THEORYTICAL APPROACH ON APPLICATION OF GENERALIZED CLOSED SETS ENVIRONMENTAL LIFESTYLE USING r-NEIGHBOURHOOD IDEALSPACES

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Abstract. We introduce in this paper, the new notion of $R_Ig$-closed sets and obtain some of its characterizations in ideal r-Neighbourhood space and give nice results on $R_Ig$-closed sets with examples. Finally, we discuss application of $R_Ig$-closed sets.

Keywords: Relation, $I_r$-closed set, $R_Ig$-closed set, $R_Ig$-open set.

1. Introduction

In 1970, the notion of generalized closed sets introduced by Levine [5] and various Generalized concepts in topology were introduced by Levine [5] and various authors were making modifications in generalized concepts. Lin[6] and Yao[9] introduced the concept of rough sets in neighbourhood system and Hosny[2] generated different topologies by the concept of idealization of $j$-approximation spaces.

In this paper, we introduce the new notion of $R_Ig$ closed sets and obtain some of its characterizations in ideal r-Neighbourhood space and give nice results on $R_Ig$ closed sets with examples. Finally, we discuss application of $R_Ig$ closed sets with an example.

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2. Preliminaries

**Definition 2.1.** [3] A non-empty collection $I$ of subsets of a set $U$ is called an ideal on $U$, if it satisfies the following conditions.

1. $A \in I$ and $B \in I \Rightarrow A \cup B \in I$.
2. $A \in I$ and $B \subset A \Rightarrow B \in I$.

**Definition 2.2.** [6] Let $U$ be a non-empty finite set and $R$ be an arbitrary binary relation on $U$. The $r$-neighbourhood of $x \in U$ ($N_R(x)$) is defined as $r$-neighbourhood: $N_R(x) = \{y \in U : xRy\}$.

**Definition 2.3.** [6] Let $U$ be a non-empty finite set and $R$ be an arbitrary binary relation on $U$ and $\sum : U \rightarrow P(U)$ be a mapping which assigns for each $x \in U$ its $r$-neighbourhood in $P(U)$. The triple $(U, R, \sum)$ is called a $r$-neighbourhood space (in briefly, $r$-NS).

**Theorem 2.4.** [6] Let $U$ be a $r$-NS and $A \subset U$. Then, the collection $\tau = \{A \subset U : \forall p \in A, N_R(p) \subset A\}$ is a topology on $U$.

**Definition 2.5.** [2] Let $U$ be a $r$-NS and a subset $A \subset U$ is called $r$-open set if $A \in \tau$ and the complement of $r$-open set is called $r$-closed set.

**Definition 2.6** [2] Let $U$ be a $r$-NS and $I$ be an ideal on $U$. Then, the collections $\tau^I = \{A \subset U : \forall p \in A, N_R(p) \in I\}$ is a topology on $U$.

The $r$-Neighbourhood space with an ideal $I$ is called ideal $r$-NS.

**Definition 2.7** [2]

1. Let $U$ be an ideal $r$-NS and a subset $A \subset U$ is called $I_r$-open set if $A \in \tau^I$ and the complement of $I_r$-open set is called $I_r$-closed set.
2. $R^I_r(A) = \{G \in \tau^I : G \subset A\}$ ($R^I_r(A)$ is called $I_r$-lower approximations)
3. $\overline{R^I_r(A)} = \{F \in (\tau^I)^c : A \subset F\}$ is called $I_r$-upper approximations.

**Lemma 2.8.** [2] Let $U$ be an ideal $r$-NS and $A, B \subset U$. Then

1. $R^I_r(A) \subset A \subset \overline{R^I_r(A)}$.
2. $A \subset B \Rightarrow \overline{R^I_r(A)} \subset \overline{R^I_r(B)}$.
3. $A \subset B \Rightarrow \overline{R^I_r(A)} \subset \overline{R^I_r(B)}$.
4. $\overline{R^I_r(R^I_r(A))} = \overline{R^I_r(A)}$.
5. $\overline{R^I_r(A)} = \overline{R^I_r(A)}$.

3. RI- $g$-CLOSED SETS

**Definition 3.1** Let $U$ be an ideal $r$-NS and a subset $A_r$ is said to be $R_1$-$g$-closed set if $\overline{R^I_r(A_r)} \subset O_r$ whenever $A_r \subset O_r$ and $O_r$ is $I_r$-open.

The complement of $R_1$-$g$-closed set is called $R_1$-$g$-open set.
Example 3.2 Let $U=\{\text{Food, Health, Exercise, Sleep}\}$ and we get the relation $R=\{(\text{Food, Food}), (\text{Food, Health}), (\text{Health, Health}), (\text{Health, Exercise}), (\text{Exercise, Exercise}), (\text{Exercise, Sleep}), (\text{Exercise, Health}), (\text{Sleep, Sleep}), (\text{Sleep, Health})\}$ and the ideal $I=\{\emptyset, \text{Health}\}$.

The neighbourhood of Food is $\{\text{Food, Health}\}$, neighbourhood of Health is $\{\text{Health, Exercise}\}$, neighbourhood of Exercise is $\{\text{Health, Exercise, Sleep}\}$ and neighbourhood of Sleep is $\{\text{Health, Sleep}\}$.

The members of $I_r$-open sets are $\{\text{Food}\}, \{\text{Sleep}\}, \{\text{Food, Sleep}\}, \{\text{Exercise, Sleep}\}, \{\text{Food, Exercise, Sleep}\}, \{\text{Health, Exercise, Sleep}\}, \emptyset, U$ and the members of $I_r$-closed sets are $\{\text{Food}\}, \{\text{Health}\}, \{\text{Food, Health}\}, \{\text{Health, Exercise}\}, \{\text{Food, Health, Exercise}\}, \{\text{Food, Health, Exercise, Sleep}\}, \{\text{Health, Exercise, Sleep}\}, \emptyset, U$.

<table>
<thead>
<tr>
<th>Subset of $U$ ($A_r$)</th>
<th>$N_r(P), p \in A_r$</th>
<th>$R_r^I(A_r)$</th>
<th>$A_r \subseteq O_r$ and $O_r$ is $I_r$-open</th>
<th>Member of $R_g^I$ closed</th>
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<tr>
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<td>{Food, Health}</td>
<td>Singleton member {Food}</td>
<td>{Food}, {Food, Sleep}, {Food, Exercise, Sleep}</td>
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<tr>
<td>{Health}</td>
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<td>{Health, Exercise}</td>
<td>{Exercise, Sleep} and {Food, Exercise, Sleep} and {Health, Exercise, Sleep}</td>
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<th>Subset of $U$ ($A_r$)</th>
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<td></td>
</tr>
<tr>
<td>Subset of U (Ar)</td>
<td>N_r(P), p∈ Ar</td>
<td>( \overline{R^f_r}(A_r) )</td>
<td>( A_r \subseteq O_r ) and ( O_r ) is I_r-closed</td>
<td>Member of ( R^f_r ) closed</td>
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<tr>
<td>( \emptyset )</td>
<td>( \emptyset )</td>
<td>( \emptyset )</td>
<td>All I_r-open sets</td>
<td>Yes</td>
</tr>
<tr>
<td>U</td>
<td>U</td>
<td>U</td>
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**Theorem 3.3** Every I_r-closed set is R_r-g-closed.

**Proof.** The indication comes from the reality that \( \overline{R^f_r}(A_r) = A_r \).

**Remark 3.4** In the Example 3.2, the member \{Exercise\} in R_r-g-closed set but not in I_r-closed.

**Theorem 3.5** Let U be an ideal r-NS and \( A_r \subseteq U \). Then \( x_r \in \overline{R^f_r}(A_r) \) if and only if \( O_r \cap A_r \neq \emptyset \) for every I_r-open set \( O_r \) containing \( x_r \).

**Proof.** Let \( O_r \) be an I_r-open set such that \( x_r \in O_r \) and \( O_r \cap A_r = \emptyset \). Then \( A_r \subseteq U - O_r \) and \( U - O_r \) is I_r-closed set and hence \( \overline{R^f_r}(A_r) \subseteq U - O_r \). Since \( x_r \notin U - O_r \), which implies \( x_r \notin \overline{R^f_r}(A_r) \). Conversely, let \( x_r \notin \overline{R^f_r}(A_r) \). Then there exists an I_r-closed set \( F_r \) such that \( A_r \subseteq F_r \) and \( x_r \notin F_r \). Hence \( (U - F_r) \cap A_r = \emptyset \).

**Theorem 3.6** If \( A_r \) and \( B_r \) are R_r-g-closed then
(1) \( A \cup B_i \) is \( R_I \)-g-closed.

(2) \( A \cap B_i \) is \( R_I \)-g-closed.

**Proof.** The reality, \( R_I^f(A \cup B_i) = R_I^f(A_i) \cup R_I^f(B_i) \) and \( R_I^f(A \cap B_i) \subseteq R_I^f(A_i) \cap R_I^f(B_i) \) gives the proof.

**Remark 3.7** The collection of \( R_I \)-g-closed sets form a topology.

**Theorem 3.8** If \( A_i \) is \( R_I \)-g-closed and \( A_i \subseteq B_i \subseteq R_I(A_i) \) then \( B_i \) is \( R_I \)-g-closed.

**Proof.** Let \( B_i \subseteq O_i \) and \( O_i \) be \( I_r \)-open. Then \( A_i \subseteq U \) and \( A_i \) is \( R_I \)-g-closed. Therefore \( R_I^f(A_i) \subseteq U \) which implies \( R_I^f(B_i) \subseteq U \). Hence \( B_i \) is \( R_I \)-g-closed.

**Theorem 3.9** Let \( U \) be an ideal r-NS and \( A_i \subseteq U \). Then \( A_i \) is \( R_I \)-g-open if and only if \( F_i \subseteq R_I^f(A_i) \) whenever \( F_i \subseteq A_i \) and \( F_i \) is \( I_r \)-closed.

**Proof.** Let \( A_i \) be \( R_I \)-g-open and \( F_i \subseteq A_i \) and \( F_i \) be \( I_r \)-closed. Then \( U \setminus A_i \subseteq U \setminus F_i \) and \( U \setminus F_i \) is \( I_r \)-open. Since \( U \setminus A_i \) is \( R_I \)-g-closed, \( R_I^f(U \setminus A_i) \subseteq U \setminus F_i \) and \( U \setminus R_I^f(A_i) = R_I^f(U \setminus A_i) \subseteq U \setminus F_i \). Hence \( F_i \subseteq R_I^f(A_i) \).

Conversely, let \( U \setminus A_i \subseteq O_i \) where \( O_i \) is \( I_r \)-open. Then \( U \setminus O_i \subseteq A_i \) and \( U \setminus O_i \) is \( I_r \)-closed. By hypothesis, we have \( U \setminus O_i \subseteq R_I^f(A_i) \) and hence \( R_I^f(U \setminus A_i) = U \setminus R_I^f(A_i) \subseteq O_i \). Hence \( A_i \) is \( R_I \)-g-open.

**Theorem 3.10** Let \( U \) be an ideal r-NS and \( A_i \subseteq U \).

(1) \( A_i \) is \( R_I \)-g-closed,

(2) \( R_I^f(A_i) \subseteq O_i \) such that \( A_i \subseteq O_i \) and \( O_i \) is \( I_r \)-open,

(3) \( R_I^f(A_i) \cap F_i = \emptyset \) whenever \( A_i \cap F_i = \emptyset \) and \( F_i \) is \( I_r \)-closed.

The statements (1), (2) and (3) are equivalent.

**Proof.** (1) \( \iff \) (2) Directly we get by the definition 3.1.

(2) \( \implies \) (3) Let \( A_i \cap F_i = \emptyset \) and \( F_i \) be \( I_r \)-closed. Then \( A_i \subseteq U \setminus F_i \) and \( U \setminus F_i \) is \( I_r \)-open. By (2), \( R_I^f(A_i) \subseteq U \setminus F_i \). Hence \( R_I^f(A_i) \cap F_i = \emptyset \).

(3) \( \implies \) (1) Let \( A_i \subseteq O_i \) where \( O_i \) is \( I_r \)-open. Then \( A_i \cap U \setminus O_i = \emptyset \) and \( U \setminus O_i \) is \( I_r \)-closed. By (3), \( R_I^f(A_i) \cap U \setminus O_i = \emptyset \) which implies that \( R_I^f(A_i) \subseteq O_i \). Hence \( A_i \) is \( R_I \)-g-closed.

**4. Application**

In the example 3.2, the members of \( I_r \)-open sets are \{Food\}, \{Sleep\}, \{Food, Sleep\}, \{Exercise, Sleep\}, \{Food, Exercise, Sleep\}, \{Health, Exercise, Sleep\}, \emptyset, \U
t and the members of \( I_r \)-closed sets are \{Food\}, \{Health\}, \{Food, Health\}, \{Health, Exercise\}, \{Food, Health, Exercise\}, \{Health, Exercise, Sleep\}, \emptyset, \U.
The members of $R_I g$-open sets are $\{\text{Food}\}$, $\{\text{Exercise}\}$, $\{\text{Sleep}\}$, $\{\text{Food, Exercise}\}$, $\{\text{Food, Sleep}\}$, $\{\text{Exercise, Sleep}\}$, $\{\text{Food, Exercise, Sleep}\}$, $\emptyset$, $U$ and the members of $I_r$-closed sets are $\{\text{Food}\}$, $\{\text{Health}\}$, $\{\text{Food, Health}\}$, $\{\text{Health, Exercise}\}$, $\{\text{Food, Health, Exercise}\}$, $\{\text{Health, Exercise, Sleep}\}$, $\emptyset$, $U$.

Therefore the members in difference of $I_r$-open sets and $R_I g$-open sets are $\{\text{Exercise}\}$ and $\{\text{Food, Exercise}\}$ and hence the common activity is Exercise and hence Exercise is base for all human activities.

5. Conclusion

Further study about r-neighbourhood space, may give many solutions for the real life problems.

References