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Structural theory as a research language for open complex systems, including social ones

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Abstract. The application of structural theory allows to see the same universal construction, called a “model,” applicable to a broad class of modeling areas. Everything in the world is a model in this theory: an agent, a complex of agents, and a complex of complexes. We can say that the widely known formula: “One in all and all in One” is true in this area, as well as the well-known aphorism of H. Poincare: “Mathematics is the art of calling different things by the same name.” The application of geometric theory is the consideration of morphisms of base sets and the assigning invariants that allow a complex system to remain under such morphisms. The proposed language for the complex systems behavior makes it possible to draw several interesting conclusions about the nature of complex open systems, simply due to its means of distinguishing entities. A complex system must sacrifice a part of its power (the most expensive and liquid that it has) to preserve its laws of life (maintain its structure and set of axioms). Otherwise, it will not be able to remain itself. It is interesting to remember that many religions teach that this world stands on the sacrifice.

1 Introduction

Let us outline the range of systems this work is devoted to, by listing some of their fundamental properties:

• Fractality. That means that the components of the complex system may be also complex systems [1], see Fig. 1.

• Behavior. A complex system has behavior -- the ability to react in known methods on some conventional events of the inner and outer environment.

• Openness and Nonequilibrium. Complex systems are dissipative [2]. They exchange matter, energy, and information streams with the surrounding world and are out from the thermodynamic equilibrium.

• Sustainable Dynamic Equilibrium. A complex system is in sustainable equilibrium while it is successful. However, its stability is dynamic. It should run for running like a bicycle. Moreover, as a rule, it has a particular control system to keep this sustainable equilibrium, spending a part of the system's power for this purpose.

• Three Worlds. A genuinely complex system is not just a collection of material objects (as a pile of bricks is not a house). Its information components – the structure and programs –

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play an essential role in material components organizing. Also, a set of ideas-axioms is necessary to define the system's goals, objectives, and methods. We know the consideration of complex systems in these three aspects since Plato.

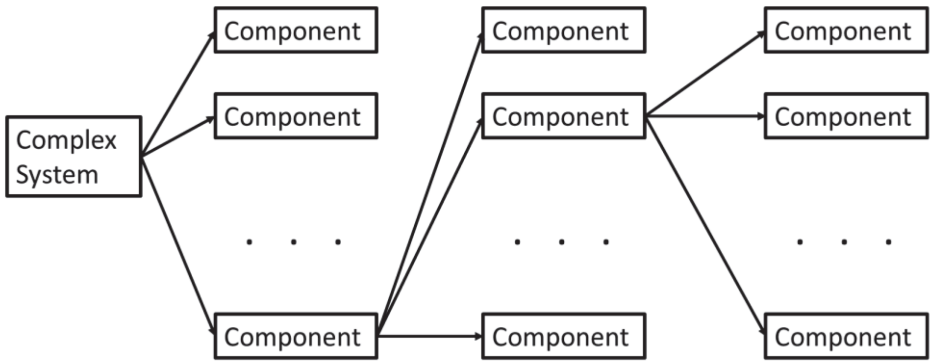

Fig. 1. Fractality of Complex Systems.

The works [3, 4] are developed the Model Synthesis theory. It justifies end-to-end technology for describing, synthesizing, and implementing agent models. Model Synthesis formalizes the concept of the behaving agent -- an elementary complex system. This formalization is not one more item from the endless list of heuristic agent constructs but a consequence of the necessary conditions for modeling. Firstly – from the closeness hypotheses for the model. Secondly, the work [3] found sufficient conditions (close to the necessary ones) for the possibility of model constructing – a kind of the existence theorem.

A universal agent, called a model in the Model Synthesis, has the following two essential properties:

• The operation of uniting a finite number of model-components into the model complex is closed (does not go beyond the family of models).

• The organization of simulation calculations is the same for all representatives of the family. It means a possibility of the universal computer program creation, running any simulation model, and focusing on parallel calculations.

These properties allow building fractal agent systems without having to worry about how to organize their calculations later. A vital consequence of the Model Synthesis is that we can compare almost any complex agent system: a country, a theater of war operations, a holding company, a factory, a university, a political party, a social stratum with a mathematical object – “model,” – at least in a thought experiment.

We describe a family of model-components as N. Bourbaki's species of structure [5]. Why at all species of structure? The species of structure description consists of three parts: base sets, typical characterizations, and axioms.

These three sections of the species of structure represent the mentioned three worlds to consider complex systems. The first section – the principal base sets – is the “material world”' of the species of structure, where it is realized, but from which it is independent enough since this world is a subject of mappings (morphisms). The second section – typical characterizations – actually defines the structure on principal base sets. It is from the information world and determines how the elements of the base sets are interconnected. Finally, the third section – axioms – belongs to the “world of ideas.” The species of structure language is either suitable for open complex systems description: it focused on the typical characterizations transition when mapping the base sets. We can describe the variable composition of the open systems by conserving the structure morphisms.

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2 The “model” species of structure – a universal modeling agent

Let's now proceed to the description of the “model” species of structure.

We do this differently than in [4], – by one species of structure, but with an auxiliary base set $\mathbb{N}$ – natural numbers set. The species of structure, in this case, is denoted as $\Sigma(\mathbb{N},\xi)$ – there are auxiliary base sets and axioms in the parentheses, which are not the subjects for future morphisms.

$$\Sigma(\mathbb{N},\xi) =< \boldsymbol{X}, M, E, \{M_j\}_{j=1}^{N}, \{E_j\}_{j=1}^{N}, \mathbb{N};$$

$$N \subset \mathbb{N},$$

$$\boldsymbol{x} \subset \boldsymbol{X}, \boldsymbol{a} \subset \boldsymbol{X}, \tag{1}$$

$$s \subset M, f \subset M, \tag{2}$$

$$\{m_{j,real} \subset M_j \times M\}_{j=1}^{N}, \tag{3}$$

$$\{e_{j,real} \subset E_j \times E\}_{j=1}^{N}, \tag{4}$$

$$\{m_{j,in} \subset M_j \times \mathfrak{B}(\boldsymbol{X})\}_{j=1}^{N}, \tag{5}$$

$$\{m_{j,out} \subset M_j \times \mathfrak{B}(\boldsymbol{X})\}_{j=1}^{N}, \tag{6}$$

$$\{e_{j,in} \subset E_j \times \mathfrak{B}(\boldsymbol{X})\}_{j=1}^{N}, \tag{7}$$

$$\{sw_j \subset E_j \times M_j \times M_j\}_{j=1}^{N}, \tag{8}$$

$$\{m_j^0 \subset M_j\}_{j=1}^{N}, \tag{9}$$

$$\{p_j \subset \mathrm{B}(M_j) \times \mathrm{B}(E_j) \times M_j \times \times \mathfrak{B}(E_j \times M_j \times M_j)\}_{j=1}^{N}; \tag{10}$$

$$\xi: |N| = 1,$$

$$R_1: (\boldsymbol{x} \cup \boldsymbol{a} = \boldsymbol{X}) \& (\boldsymbol{x} \cap \boldsymbol{a} = \emptyset),$$

$$R_2: (s \cup f = M) \& (s \cap f = \emptyset),$$

$$R_3: \left\{ \left(\forall m \in M_j \right) \left(\exists ! \tilde{m} \in M \right) \left(\{ m, \tilde{m} \} \in m_{j,real} \right) \right\}_{j=1}^{N},$$

$$R_4: \left\{ \left(\forall e \in E_j \right) \left(\exists ! \tilde{e} \in E \right) \left(\{ e, \tilde{e} \} \in e_{j,real} \right) \right\}_{j=1}^{N},$$

$$R_5: \left\{ \left(\forall m \in M_j \right) \left(\exists ! r \in \mathfrak{B}(\boldsymbol{X}) \right) \left(\{ m, r \} \in m_{j,in} \right) \right\}_{j=1}^{N},$$

$$R_6: \left\{ \left(\forall m \in M_j \right) \left(\exists ! r \in \mathrm{B}(\mathbf{x}) \right) \left(\{ m, r \} \in m_{j,out} \right) \right\}_{j=1}^{N},$$

$$R_7: \left\{ \left(\forall e \in E_j \right) \left(\exists ! r \in \mathfrak{B}(\boldsymbol{X}) \right) \left(\{ e, r \} \in e_{j,in} \right) \right\}_{j=1}^{N},$$

$$R_8: \left\{ \left(\left(\forall e \in E_j \right) \left(\exists ! r \in M_j \times M_j \right) \left(\{ e, r \} \in sw_j \right) \right) \& \right.$$

$$\left. \& \left(\left(\{ e, r \} \in sw_j, \{ \tilde{e}, \tilde{r} \} \in sw_j, r = \tilde{r} \right) \Rightarrow \left(e = \tilde{e} \right) \right) \right\}_{j=1}^{N},$$

$$R_9: \left\{ p_j = \{ M_j, E_j, m_j^0, sw_j \} \right\}_{j=1}^{N},$$

R_{10}: axiom of the uniqueness of the model characteristics calculating,
R_{11}: axiom of the organization of simulation calculations>.

Here $\mathfrak{B}(\cdot)$– the set of all subsets of the set in brackets. Everywhere $\{\ldots_j\}_{j=1}^{N}$ means that the contents of the braces are repeated with comma N times, while the index j is replaced by numbers $1, \ldots, N$. For example, $\{M_j\}_{j=1}^{N}$ is a shortened version of the record $M_1, \ldots, M_N$.

Principal base sets of the species of structure are $\boldsymbol{X}, M, E, \{M_j\}_{j=1}^{N}, \{E_j\}_{j=1}^{N}$. Further, $\boldsymbol{X}$ – is the set of characteristics of the model, occasionally, we will divide it into two subsets $\boldsymbol{X} = \{\boldsymbol{x}, \boldsymbol{a}\}$, where $\boldsymbol{x}$ − are the internal characteristics of a model that, following the closeness hypothesis completely determine its status, and, $\boldsymbol{a}$ − its external characteristics, that by the same closeness hypothesis completely defines its interaction with all the rest world. M − the set of different implementations of methods-elements and $M = \{s, f\}$ − slow s, realizing a smooth dependence of the internal characteristics of the model from its internal and external characteristics, and fast f − which implement the internal model characteristics gaps. E − the set of different implementations of methods-events, associated with the model. Events are what our model must respond to. The method-event – is a method that gets as an input a subset of model characteristics $Y \subset X$, and gives as an output a non-negative number, indicating that the event occurred if the output is zero, or the forecast time-to-event if the output is positive.

$\mathbb{N}$ – an auxiliary base set of natural numbers, from which the typical characterization $N \subset \mathbb{N}$ selects a subset N, which turns out to be just a natural number, with the help of the auxiliary axiom $\xi: |N| = 1$ (meaningfully it is the number of processes in the model).

The typical characterizations (10) and axioms R_9 (indirectly, their definitions also include typical characterizations (8) with axioms R_8, determining the rules for elements switching in processes and typical characterizations (9), determining the initial elements of the processes) formally determine the processes.

Each process p_j, $j = 1, \ldots, N$, defined by (10) and R_9, successively carries out some finite set of elementary actions M_j possible for it, which we will call the set of its methods-elements. The execution of one or another method-element depends on the situations E_j arising in the system to which the process responds. They will be called the set of its methods-events. Typical characterizations (3), (4) shows, from where the methods-elements and methods-events are taken, (5) – (7) shows what characteristics of the model are transmitted to methods-elements and accepted from them, typical characterization (8) sets the rules for switching elements under the influence of the events. Axioms R_1 – R_8 specify the typical characterizations (1) – (8).

The model's most important property is its behavior, i.e., the ability to respond to specific pre-known influences – events (4) of the internal and external environment by one or another known way – method-element (3). The methods-elements (in a programming sense, i.e., routines that implement the algorithm of the action) execute elementary actions of the model. The methods-events – subprograms that respond to a particular combination of model characteristics – realize elementary reactions of the model. Both methods and events, along with characteristics, are included in the base sets of the model.

The R_{10} axiom of the uniqueness and unambiguity of the model characteristics calculating is a consequence of the closeness hypothesis [6]. The unambiguity of mapping the characteristics of the model at the left point of the modeling step into the characteristics on the forecast segment makes it possible to parallelize the process of calculating this mapping without recording conflicts. This consequence of the closeness hypothesis applies to lower-level models. Special rules are introduced [4] for a combination of models into a complex in order for preserving the uniqueness and unambiguity of its computational process.

How the R_{11} – axiom of the organization of simulation calculations works? This axiom is also a consequence of the closeness hypothesis. No matter what the field of the modeling is (a technical system, the struggle of populations for survival, the epidemic spread, or social processes), the entire dynamics of the model is very simple and the same for any subject area. This is a four-stroke cycle, and similar to the Carnot cycle, or the work of the combustion engine.

First, we choose a standard step of the simulation Δt. At the first step, we know the internal characteristics' initial values and the processes' initial methods-elements. Secondly, we get current method elements of all processes and all of the internal characteristics of the model at the beginning of the simulation step. Third, observability of the external characteristics at any moment is assumed.

Further,

1. First, we calculate all the events associated with the current elements of all the processes. The rules of switching determine the correlation of events with the current elements of the processes. We can compute the events in parallel, but to promote the computing process further, we should wait to complete the computation of all the events. If there are events that occurred, we check whether there are transitions to the fast elements. If there are any - the fast elements run (they become current). We can also compute them in parallel, but we should wait to complete all their calculations to advance the computing process further. Then we return to the beginning of item 1. If there are no transitions to fast elements - we transit to the new slow elements and then return to the beginning of item 1.
2. If there is no events occurrence, we select the nearest $\Delta\tau$ from all the forecasts of the events.
3. If the standard step of modeling Δt does not exceed the predicted time to the nearest event $\Delta t \leq \Delta\tau$, – we compute the current slow elements with the normal step Δt.

Otherwise, we compute them with the step to the nearest predicted event $\Delta\tau$. We can compute slow elements in parallel, with the expectation of completion of the latter.
4. Return to the beginning of item 1.

So we may write a universal computer program under the rules described above, which would carry out simulation calculations for any member from the models' family.

3 The Geometric Theory of Behavior

We obtain the behavior of our complex system, having described all models' behavior and having constructed the synthesis of all the complexes, including the complex of the highest level. Morphisms may modify the base sets of the model, changing elementary actions and reactions up to the opposite.

Morphisms (even homeomorphisms – continuous in both directions isomorphisms in the topological spaces category) can lead the system to the opposite of the original stage, step by step imperceptibly and in a short time. Therefore, the question naturally arises about invariants that limit the entire set of possible morphisms. How do invariants work? The invariant, an axiom in the language of describing the structure of a complex system, distinguishes from the group of morphisms a subgroup of admissible morphisms preserving this invariant. Invariants that restrict the transformation of behavior in society are religious commandments, moral and cultural values, traditional and civil norms of behavior – all that we can call the Law with a capital letter. We can classify the types of collective behavior of different cultures [7], basing on invariants.

As mentioned above, complex systems are in dynamic equilibrium: to work, they must work (such as a four-stroke engine or any enterprise). For maintaining its dynamic equilibrium, a complex system must firstly preserve the potential of dynamic equilibrium - to perform a specific minimum work per unit of time (for example, rotating at least 800 revolutions per minute for the engine, paying salaries, utilities, renting premises, etc., for the enterprise). Secondly, a complex system carries out regular activities to maintain the structure and invariants (laws of existence). We called these activities a cult in [8]. For example, to keep a car alive, we need periodically to undergo maintenance, inspection, refuel, etc. Periodically programming of "adherence to certain symbolic systems" (as anthropologists formulate this [9]) is needed in more complex cases of social systems.

A complex system must spend part of its power on maintaining the laws of its existence in three planes - physical, informational, and ideological (preserving the axioms of its existence). If a complex system for a particular time cannot maintain its potential of dynamic equilibrium and support cult activities – it ceases its presence in the former stage. Then, successful competitors most likely will get for nothing its base sets (all kinds of assets).

As a result, we can say that in complex open systems, conservation of laws instead of conservation laws prevails [8], in contrast to "simple" closed physical systems. The essence of conservation of laws and conservation laws is the same: conservation laws are invariants of certain symmetries: energy – the uniformity of time, momentum – the uniformity of space, etc.

The geometric theory provides a mathematical language environment for discourse in the subject field of modeling complex systems behavior, capable of identifying subtle differences of the considered entities that are sometimes lost when discussed in natural language.

4 A Structural Language for Discourse in the Complex Systems Domain

The universal agent of Model Synthesis, called a model, has the following two fundamental properties. The algorithm of simulation calculations is the same for any model. So, the universal computer program focused on parallel computations can perform it. A models' family is closed under combining a finite number of models into a model-complex, i.e., such a complex itself has a model species of structure.

These properties allow you to build a complex fractal agent-based system without worrying about organizing its calculations. Therefore, no matter how sophisticated a complex system is, its model can be simple and uniform for a vast class of systems. It consists of two parts: an invariable dynamic part – a four-step program for the computations organizing. And a database – a static component with invariable types of tables.

We can describe the family of models as a species of structure in the sense of N. Bourbaki [5]. The species of structure description consists of three parts: base sets, typical characterizations, axioms. These three parts represent those mentioned above, "three worlds," we proposed considering complex systems.

The first part - the principal base sets – is the "material world" of the species of structure it embodies. The structure is somewhat independent of the base sets since they are subjects to transformations (morphisms).

The second part – typical characterizations – actually defines the structure on the base sets. It is from computer science and determines how the elements of the base sets interact with each other.

Finally, the third part – axioms – belongs to the "world of ideas."

As will be shown in the next sections of this work, the species of structure language is also suitable for describing open systems: it focuses on typical characterizations, persisting under morphisms of base sets. We can characterize the open systems by morphisms of base sets that preserve their structure.

Why is there a section 2 with a lot of formulas in this work – in subsequent sections, the discourse is conducted at a fairly "humanitarian" level? To explain this fact, we will give the following analogy.

N. Bourbaki in [5] proposed languages of three levels to describe all kinds of mathematical objects (long before the advent of programming and algorithmic languages of different levels).

- It is a language with the original seven special characters, plus a natural alphabet at the lowest level. The real Bourbaki's description of a mathematical object exists in this language. The proofs in this language are extraordinarily simple, but it is completely unsuitable for the ordinary human mind – there are "too many letters" in it to be able to grasp any meaningful concept described in it. The situation is very similar to binary arithmetic. Arithmetic itself is super simple, but the abundance of signs makes it suitable only for a computer, but not for a person.
- At the intermediate level – the language of abbreviations, where the usual mathematical symbols (for example, quantifiers $\exists$, $\forall$, $\emptyset$, and other mathematical constructions) are described in the Bourbaki's language of the lower level.
- Finally, a high-level language. Almost ordinary mathematical expressions are written on it, theorems are formulated and proved. It is quite possible to perceive them that way, at least there are a number of works (for example, this one, or [4]) where Bourbakian constructions, for example, species of structure, are considered on the basis of traditional, but not Bourbakian axiomatics of the set theory. However, all Bourbakian constructions, along with the usual general mathematical perception, allow for a specific Bourbakian

interpretation – they can be interpreted into a lower-level language, where they all acquire a completely formal Bourbakian meaning. However, they become completely inaccessible to ordinary human perception, due to too large a set of signs.

The theory of Model synthesis proposed above allows us to formally describe almost any agent system as a model species of structure. Including social systems that are traditionally the subject of the study of descriptive sciences, for example such as country, economy, enterprise, military operations, political party, social stratum, a set of this article readers.

The species of structure "model" is the lower-level language of such descriptions. Next, you can enter an intermediate level language. For example, the behavior of a complex system is understood as a set of its skills (3), R_3; events (4), R_4 – situations to which the system is able to respond; behavior program (8) – (10), R_8, R_9.

In [10] it is shown that the class of programs defined by the typical characterizations (8) – (10) and the axioms R_8, R_9 is algorithmically complete. Therefore, for example, in a high-level language, we can talk about the "preservation of laws" in a complex open system ([8] or section 4 of this work), as a rather complicated algorithm for the system controlling, nevertheless described by (8) – (10) and R_8, R_9.

As examples of humanitarian phrases that allow formal interpretation in the language of model species of structures, we can cite here well-known aphorisms not only from humanitarian discourse, but also from the fields of philosophy, religion, esotericism:

• "One in all and all in One." – H. Poincare said: "Mathematics is the art of calling different things by the same name." Everything is a model: any agent, any complex of agents, complex of complexes, and everything in the world (virtual, of course).

• "What drives everything in the world – is motionless and unchanging." – A relatively simple four-step program implements all the dynamics of any complex system (Four steps in the section 2, or more in [6]).

• "This world stays upon the sacrifice" – is the Law of sustainable development of complex open systems (conservation of laws instead of the conservation laws [8]). The system needs to spend a part of its power to conserve the set of its typical characterizations and axioms under morphisms of base sets, to remain itself under these morphisms. Otherwise, it will collapse or turn into something qualitatively different.

• "The cult is the base of the culture" – P.A. Florensky [11]. To preserve the desired behavior (the culture) of a complex open system, under its base sets morphisms, the periodic activities (the cult) are needed to program its agents' behavior.

5 Conclusion

Let us list the main ideas of this work.

It offers the universal structure and proves its applicability for modeling the broad class of systems that possess behavior [3, 4]. This structure was named here a model. A family of species of structure in the sense of N. Bourbaki [5] represents its static part. A relatively simple universal program that organizes the performance of any representative of the family mentioned above gives the necessary dynamics to the static structure of the model. Any system of differential equations can be a particular type of model.

The geometric theory of behavior provides a mathematical language environment for discourse in the domain of complex systems modeling, i.e. the ability to identify rather subtle differences of the entities under consideration, which are usually lost, mixed up during their humanitarian discussion in natural language.

Using the language of geometric theory, we show that to maintain the given behavior of a complex system, it is not enough to take physical actions to preserve the potential of dynamic equilibrium. An ideological-informational system for programming the conservation of invariants providing a given behavior is needed. In this work, it is called a cult. The cult, along with the potential of dynamic equilibrium maintaining, ensures the conservation of the laws of a complex system's existence. The phenomenon of conservation of laws in a complex system confirms the opinion of P.A. Florensky that the cult is the basis of the culture as the system of social behavior [11].

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