Statistical distributions for modeling mineral liberation

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Abstract. Using the algorithm and the results of laboratory studies on grinding of loparite ores from the Lovozero deposit in a ball and rod mill, simulation grinding models were designed based on the transformation of a discrete function of the density distribution of outputs of mineral particle size classes. The model’s adequacy was quantitatively analysed carried out by calculating the determination coefficient without taking into account the constant component of the distribution; for a qualitative assessment of the result the authors used the Cheddock scale. The authors studied the minerals disintegration at ball and rod grinding with the following verification of the proposed algorithm. To identify the distribution of loparite by size classes in the initial ore samples and in the samples obtained in the disintegration taking place at different productivity, the X-ray diffraction analysis and the combined (weight, optical and geometric) methods were used.

1 Introduction

The improvement of mineral processing technologies determines the need to develop optimal technological flowsheets of mineral processing, taking into account the actual deterioration of the quality of ore raw materials. It can be possible through the creation of digital twins of processing production: complexes of mathematical models describing the state "as is" in a simulation object and providing control action to ensure the state "as required" [1, 2]. A special type of mathematical models are simulation models, which reproduce the behavior of the real system in time and allow obtaining detailed statistics of the system functioning depending on the input data [3, 4].

Research of potential of mathematical modeling of processing processes is widely and diversely presented today in scientific literature. For example, work [5] is devoted to development of a kinetic model of minerals extraction by grinding. Based on the population balance theory, a model is proposed here, which uses the notion of a continuous spectrum of initially bound particles in ore material fractions. The model has been validated with copper ores in both laboratory and industrial tests. Of interest is the work [6], where the separation coefficient and disintegration coefficient are introduced to evaluate the results of mineral processing, and the evaluation itself shows whether the quality of the separation product is affected by the separation process itself or by the degree of

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disintegration of minerals achieved. In [7], an attempt is made to predict changes in the distribution of minerals during grinding using the probabilistic method. And it considers the probability that a feed particle of a certain composition and size will form a product particle of a certain size and composition. Work [8] draws attention to the fact that the processes of particle size reduction are one of the most intensively studied areas of mineral processing, where research is mostly aimed at reducing the operating costs of preparatory stages and less at developing practical methods that facilitate the accelerated release of minerals from each other. The objective of work [9] was to develop a set of cost functions for major grinding mill equipment. These models were developed using two different techniques: univariate regression (UVR) and multivariate regression (MVR) based on principal component analysis (PCA). The paper [10] aimed to reveal the liberation behaviours to define a methodology for calculating the liberation distribution functions of the minerals. Within the scope, grinding tests were performed with complex copper ore, which was subjected to the ball and stirred milling. In [11], presents a strategy based on using flotation studies to model flotation and grinding via an integrated approach. The methodology, which is an approximate method that allows one to study of the effects of grinding on flotation circuits, is applied to a copper sulfide mineral with appropriate results. In [12] investigated ultra-fine coal grinding performance of four low- to moderate-cost grinding media in a laboratory stirred mill. The mathematical models generated based on population balance modeling provided an accurate forecast of the particle-size distribution. A step-wise algebraic routine in [13] is used to fit a dynamic non-linear model, specifically developed for process control, to steady-state process data of an industrial single-stage grinding mill circuit. The results indicate that the model provides a qualitatively accurate response of the main process variables. The model is for model-based predictive process control.

2 Brief description of the material to be grinded

Loparite in the original ore was both in a free form, and in aggregates with nepheline, feldspars and aegirine. The total content of liberated loparite in the original ore was 29.41%. In material larger than 1.0 mm loparite was only as inclusions in nepheline grains, less often feldspars and aegirine (Figure 1 a, b). Liberated grains of loparite appear in the -1.0+0.63 mm class and are more often twinned crystals with partially preserved cut. Aggregates of loparite with rock-forming minerals are predominantly poor, 6-10% of the grain volume, less often medium, up to 20-25% of the grain volume (Figure 1 c). Grains of loparite with admixture and crusts of secondary minerals were encountered. In the material larger than 0.63 mm, it was about 85-86%. Loparite aggregates are predominantly rich (55-85%) to 95% of the grain volume and medium (25-55%) (Figure 1 d, e, f). In material less than 0.10 mm, the content of loparite aggregates is small, about 1.5-2%, and aggregates are predominantly poor (5-25%) and rich (55-85%).

When the ore is grinded, the liberated loparite grains acquire angular, "splintering" appearance. The morphology of aggregates remains similar to that of the original ore (Figure 2 a, b).

The authors studied the processes of loparite ore grinding in laboratory ball and rod mills of equal power (1.1 kW) at constant mass loading of the original ore (1.2 kg) and at different values of grinding time. Table 1 shows the distribution of loparite in size classes and kinetics of its liberation for two (selectively) grinding regimes.
Fig. 1. Mineral particles of loparite ore before grinding: a - general view of the ore particles larger 1.0 mm; b - loparite (black with metallic luster) in combination with nepheline, feldspars and aegirine, size class -1.6+1.0 mm; c - loose loparite grains (upper row) and its aggregates in the original ore, size class -1.0+0.63 mm; d - very rich, rich and medium loparite aggregates, size class -0.63+0.40 mm; e - free loparite grains and its rich and poor aggregates of size class -0.40+0.20 mm; f - rich and medium sized loparite aggregates in the size class of -0.20+0.10 mm.

3 Substantiation of the simulation algorithm for mineral particle disintegration operations

The material composition of the disintegration feed can be represented as a set of narrow fractions, in general, polymineral particles, distinguished by their belonging to different size classes and to different content classes of the useful component. Representation of each fraction is characterized by the ratio of its mass to the total mass of all fractions. In the course of disintegration polymineral particles are destroyed and their mass is redistributed into other fractions with smaller particle size. In this case, at all stages of disintegration in the total mass of particles there are such, in which the share of content of useful component...
is equal to one, and such where it is equal to zero. Therefore, when describing the set of particles undergoing disintegration, it should be taken into account that $\beta^g_1 = 1$, and $\beta^g_M = 0$, where $M$ – number of content classes of useful component; $\beta^g_M$ - the final values of mass (g) content of useful component in the content classes. This is also consistent with the interpretation of the integral B (beta) function, which gives infinitely large values on the boundaries of the region of existence $[0; 1]$ and its application for simulation modeling purposes is restricted to the internal values of this region. This limitation can be found in more detail in [14].

![Figure 2](image1.png)  
**Fig. 2.** Mineral particles of loparite ore after grinding in a rod mill, grinding time 8 min: a - loparite in the material larger 0.40 mm; b - loparite in the material -0.40+0.20 mm as loose grains and in aggregates.

### Table 1. Kinetics of loparite liberation.

<table>
<thead>
<tr>
<th>%</th>
<th>Size class, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.045</td>
</tr>
<tr>
<td>γ</td>
<td>4.55</td>
</tr>
<tr>
<td>Lp</td>
<td>1.23</td>
</tr>
<tr>
<td>β</td>
<td>100</td>
</tr>
<tr>
<td>γ</td>
<td>19.12</td>
</tr>
<tr>
<td>Lp</td>
<td>1.37</td>
</tr>
<tr>
<td>β</td>
<td>100</td>
</tr>
<tr>
<td>γ</td>
<td>50.76</td>
</tr>
<tr>
<td>Lp</td>
<td>1.12</td>
</tr>
<tr>
<td>β</td>
<td>100</td>
</tr>
</tbody>
</table>

| γ      | 16.57  | 5.01   | 11.05  | 14.66  | 19.14  | 20.04  | 13.53  | -      | -      | -      | -      |
| Lp     | 1.23   | 0.93   | 0.87   | 1.55   | 1.51   | 1.07   | 0.64   | -      | -      | -      | -      |
| β      | 100    | 99     | 98     | 95     | 90     | 80     | 15     | -      | -      | -      | -      |

| γ      | 38.53  | 17.18  | 19.57  | 21.86  | 2.86   | -      | -      | -      | -      | -      | -      |
| Lp     | 1.05   | 1.17   | 0.83   | 1.43   | 1.05   | -      | -      | -      | -      | -      | -      |
| β      | 100    | 99     | 98     | 95     | 90     | -      | -      | -      | -      | -      | -      |

$\gamma$, % - yield of size class; $Lp$, % - loparite content; $\beta$, % - liberated loparite content.
It is also important that during disintegration the total amount of components included in the feeding does not change. This is a preparatory operation, in which separation does not occur, and the total content of the useful component $\beta_0$ in the mass of mineral particles does not change during disintegration simultaneously with the fact that the redistribution of the useful component in size classes and content classes proceeds uninterruptedly. Yield values of fractions in the size classes of fine particles increase; within these classes as the particle sizes decrease, the fraction of particles rich in useful component and particles depleted of it increases. Since disintegration proceeds simultaneously with reduction of the mineral particle size, it is fair to ask what should be the degree of particle size reduction for disintegration to be optimal. The answer to this question is the possibility to determine the dissemination of the useful component $\beta_0$ in aggregates of particles, i.e. the most probable size of mono-mineral grains of the useful component. Note that it is this value adopted by the authors as the basis for determining the parameters B (beta) distribution.

This allows for each of m classes of useful component content to obtain a density function of the distribution of particle size classes yields after particle size reduction $y_{mi}^{sim}$. This distribution for each m will have the form of triangular $i \times i$ matrices, where the particles of the i-th size class are distributed into the j-th size classes so that $1 \leq j \leq i$. The diagonal elements of these matrices represent the mass fractions of the initial narrow fractions, indicating the number of particles which, during reduction of coarseness, pass into finer particle size classes. In terms of implementation of the particle size reduction operation, this can be both a consequence of the technology and the processed ore peculiarities. Two other aspects precede the application of the B (beta) distribution in view of the above. First, when forming the mentioned matrices, among other things, the yield values of size classes for $m = 1$ and $m = M$ are obtained. These sets of values $y_{11}^{sim}, y_{12}^{sim}, ..., y_{1N}^{sim}$ and $y_{M1}^{sim}, y_{M2}^{sim}, ..., y_{MN}^{sim}$ B (beta) are not subject to distribution. Second, similar sets for values $1 < m < M$ are subject to B (beta) distribution in part of values minus those of corresponding diagonal elements, which will have to be taken into account later in results of B (beta) distribution by summation to elements of m lines.

For each $1 < m < M$ in each size class, the parameters B (beta) of the distribution are determined. For the convenience of further discussion, we assign them the designations $a$ and $b$, respectively. Then,

$$a_{mi} = \beta_m^a \frac{d_i^a}{d_0}, \quad b_{mi} = (1 - \beta_m^a) \frac{d_i^b}{d_0}. \quad (1)$$

Then, for each size class in each $1 < m < M$ content class, the values of integrals are calculated:

$$I_{mi}(\beta) = \int_{\beta_{m+1}}^{\beta_m} \frac{\beta^{a_{mi}-1}(1 - \beta)^{b_{mi}-1}}{B(a_{mi}, b_{mi})} d\beta. \quad (2)$$

At the next step, for each $1 < m < M$ we form $m \times i$ matrix of redistribution of particles of m content class to M content classes as a result of particle size reduction, actually - the matrix of the result of disintegration. To do this, for example, in the distribution of the i-th size class, let's introduce in addition l - index of the useful component content class $1 < l < M$. Then in further considerations we can say that in the i-th class of particle size, the particles belonging to m class of content are allocated to each l - class of content. In this case, when $l = m$, additive term, equal to the corresponding
diagonal element of the density function matrix of the yield distribution of mineral particle size classes after reducing their size $y_{mi}^{\text{sim}}$, should be taken into account.

Let us introduce the notation of the calculated element: $y_{mi}^{\text{sim}}$. In this designation: $\gamma$ - indicates that the element in the sense is the mass fraction of particles involved in the operation; $m$ - defines the content class, which is the source of particles distributed; $i$ - defines that in this $m$ content class particles of the $i$-th size class are distributed; $l$ - indicates to which content class in the current size class particles from $m$ content class are redistributed during disintegration; $\text{sim}$ - indicates that the element value was obtained from the simulation.

Then:

$$
\gamma_{mi}^{\text{sim}} = (1 - l_{mi}(\beta_2)) \cdot (y_{mi}^{\text{sim}} - y_{mi} f_{mi}^{n});
$$

$$
\gamma_{mi}^{\text{sim}} = (l_{mi}(\beta_i) - l_{mi}(\beta_{i+1})) \cdot (y_{mi}^{\text{sim}} - y_{mi} f_{mi}^{n});
$$

$$
\gamma_{mi}^{\text{sim}} = (l_{mi}(\beta_m) - l_{mi}(\beta_{m+1})) \cdot (y_{mi}^{\text{sim}} - y_{mi} f_{mi}^{n}) + y_{mi} f_{mi}^{n};
$$

$$
\gamma_{mi}^{\text{sim}} = l_{mi}(\beta_M) \cdot (y_{mi}^{\text{sim}} - y_{mi} f_{mi}^{n}),
$$

where $f_{mi}^{n}$ - normalized values of weights of the discrete particle size class yield density function obtained by applying the Gauss-Laplace distribution to simulations of mineral particle size reduction [15] and

$$
\sum_{i=1}^{M} y_{mi}^{\text{sim}} = y_{mi}^{\text{sim}}.
$$

The final step of the algorithm consists in formation of predicted values of yields of narrow fractions of mineral particles at the end of disintegration. Let’s denote it $y_{mi}^{\text{B}}$. Then for each size class we get the following expressions of the predicted values:

$$
\gamma_{1i}^{\text{B}} = \gamma_{1i}^{\text{sim}} + \sum_{m=2}^{M-1} y_{mi}^{\text{sim}},
\gamma_{li}^{\text{B}} = \sum_{m=2}^{M-1} y_{mi}^{\text{sim}};
\gamma_{Mi}^{\text{B}} = \sum_{m=2}^{M-1} y_{mi}^{\text{sim}} + y_{mi}^{\text{sim}}.
$$

4 Conclusion

The authors have developed simulation models for prediction of narrow loparite fractions yield after disintegration operation at different modes using the given algorithm. Figures 3 - 7 show distribution diagrams of narrow loparite fractions obtained as a result of laboratory studies and by simulation modeling.

The obtained simulation models for predicting the yield of narrow loparite fractions at different grinding regimes, on the one hand, reflect the tendency consisting in a natural redistribution of the useful component in the size and content classes, on the other hand, correspond to the concepts of the mineral processing theory about the liberation of mineral grains during grinding. It should be noted that the low content and heterogeneous nature of loparite distribution in the initial ore did not affect the adequacy of the model.

Following the disintegration the separation of the mineral particle flow in the general case involves the staged separation of the tailings, middlings and concentrate, which is carried out by consecutive classifiers and separators. Obtained predictive models of changes in the material composition actually allow predicting the operation of this
equipment and essentially determine the controlling influence on the technological flowsheet as a whole.

**Fig. 3.** Distribution of yields of narrow loparite fractions in the ore.

**Fig. 4.** Disintegration in a ball mill, grinding time 8 min: a - distribution of fractions; b - simulation model of the distribution of fractions.

**Fig. 5.** Disintegration in a ball mill, grinding time 32 min: a - distribution of fractions; b - simulation model of the distribution of fractions.
Fig. 6. Disintegration in a rod mill, grinding time 4 min: a - distribution of fractions; b - simulation model of the distribution of fractions.

Fig. 7. Disintegration in a rod mill, grinding time 16 min: a - distribution of fractions; b - simulation model of the distribution of fractions.

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