Modelling and calculation of stimulated oscillation for a crushing plant with vibration

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Abstract. The paper presents the results of the mathematical modelling of external stimuli to improve the efficiency of the removal of grain material from the grinding zone. Thus, the authors proposed to improve the evacuation conditions of the finished product from the working chamber of the crushing plant, as one of these solutions to improve the grinding process in crushers. As a result of creation of external vibrating influence. For this purpose, the calculation chart for modelling the proceeding process of the crushing plant body movement is created which takes into account weight and sizes of the crusher, weight, angular velocity of rotation and eccentric piece location, as well as rigidity of an elastic suspension on which the crusher is established. For the solution of the posed problem, we used methods of analytical mechanics, the Lagrange equation of the second kind, for the mechanical system with two degrees of freedom. As a result of calculations amplitude-frequency characteristics of mechanical system are determined. The conclusion about the choice of the angular velocity of the eccentric rotation for creation of necessary external stimulating influence on the crushing system and improvement of conditions of evacuation of a ready product from a working zone is given. The resonance frequencies of first- and second-order system vibrations and the vibrator frequency are determined to achieve resonance frequencies. Keywords: vibration, modelling, Lagrangian equations of the second kind, grinding, crushing plant.

1 Introduction

A large proportion of the grain materials used in the production of concentrated feed on farms is processed on hammer-type crushers. This is due to their high reliability, simple design, and high productivity. But in addition to the generally acknowledged advantages of hammer-type crushers, there are also disadvantages. These include: a significant (up to 25%, and

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sometimes more) of the flour fraction in the sod, high power consumption of the grinding process (6.0 - 18.0 kW‧h/t) [1, 2].

To solve the issue of turf grinding in hammer mills, engineers proposed to implement various technical solutions. Such as improvement of geometry and mechanism of hammers suspension, optimisation of their location on the rotor, application of stepped (cascade) crushing scheme, intensification of crushed product evacuation from the crusher working chamber and so on. The implementation of these technical solutions in experimental designs also contributes to reducing the energy intensity of the grinding process [3-5].

Power costs (losses) in the drive of hammer crusher are caused by friction in the bearings and mechanisms of the drive. They can be determined both by calculation methods and experimentally during the operation of hammer crusher without a rotor in the crushing chamber. The drive elements of the hammer crusher are assembled from standard components, the performance characteristics of which are calculated in accordance with standard procedures. Therefore, no significant reductions in drive power can be expected.

The power consumption for air circulation through the crushe r (fan effect of the crushe r rotor) depends on the design of the rotor and the tine (hammers). Some modifications of hammer crushers deliberately increase the fan effect and use it to transport grain to the crushing chamber and evacuate the sod. The fan effect can be calculated or experimentally calculated as a percentage of the calculated capacity for a hammer crushe r with and without rotor in the crushing chamber. The power costs are influenced by the rotor design, rotor velocity, and the medium (density of the medium) in which the rotor rotates. Optimisation of these parameters is a promising research area.

The power input for grinding and milling of grain is determined by the physical and mechanical characteristics of the milled grain material as well as by the given milling conditions. It can be determined experimentally as the difference between the power obtained in the grinding of the grain material and the hammer crushe r with the rotor in the crushing chamber. As the research work area is considered, the creation of an external driving force (vibration) is considered, through the use of a vibrating device to intensify the separation effect through the separating surface of the crushe r, with adjustable frequency and phase characteristics. The working direction of research is caused by considerable quantity of the researches in which positive influence of controlled vibration on technological processes is noticed [6-14], and application of mathematical methods of modeling essentially facilitates process of scientific research [1-4].

The purpose of the study is to provide a theoretical justification for the parameters and operating modes of a crushing plant exposed to an external driving force (vibration), in order to improve the conditions for the evacuation of the crushed product from its working chamber.

2 Materials and methods

Feeding grain material into the crushing chamber leads to a sharp increase in the movement of the crushe r body in all three coordinate axes. Therefore, we will make assumptions when analysing the movement of the crushe r body:

- the support springs allow only vertical displacements of fixing points to the plate, on which the crushe r is mounted;
- the vibrations occur in the plane (y,z) (Fig. 1);
- deviations from the equilibrium position at oscillations are small, therefore in compiling expression of kinetic and potential energy we take into account values of the first and the second order of smallness only, and under the sign of trigonometric functions we discard small terms [1-2]:
\[
\sin(\omega t + \varphi) \approx \sin(\omega t);
\]
\[
\cos(\omega t + \varphi) \approx \cos(\omega t);
\]

– oscillation damping forces are not considered.

**Fig. 1.** Diagram of crushe r body oscillations in the plane \((y, z)\).

Let us choose as generalised coordinates the vertical displacement of the pole \(O\) along the \(z\)-axis (the pole is the projection of the centre of mass on the base plate) and \(\varphi\) as the angle of rotation of the crusher around the pole. The generalised coordinates are taken from the static equilibrium position.

The rotation of the eccentric with a constant angular velocity \(\omega\) is a non-stationary relationship. The rotation of the crusher rotor with constant angular velocity \(\omega_p\) with respect to the axis parallel to \(y\) is also a non-stationary relationship. The gyroscopic moment of the crusher rotor that occurs when changing the angle \(\varphi\) is collinear to the \(z\)-axis, so it does not influence the selected generalized coordinates, which will appear when calculating the derivatives in the left part of the Lagrange equations [1-4].

### 3 Results and discussion

The system has two degrees of freedom, so to describe it let us form two Lagrange equations of the second kind. Let us calculate the kinetic energy of the system and express it through generalized coordinates.

\[
T = T_M + T_m;
\]

(1)

here \(T_M\) is kinetic energy of a crushe r without eccentric mass \(M\), \(T_m = \kappa\) is the kinetic energy of the eccentric considered as a material point of mass \(m\). The first term is defined by König’s theorem:

\[
T_M = \frac{M(\dot{z}^2 + (\dot{\varphi}h)^2)}{2} + \frac{J_{xc}\dot{\varphi}^2 + J_{xr}\dot{\varphi}^2}{2};
\]

(2)

where the first term reflects translational motion together with the centre of mass and the second term rotates around the centre of mass, with \(J_{xc}\) – moment of inertia of mass \(M\), including the rotor, about an axis parallel to the \(x\)-axis passing through the centre of
mass, \( J_{xy} \) is the moment of inertia of the crusher rotor, about its axis of rotation, parallel to the axis \( y \).

\[
T_m = \frac{mv^2}{2};
\]  

(3)

here \( v \) is an absolute velocity vector of the eccentric \( \vec{v} = \vec{v}_e + \vec{v}_r \); 
\( \vec{v}_e \) – transfer velocity together with the centre of the eccentric shaft in translational motion; \( \vec{v}_r \) – Relative rotation velocity of the eccentric in progressive axes when changing \( \varphi \) and \( \omega t \) angles:

\[
v_{ez} = \dot{z} - l_3 \dot{\varphi};
\]

\[
v_{ey} = b \dot{\varphi};
\]

\[
v_{rz} = a\omega \cdot \sin(\omega t + \varphi) \approx a\omega \cdot \sin(\omega t);
\]

\[
v_{ry} = a\omega \cdot \cos(\omega t + \varphi) \approx a\omega \cdot \cos(\omega t);
\]

\[
v^2 = (v_{ez} + v_{rz})^2 + (v_{ey} + v_{ry})^2;
\]

\[
v^2 = (\dot{z} - l_3 \dot{\varphi} + a\omega \cdot \sin(\omega t))^2 + (b \dot{\varphi} + a\omega \cdot \cos(\omega t))^2;
\]

Then we obtain:

\[
T_m = \frac{m}{2} ((\dot{z} - l_3 \dot{\varphi} + a\omega \cdot \sin(\omega t))^2 + (b \dot{\varphi} + a\omega \cdot \cos(\omega t))^2).
\]

Thus, the kinetic energy of the system is:

\[
T = \frac{M(\dot{z}^2 + (\dot{\varphi} h)^2)}{2} + \frac{J_{xx} \dot{\varphi}^2 + J_{yy} \omega^2}{2} + \frac{m}{2} ((\dot{z} - l_3 \dot{\varphi} + a\omega \cdot \sin(\omega t))^2 + (b \dot{\varphi} + a\omega \cdot \cos(\omega t))^2).
\]  

(4)

The potential energy \( P \) depends only on the deformation of the springs, measured from the static equilibrium position, since gravity forces are compensated by the magnitude of their pre-strain.

\[
P = \frac{1}{2} k_1 (z - \varphi l_1)^2 + \frac{1}{2} k_2 (z + \varphi l_2)^2;
\]  

(5)

Compose a Lagrange function \( L \) by substituting the expressions Error! Reference source not found. and Error! Reference source not found.:

\[
L = T - P = \frac{M(\dot{z}^2 + (\dot{\varphi} h)^2)}{2} + \frac{J_{xx} \dot{\varphi}^2 + J_{yy} \omega^2}{2} + \frac{m}{2} ((\dot{z} - l_3 \dot{\varphi} + a\omega \cdot \sin(\omega t))^2 + (b \dot{\varphi} + a\omega \cdot \cos(\omega t))^2) - \frac{1}{2} k_1 (z - \varphi l_1)^2 - \frac{1}{2} k_2 (z + \varphi l_2)^2.
\]  

(6)

Let us write the Lagrangian equations of the second kind for the coordinates \( z \) and \( \varphi \) as:
\[
\begin{align*}
\frac{d}{dt}\left(\frac{\partial L}{\partial z}\right) - \frac{\partial L}{\partial z} &= 0; \\
\frac{d}{dt}\left(\frac{\partial L}{\partial \phi}\right) - \frac{\partial L}{\partial \phi} &= 0; \\
\end{align*}
\]  

(7)

We find the terms in the first equation Error! Reference source not found.: 
\[
\frac{\partial L}{\partial z} = M\ddot{z} + m(\dot{z} - l_3\dot{\phi} + a\omega \cdot \sin(\omega t)); \\
\frac{d}{dt}\left(\frac{\partial L}{\partial z}\right) = (M + m)\ddot{z} - ml_3\ddot{\phi} + ma\omega^2 \cos(\omega t); \\
\frac{\partial L}{\partial z} = -(k_1 + k_2)z - (k_2l_2 - k_1l_1)\phi; \\
\]

and restrict ourselves to the first order of small values in the first summand and group the coefficients in the second at generalised coordinates
\[
\frac{d}{dt}\left(\frac{\partial L}{\partial z}\right) = (M + m)\ddot{z} - ml_3\ddot{\phi} + ma\omega^2 \cos(\omega t); \\
\frac{\partial L}{\partial z} = -(k_1 + k_2)z - (k_2l_2 - k_1l_1)\phi; \\
\]

Then the differential equation for the coordinate \(z\) will be:
\[
(M + m)\ddot{z} - ml_3\ddot{\phi} + ma\omega^2 \cos(\omega t) + (k_1 + k_2)z + (k_2l_2 - k_1l_1)\phi = 0. \tag{8}
\]

Similarly, for the coordinate \(\phi\), we will form a second equation in the system Error! Reference source not found.: 
\[
\frac{\partial L}{\partial \phi} = (Mh^2 + J_{xc})\ddot{\phi} - ml_3(\dot{z} - l_3\dot{\phi} + a\omega \cdot \sin(\omega t)) \\
+ mb(b\dot{\phi} + a\omega \cdot \cos(\omega t)); \\
\frac{d}{dt}\left(\frac{\partial L}{\partial \phi}\right) = (Mh^2 + J_{xc})\ddot{\phi} - ml_3(\dot{z} - l_3\dot{\phi} + a\omega \cdot \cos(\omega t)) \\
+ mb(b\dot{\phi} - a\omega \cdot \sin(\omega t)).
\]
\[
\frac{\partial L}{\partial \phi} = k_1l_1(z - \phi l_1) - k_2l_2(z + \phi l_2); \\
\frac{\partial L}{\partial \phi} = (k_1l_1 - k_2l_2)z - (k_1l_1^2 + k_2l_2^2)\phi; \\
\]

\[
(Mh^2 + J_{xc} + ml_3^2 + mb^2)\ddot{\phi} - ml_3\ddot{z} - ml_3a\omega^2 \cdot \cos(\omega t) - mba\omega^2 \cdot \sin(\omega t) + (k_2l_2 - k_1l_1)z + (k_1l_1^2 + k_2l_2^2)\phi = 0. \tag{9}
\]

Let us represent differential equations (8) and (9) in matrix form
\[ [M][\ddot{q}] + [K][q] = \{A_c\} \cos(\omega t) + \{A_s\} \sin(\omega t); \] 

Heer \( \{\dot{q} \}\) – common coordinates, \( \{\ddot{q} \}\) – generalised accelerations;

\[ [M] = \begin{bmatrix} (M + m) & -ml_3 \\ -ml_3 & (Mh^2 + J_{xc} + ml_3^2 + mb^2) \end{bmatrix}; \] 

\[ [K] = \begin{bmatrix} (k_1 + k_2) & (k_2l_2 - k_1l_1) \\ (k_2l_2 - k_1l_1) & (k_1l_1^2 + k_2l_2^2) \end{bmatrix}; \]

\( \{A_c\}, \{A_s\}\) – vector-column of the amplitudes of the forcing forces at the cosine and sine functions, respectively:

\[ \{A_c\} = \begin{bmatrix} -ma\omega^2 \\ ml_3a\omega^2 \end{bmatrix}; \]

\[ \{A_s\} = \begin{bmatrix} 0 \\ mb\omega^2 \end{bmatrix}. \]

The partial solution of the matrix equation (10), corresponding to the steady-state oscillation mode, will be found as a function in the right-hand side:

\[ \{q\} = \{C\} \cos(\omega t) + \{S\} \sin(\omega t); \]

We substitute this solution (15) into equation (10) to determine the coefficients in the vectors \( \{C\} \) and \( \{S\} \) and equate the coefficients with the same time functions, we get:

\[ \cos(\omega t): \quad -\omega^2[M]\{C\} + [K]\{C\} = \{A_c\}; \]
\[ \sin(\omega t): \quad -\omega^2[M]\{S\} + [K]\{S\} = \{A_s\}; \]

then

\[ \{C\} = [-\omega^2[M] + [K]]^{-1}\{A_c\}; \]

\[ \{S\} = [-\omega^2[M] + [K]]^{-1}\{A_s\}; \]

The sine and cosine functions are phase-shifted by an angle of \( \pi/2 \), so the coefficient values obtained are \( \{C\} \) and \( \{S\} \) from formulae (16) and (17) must be squared and added under the root to obtain the amplitudes for each of the generalised coordinates.
\[ Z(\omega) = \sqrt{C_1^2 + S_1^2}; \]  
(18)

\[ \Phi(\omega) = \sqrt{C_2^2 + S_2^2}; \]  
(19)

Changing in formulas (16), (17) values \( \omega \) by expressions (18), (19) amplitude-frequency characteristics of system are constructed. As initial data for calculations following values are taken:

1. Stiffness of springs \( k_1, k_2 \) [N/m] (each of the four springs has a stiffness of \( k=1633 \) kN/m), then \( k_1=k_2=3266 \) kN/m.
2. Horizontal distance from the spring attachment points to \( C \) - centre of gravity of the entire system without eccentric (Fig. 1.) \( l_1 =0.289 \) m, \( l_2 =0.236 \) m, and from projection \( C \) to the eccentric rotation axis \( l_3 =0.179 \) m.
3. Vertical distances from the mounting plane of the springs on the crusher to the centre of gravity of the structure \( h=0.263 \) m.
4. Vertical distances from the mounting plane of the springs on the crusher to the eccentric shaft \( b=0.105 \) m.
5. Eccentricity (distance from the axis of rotation to the centre of mass of the eccentric piece) \( a=0.014 \) m.
6. Weight of the entire system without eccentrics \( M=210.9 \) kg.
7. Eccentric weights \( m=1.558 \) kg.
8. Moment of inertia of the entire structure (without eccentrics) about the axis \( x \) passing through the centre of mass \( J_{xc}=9.71 \) kg \( \cdot \) m\(^2\).
9. Operating range of the eccentric shaft angular rotation velocity \( \omega =293 \pm 105 \) [rad/s].

The calculations were carried out in a Mathcad programme.

At \( \omega =293 \) rad/s the amplitude along the z-axis is \( Z=3.723 \cdot 10^{-4} \) m, and by the angle \( \varphi - \Phi =1.669 \cdot 10^{-4} \) rad. Figures 2 and 3 show the amplitude-frequency characteristics of a mechanical system. The analysis of the graphs shows that the system has two natural frequencies of oscillation \( \omega_1 =135 \) rad/s and \( \omega_2 =176 \) rad/s.

![Fig. 2. Dependence \( Z(\omega) \).](image-url)
4 Conclusions

1. Resonance is observed for both coordinates at these frequencies, with resonance at the first natural frequency being most pronounced for angle $\phi$ and weakly pronounced at the second. For the $z$ coordinate the pattern is more uniform in energy distribution, but there is a more significant increase in amplitude at the first frequency.

2. To achieve resonance oscillations, it is advisable to set the vibrator frequency equal to $\omega_1 = 135$ rad/s.

References


