} + \gamma) = 0 \), where \( \varphi_{14} + \gamma_{4} \leq \theta_{12} + \gamma \leq \varphi_{12} + \gamma_{2} \). In this case, from the equality corresponding to (3), it follows that

\[
\frac{du_{12y}}{d(\theta_{12} + \gamma)} = -\nu_m \frac{d\xi_{12}}{d(\theta_{12} + \gamma)} \cdot \frac{dy_{12}}{d(\theta_{12} + \gamma)}
\]

or

\[
\frac{du_{12y}}{d(\theta_{12} + \gamma)} = -\nu_m \frac{R_1 \cos(\varphi_{12} + \gamma_{2})}{h_{12}^0(1 + k_1 \lambda_{12})} \cdot \frac{\sin^2(\theta_{12} + \gamma)}{(k_1 \lambda_{12} \cos(\varphi_{12} + \gamma_{2}) + \cos^3(\theta_{12} + \gamma)).}
\]

Let us transform it taking into account similar expression (18)

\[
\frac{du_{12y}}{d(\theta_{12} + \gamma)} = a_{12} \frac{(\theta_{12} + \gamma)^2}{m_{12}^2 - (\theta_{12} + \gamma)}. \tag{21}
\]

The solution to this equation is

\[
u_{12y} = \left[ a_{12} \frac{(\theta_{12} + \gamma)^2}{3m_{12}^2} + C_{13} \right].
\]

or into account condition \( u_{12y}(\varphi_{14} + \gamma_{4}) = 0 \)

\[ u_{12y} = b_{12}((\varphi_{14} + \gamma_{4})^3 - (\theta_{12} + \gamma)^3), \quad \varphi_{14} + \gamma_{4} \leq \theta_{12} + \gamma \leq \varphi_{12} + \gamma_{2}. \tag{22} \]

Hence, we have

\[ u_{12\theta} = -b_{12}((\varphi_{14} + \gamma_{4})^3 - (\theta_{12} + \gamma)^3), \quad \varphi_{14} + \gamma_{4} \leq \theta_{12} + \gamma \leq \varphi_{12} + \gamma_{2}. \tag{23} \]

Generalizing formulas (17), (20), and (23), we obtain

\[
\begin{cases}
\nu_{11\theta} = -b_{11}((\varphi_{11} + \gamma_{1})^3 + (\theta_{11} + \gamma)^3), \quad -(\varphi_{11} + \gamma_{1}) \leq \theta_{11} + \gamma \leq 0, \\
\nu_{12\theta} = -b_{12}((\varphi_{14} + \gamma_{4})^3 - (\theta_{12} + \gamma)^3), \quad 0 \leq \theta_{12} + \gamma \leq \varphi_{12} + \gamma_{2},
\end{cases}
\tag{24}
\]

where \( \varphi_{14} + \gamma_{4} = \varphi_{1}(\varphi_{12} + \gamma_{2}), \quad 0 < \varphi_{1} \leq 1. \)
The filtration rate of fluid flowing through the contact curve of the upper roll is determined likewise.

It has the following form:

\[
\begin{cases}
    u_{21\theta} = b_{21}((\phi_{21} - \gamma_1)^3 + (\theta_{21} - \gamma)^3), \quad -(\phi_{21} - \gamma_1) \leq \theta_{21} - \gamma \leq 0, \\
    u_{22\theta} = b_{22}((\phi_{24} - \gamma_4)^3 - (\theta_{22} - \gamma)^3), \quad 0 \leq \theta_{22} - \gamma \leq \phi_{22} - \gamma_2,
\end{cases}
\]

(25)

where \( \phi_{24} - \gamma_4 = \zeta_2(\phi_{22} - \gamma_2) \), \( 0 < \zeta_2 \leq 1 \),

\[
b_{21} = \frac{v_n R_2 \cos(\phi_{21} - \gamma_1)}{3h_{21}^0(1 + k_{21}h_{21})(1 + k_{21}h_{21} \cos(\phi_{21} - \gamma_1))},
\]

\[
b_{22} = \frac{v_n R_2 \cos(\phi_{22} - \gamma_2)}{3h_{22}^0(1 + k_{22}h_{22})(1 + k_{22}h_{22} \cos(\phi_{22} - \gamma_2))}.
\]

3 Results

Analytical dependencies are determined that describe the patterns of changes in filtration rates in the pressing area.

4 Conclusions

From the analysis of calculated data and graphs (Figures 2-3), it follows that:

- the fluid filtration rate along the \( O\xi \) axis at the boundaries of the compression zone is zero, and inside the zone, it takes negative values. It has a minimum at a point determined by angle of \( \theta_{11} + \gamma = 0,63(\phi_1 + \gamma_1) \) (\( \phi_1 \) is the gripping angle);

![Fig. 2. Graph of changes in filtration rates](image)

Fig. 2. Graph of changes in filtration rates \( u_\chi \) : 1\( - \zeta = 0,33; \ 2 - \zeta = 0,67; \ 3 - \zeta = 1 \).

![Fig. 3. Graph of changes in filtration rates](image)

Fig. 3. Graph of changes in filtration rates \( u_\theta \) of the lower roll
1\( - \zeta = 0,33; \ 2 - \zeta = 0,67; \ 3 - \zeta = 1 \).
• pattern of changes in filtration rates along the $Ox, Oy$ axes, and the $\theta$ angle in the restoration zone depend on number $\zeta_i$, which determines the position of the point where the fluid changes direction. The closer $\zeta_i$ to zero, the longer the part of the strain restoration zone where the fluid flows from the roll coating back into the wet material. As the number $\zeta_i$ increases from zero to one, the extent of the filtration rate change along the $Ox$ axis increases. At $\zeta_i = 1$, the filtration rate $u_x$ inside the restoration zone is positive and has the maximum at the point determined by an angle of $\theta_2 + \gamma = 0.63(\varphi_2 + \gamma_2)$ ($\varphi_2$ – is the exit angle). It is zero at the beginning and end of the strain restoration zone.

References

5. Z. Rakhimova, G. Bahadirov, M. Musirov et al, Cite as: AIP Conference Proceedings 2637, 060006 (2022)
8. D. McDonald, R.J. Kerekes, J. Zhao, BioResources 15, 7319-7329 (2020)
13. N.E. Novikov, Pressing paper web (Forest industry, Moscow, 1992)