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Distribution of the temperature field in the rod, taking into account the correction for heat loss through its side surface

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Abstract. The article deals with heat transfer through a rod with a small cross-section. The rod is assumed to be so thin that the temperature at all points of its cross-section can be considered the same. When finding a solution, the power series decomposition method is used. Formulas were derived not only for finding the temperature field, taking into account the correction for heat loss through the lateral surface of the rod, but also for determining the coefficient of external heat transfer and the coefficient of thermal conductivity from known values of temperatures and heat flows at its ends.

1 Introduction

Today, heat exchange processes play an important role, both in the technical sphere and in nature. The problem of heat transfer control is one of the most urgent in modern power engineering, aerospace engineering, chemical industry and many other areas of energy technology. To date, an impressive layer of work has been devoted to the study of heat exchange processes. Of particular scientific interest are works describing modern heat exchange elements of heat exchange equipment with a detailed description of the methods of their manufacture, as well as the management of a heat exchanger with a variable heat exchange surface area. To date, a new direction is being formed, implying the inclusion of signs and properties of the material in the mathematical models being developed. Thus, functional dependencies are increasingly being included in the modelling of heat and mass transfer processes.

At the same time, the calculation of heat loss itself is used in the design of heating systems for buildings and structures. It is made in order to determine the power of the system itself. The energy calculation is reduced to calculating the heat transfer coefficients of the external elements of a residential building, which can be of various shapes.

Heating and cooling of structures directly depends on the shape and surface area of the elements themselves. In this case, the surface itself can be increased either by finning, or by replacing rods of circular cross-section, which have a minimum area, with other rods with an increased cross-section, for example, with an oval or elliptical cross-section [1-4].

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The main provisions of the theory of heat and mass transfer have been developed for a long time. Fundamentals of thermal conductivity and the use of this quality to describe the states of various bodies, as well as features of use for technological tasks aimed at transferring mass and heat [5-7]. For example, many works have been devoted to the issues of thermal calculation of elliptical bodies for the case of the presence of internal heat sources under various conditions [8, 9].

To solve problems related to finding the temperature field, a differential equation of thermal conductivity is introduced. A differential equation is usually understood as a mathematical dependence, expressed by a differential equation, between the physical quantities characterizing the phenomenon under study, and these physical quantities are functions of space and time. Such an equation characterizes the course of a physical phenomenon at any point of the body at any time.

The differential equation of thermal conductivity gives the relationship between temperature, time and coordinates of the elementary volume. It mathematically describes the transfer of heat inside the body. In order to find the temperature field inside the body at any time, that is, to solve the differential equation, it is necessary to know the temperature distribution inside the body at the initial time (initial condition), the geometric shape of the body and the law of interaction between the environment and the surface of the body (boundary condition).

The set of initial and boundary conditions is called boundary conditions; the initial condition is called a temporal boundary condition, and the boundary condition is called a spatial boundary condition.

The initial condition is determined by setting the temperature distribution law inside the body at the initial time

The differential equation together with the initial and boundary conditions completely defines the problem, that is, knowing the geometric shape of the body, the initial and boundary conditions, it is possible to solve the differential equation to the end and, consequently, find the temperature distribution function at any time.

The article deals with heat transfer through a rod with a small cross-section, taking into account the correction for heat loss through the lateral surface of the rod.

2 Main Part

Consider the case of heat transfer through a rectilinear rod of radius r. There are no heat sources in the rod $q_1 = 0$, as well as in the coating layer $q_2 = 0$. The external temperature is the same along the entire surface of the rod Te = const (fig. 1).

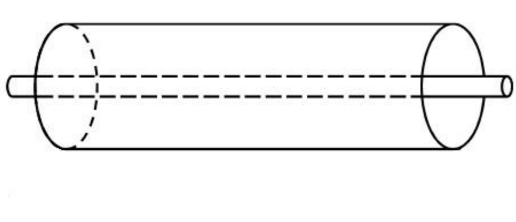

Fig. 1. Rectilinear rod with a coated shell.

The heat transfer equation in this case has the form

$$\frac{d^2T}{dx^2} + \mu^2(T_e - T) = 0 \tag{1}$$

Transform equation (1)

$$\frac{d^2T}{dx^2}-\mu^2T=-\mu^2T_e \tag{2}$$

We will look for the solution of this differential equation in the form

$$T=T(x)+T_0 \tag{3}$$

Where T(x) is the solution of a homogeneous equation

$$\frac{d^2T}{dx^2}-\mu^2T=0 \tag{4}$$

In addition, T_0 is the solution of an inhomogeneous equation

$$\frac{d^2T_0}{dx^2}-\mu^2T_0=-\mu^2T_e \tag{5}$$

Solution of the inhomogeneous equation (5)

$$T_0=T_e \tag{6}$$

We will look for the solution of the homogeneous differential equation (4) in the form [10]

$$T=C_1sh\,\mu x+C_2ch\,\mu x \tag{7}$$

We will find the values of constants C_1, C_2 by setting the following boundary conditions. Let there be a heat flow through the rod and the temperatures at the edges of the rod are given. The boundary conditions have the form

$$T\big|_{x_1}=T_1, \quad T\big|_{x_2}=T_2. \tag{8}$$

Let's define the values of constants for the function (7). In this case. For x = 0 we have $T_1=C_2$. For x = l we have

$$T_2=C_1sh\mu l+T_1ch\mu l \tag{9}$$

Where from

$$C_1=\frac{T_2}{sh\mu l}-T_1\frac{ch\,\mu l}{sh\mu l}. \tag{10}$$

The desired function (solution of a homogeneous equation), after a series of transformations, will take the form

$$T = \frac{sh\,\mu(l-x)}{sh\mu l}T_1 + \frac{sh\,\mu x}{sh\mu l}T_2 \tag{11}$$

From the relation (16) it follows that the value of the heat flux q0 entering the rod through the plane x = 0 is equal to

$$Q_0 = -\lambda S\frac{dT}{dx}\bigg|_{x=0} = \frac{\lambda S\mu(T_1 ch\,\mu l - T_2)}{sh\mu l} \tag{12}$$

The value of the heat flux ql given to the rods through the plane x = l is equal to

$$Q_l = -\lambda S\frac{dT}{dx}\bigg|_{x=l} = \frac{\lambda S\mu(T_1 - T_2 ch\,\mu l)}{sh\mu l} \tag{13}$$

The heat loss through the lateral surface of the rod between the planes x = 0 and x = l is equal to

$$Q_0 - Q_l = \lambda S\mu(T_1 + T_2)th\frac{1}{2}\mu l \tag{14}$$

In practice, α is usually small, and therefore hyperbolic functions can be represented as a series and limited to the first members of these series. In this case, you can approximately write

$$Q_0 = \frac{\lambda S}{l}(T_1 - T_2) + \frac{1}{6}\lambda S\mu^2 l(2T_1 + T_2) \tag{15}$$

$$Q_l = \frac{\lambda S}{l}(T_1 - T_2) - \frac{1}{6}\lambda S\mu^2 l(T_1 + 2T_2) \tag{16}$$

$$Q_0 - Q_l = \frac{1}{2}\lambda S\mu^2 l(T_1 + T_2) \tag{17}$$

or

$$Q_0 = \frac{\lambda S}{l}(T_1 - T_2) + \frac{1}{6}\alpha Pl(2T_1 + T_2) \tag{18}$$

$$Q_l = \frac{\lambda S}{l}(T_1 - T_2) - \frac{1}{6}\alpha Pl(T_1 + 2T_2) \tag{19}$$

$$Q_0 - Q_l = \frac{1}{2}\alpha Pl(T_1 + T_2) \tag{20}$$

From expression (18) it is possible to obtain the value of thermal conductivity λ

$$\lambda = \frac{Q_0 l - \frac{1}{6}\alpha Pl^2(2T_1 + T_2)}{S(T_1 - T_2)} \tag{21}$$

Also, from expression (20) it is possible to obtain the value of external heat transfer α

$$\alpha = \frac{2(Q_0 - Q_l)}{Pl(T_1 + T_2)} \tag{22}$$

Then the value of thermal conductivity λ is equal to

$$\lambda = \frac{Q_0 l - \frac{1}{3} l \frac{(2T_1 + T_2)(Q_0 - Q_l)}{(T_1 + T_2)}}{S(T_1 - T_2)} \tag{23}$$

The obtained formulas (22) and (23) allow us to find the value of external heat transfer α and thermal conductivity λ from the experimentally found values of temperatures and heat fluxes.

Next, from expression (21) we express T2

$$T_2 = \frac{2(Q_0 - Q_l)}{\alpha Pl} - T_1 \tag{24}$$

Then

$$T = \mathrm{T}_1 \frac{sh\mu(l-x) - sh\mu x}{sh\alpha l} + \frac{2(Q_0 - Q_1)sh\mu x}{\alpha Plsh\mu l} \tag{25}$$

The obtained formula (25) allows us to find the temperature distribution in the rod, taking into account the correction for heat loss through its side surface.

3 Conclusions

Thus, an expression was obtained in the work to determine the temperature distribution in the rod, taking into account the correction for heat loss through its lateral surface. A differential equation of thermal conductivity has been solved, which establishes a connection between temporal and spatial changes in body temperature; it mathematically describes the transfer of heat inside the body. In the solution, the method of decomposition in a row was applied. Formulas were also obtained for determining the coefficient of external heat transfer and the coefficient of thermal conductivity from known values of temperatures and heat flows.

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