Mathematical model of heat and humidified air treatment in the injector camera

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Abstract. The article describes a mathematical model for the heat-moisture treatment of air in the nozzle chamber. The dynamics of unsteady processes of heat and mass transfer between the working substances in the apparatus and the transfer characteristics, balance equations of exchanging media are considered. A physical and mathematical description of heat and mass transfer processes in a thicker working medium and the relationship between the heat and mass fluxes through a surface unit and the mass-average temperature and potential are presented. The expression and their dependencies are presented for determining the final parameters of air in the irrigation chamber.

1 Introduction

Air conditioning covers a set of measures to ensure the required state of the indoor air environment. Depending on the parameters of the internal air, a corresponding heat and humidity treatment of the air is carried out. It may include the following processes: heating, cooling, humidification, or air drying.

In central air conditioning systems, the main unit for the implementation of these processes is the nozzle chamber. The processes of heat-moisture processing of air in nozzle chambers have a complex mechanism, including heat and mass transfer at the interface of two water-air phases. This makes it necessary to introduce various simplifications into the description of processes and is often limited only to experimental data.

The exchange of heat and mass between the working substances in the apparatus occurs in the direction of aligning the thermodynamic potentials of the substances due to changes in capacitive indicators. Such processes are transient. If the potential differences of the working media are maintained at a given level or are changed in a certain way due to external influences, then at the boundary of the working media there is a complex process of unsteady heat and mass transfer.

To consider the dynamics of such transient unsteady processes, it is necessary to determine the forces that cause heat and mass transfer, and transfer characteristics, to draw up heat and mass transfer equations and the corresponding balance equations of exchanging media.

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2 Methods

Thus, the general physical and mathematical formulation of the problem includes: the thermodynamic equation of state of exchanging media and the equations of their limiting equilibrium state; thermophysical equations of heat and mass transfer between moving media and their thermal and mass balance equations. The general system of equations can be composed in differential form, either explicitly, or in the form of a mixed system of equations, moreover, some equations are often combined [1–5].

The implementation of more accurate physical and mathematical formulations requires less experimental data. The main physical and mathematical descriptions of heat and mass transfer processes in the thickness of the working media are as follows:

1) based on the Reynolds equations;
2) based on criteria equations;
3) based on integral criterion equations.

In many engineering problems, of interest is not the distribution of parameters in exchanging media, but, for example, heat fluxes at the boundaries and their average temperatures. Therefore, the most widespread solution to these problems is the description based on one-dimensional transport, which is often called the α – model.

The basic equations in a one-dimensional description along the x axis in time τ can be obtained directly from the Reynolds equations [2]

– equation of motion for α – model

$$\frac{d\omega}{d\tau} + \frac{d(\omega)^2}{dx} = -\frac{1}{\rho} \cdot \frac{dp}{dx} + F + \frac{dR}{dx}, \quad (1)$$

where R – the losses of the total energy of the flow when moving in the channel; F – projection of the density of mass forces on the x axis; ω – flow rate; p – flow pressure; ρ – flux density

– continuity equations

$$\frac{d\rho}{d\tau} + \frac{d(\rho \cdot \omega)}{dx} = 0; \quad (2)$$

– apparent heat balance equation

$$\frac{1}{\omega} \cdot \frac{dt}{d\tau} + \frac{dt}{dx} = \frac{\alpha_t \cdot F_{sp}}{(c \cdot \rho)_t \cdot \omega \cdot f} \cdot (t_{sur} - t); \quad (3)$$

– mass balance equation

$$\frac{1}{\omega} \cdot \frac{d\theta}{d\tau} + \frac{d\theta}{dx} = \frac{\alpha_\theta \cdot F_{sp}}{(c \cdot \rho)_\theta \cdot \omega \cdot f} \cdot (\theta_{sur} - \theta); \quad (4)$$

where F_{sp} – the surface area of the exchange per 1 m of length $\left(\frac{m^2}{m}\right)$.

The relationship between the heat and mass fluxes through a surface unit and the mass-average temperature and potential establish the following relations.

$$q = \alpha_t \cdot (t_{sur} - t) \text{ and } j = \alpha_\theta \cdot (\theta_{sur} - \theta), \quad (5)$$

where θ_{sur}, θ, t_{sur}, t – potentials and temperatures, respectively, in the boundary layer near the surface and mass-average in the fluid flow.

Instead of potentials, moisture content and partial pressure are usually used when calculating the moisture flow in air. Then equations (5) take the following form

$$q = \alpha_d \cdot (d_{sur} - d) \text{ and } j = \alpha_p \cdot (p_{sur} - p), \quad (6)$$

Dimensional quantities α_t, α_θ, α_d, α_p are called heat transfer and mass transfer coefficients and take into account possible differences in real processes in their one-dimensional formulation. These coefficients are connected by complex dependences with real processes occurring in a three-dimensional flow. Exchange ratios are usually determined experimentally.

Another type of physical-mathematical model can be described using criteria equations.

For the forced movement of air washing water drops, A.V. Nesterenko [1] proposed the following relationships for determining the thermal (Nu) and diffusion (Nu') Nusselt criteria for $Re = 1 \div 220$

$$Nu = 2 + 1{,}075 \cdot Re^{0{,}48} \cdot Pr^{0{,}33} \cdot Gu^{0{,}175}, \quad (7)$$
$$Nu' = 2 + 0{,}85 \cdot Re^{0{,}52} \cdot (Pr')^{0{,}33} \cdot Gu^{0{,}135}. \quad (8)$$

To calculate the Nusselt criterion, the following criterion dependence can also be used

$$Nu = A \cdot Re_W^n \cdot Re_B^{n_1} \quad (9)$$

where A – the coefficient depending on the diameter of the nozzles; n, n_1 – are the exponents.

Author [1, 2] proposed the following relationship to determine the Nusselt criterion

$$Nu = 0{,}54 \cdot Re^{0{,}5}, \quad (10)$$

Using the Nusselt criteria, it is possible to compose expressions that determine the magnitude of the heat flux and mass flow

$$Q = Nu \cdot \frac{\lambda}{L} \cdot (t_1 - t_{liq}) \cdot F, \quad (11)$$
$$W = Nu' \cdot \frac{D'}{L} \cdot (p_1 - p_2) \cdot F, \quad (12)$$

where, t_1, t_{liq} – respectively, is the temperature of the environment and the liquid (°C), p_1 and p_2 – are the partial pressures of the vapor of the liquid and the environment $\left(\frac{kg}{m^2}\right)$, D' – is the diffusion coefficient related to the gradient of the partial pressure $\frac{m}{h}$, F – is the heat and mass transfer surface (in this case, the cross-sectional area of the chamber) (m^2), L – which determines the size (droplet diameter) (m).

For isothermal conditions, the following relation exists between the coefficients D and D'

$$D' = \frac{D}{R \cdot T}, \quad (13)$$

where R – the universal gas constant $\left[R = 8{,}314 \ \frac{Joul}{mol \cdot K}\right]$.

Equations (16) and (17) can be written in the following form

$$Q = G \cdot c \cdot (t_2 - t_{liq}) \cdot F, \quad (14)$$
$$W = G \cdot (d_1 - d_2) \cdot F. \quad (15)$$

where c – the specific heat of water equal to $4{,}187 \ \frac{kJ}{kg \cdot °C}$.

Based on equations (14) and (25), we obtained expressions for determining the final parameters of air

$$t_2 = \frac{Q}{G \cdot c \cdot F} + t_{liq}, \quad (16)$$

$$d_2 = \frac{Q}{G \cdot F} + d_1. \quad (17)$$

It should be noted that for practical calculations, the use of criterion dependencies causes certain difficulties due to the determination of the size of liquid droplets during spraying. Therefore, when determining the Reynolds number, the conditional diameter of the drop is often taken, which is a very rough assumption.

Due to the lack of methods to determine with high confidence the true size of the droplets sprayed liquids. When calculating heat and mass transfer processes, dimensionless ratios (characteristics) are used, for example, the efficiency coefficient (eq. 26).

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Heat and mass transfer in the irrigation chamber can be described by a system of integral differential equations [6 - 11]

$$\frac{di}{dy} = \int_{l_{min}}^{l_{max}} F_1 \cdot (i_w - i) \cdot dl, \quad (18)$$

$$\frac{dt}{dy} = \int_{l_{min}}^{l_{max}} F_1 \cdot (t_w - t) \cdot dl, \quad (19)$$

$$\frac{dt_w}{dy} = F_2 \cdot (i_w - i), \quad (20)$$

with initial conditions

$$\left.\begin{matrix} t = t_0 \\ i = i_0 \end{matrix}\right\}, \text{at } y = 0; t_w = t_{w_0} \text{ at } y = y_k,$$

where i – current enthalpy values $\left(\frac{kJ}{kg}\right)$; t – current air temperature (°C); t_w – current water drop temperature (°C); l – drop radius (m); y – coordinate along the air; y_k – the coordinates of the nozzle throwing this drop; F_1 and F_2 – coordinate functions y and y_k depending on the size distribution function of the droplets l, irrigation coefficient B, air and water velocities, local heat coefficient α and mass transfer β; t_{w_0} – initial temperature of the sprayed water (°C), i_0 – initial value of enthalpy; t_0 – initial value of the processed air temperature; i_w – enthalpy of saturated air at temperature t_w (enthalpy of the boundary layer of air at the surface of the droplets).

When deriving the equations, the assumption is adopted on the uniform distribution of the spectrum of the mass of water and air parameters over the cross section of the chamber. The system of equations (18), (19) and (20) is closed by a nonlinear equation

$$i_w = f(t_w) \quad (21)$$

Equation (21) describes the relationship between enthalpy and temperature of air saturated with moisture.

The system of equations (18), (20) and (21) is closed and can be solved independently of equation (19). From this it follows that the final water temperature at a given initial enthalpy of air does not depend on its temperature.

3 Results and discussion

The solution of systems (23÷25) by the small perturbation method with a parabolic approximation of equation (21) leads to the following dependence for the final air enthalpy

$$i_k - i_0 = \left(i_{w_0} - i_0\right) \cdot \left[1 + \alpha \cdot \left(i_{w_0} - i_0\right) + \beta \cdot \left(i_{w_0} - 54{,}1\right)\right] \cdot a_1, \quad (22)$$

where a_1, α, β – coefficients depending on the aerohydrodynamic situation in the rain space.

Integrating equation (19) after substituting the solutions of system (18), (20) and (21) into it, we obtain the following form of the dependence for the final air temperature in the irrigation chamber

$$t_k - t_0 = E_a \cdot \left(t_{w_0} - t\right) + 0{,}329 \cdot (a_1 - E_a) \cdot \left(i_{w_0} - i_0\right) + a_2\left(i_{w_0} - i_0\right) + b_2(i_w - 54{,}1), \quad (23)$$

where a_2, b_2 – coefficients depending on the aerodynamic situation in the rain space and rain environment; E_a – efficiency coefficient of the adiabatic process of the irrigation chamber [9-12]

$$E_a = 1 - \frac{t_k - t_{air_1}}{t_0 - t_{air_1}} \quad (24)$$

where t_{air_1} – wet air temperature (°C).

The final air temperature can be calculated by the formula

$$t_k - t_0 = E_a \cdot \left(t_{w_0} - t_0\right) + 0{,}329 \cdot \left(1 - \frac{E_a}{a_1}\right) \cdot (i_k - i_0) \quad (25)$$

Formulas (23) and (25) are valid for both polytropic and adiabatic modes of air treatment. In adiabatic processes in the irrigation chamber, equation (22) has a trivial solution, since the process proceeds along the line $i_k = const$ and $i_k = i$, and equation (25) is transformed into the well-known equality [13-15]

$$E_a = \frac{t_k - t_0}{t_{air_1} - t_0}, \quad (26)$$

where t_k – air temperature at the outlet of the chamber, t_0 and t_{air_1} – dry and wet air inlet temperature, respectively.

4 Conclusion

An analysis of the equations shows that the thermophysical properties of the gaseous and liquid coolants, the speed of their movement, the intensity of interphase heat transfer, the height of the layer of particles and their geometric dimensions affect the nature of changes in the temperature of air and liquid during their movement in the nozzle chamber. The resulting model allows you to numerically determine the parameters of the air at the outlet of the evaporative cooler; the magnitude of the heat flux and the mass flow of the substance. These parameters are the necessary information for conducting a feasibility study of the system and solving the problem of optimizing performance.

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