Rectilinear car motions taking into account the elasticity and deformability of tires

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Abstract. In this paper, the problem of developing a mathematical model of rectilinear motion of a car is solved, taking into account the elasticity and deformability of tires, as well as, for the same radii of the wheels of the car, and taking into account plane-parallel motion at a constant speed. The kinetic energy of the rectilinear motion of the car is found in the case of the same rotation of the front wheels around the axis of the pivots. Generalized forces are taken as sum of forces acting on the system and forces due to the deformation of the pneumatics. The forces acting on the system are found by virtual work method. The forces due to the deformation of the pneumatics are found as the generalized forces acting on the system under consideration, in the calculation of which all forces are taken into account, except for the tire deformation forces associated with angles and displacements. Using these kinetic energies and generalized force expressions, Lagrange's equations of the second type were used to obtain the differential equations of the rectilinear motion of the car. From the resulting mathematical model, it is possible to check and analyze the dynamics and stabilities of the system in various specific cases, i.e., under the influence of non-potential forces in the tire materials, and at large values of the tire kinematic parameters, and at high speed movements of the car. As a result of determining the stability field and stability borders at different values of constructive parameters, it will be possible to find the optimal parameters of the system and study its dynamics. Key words: mathematical model, rectilinear motion, wheels, deformation, elasticity, potential forces, kinetic energy, dynamics.

1 Introduction

The development of computer technologies associated with analytical transformations makes it possible to consider vehicle models with a large number of degrees of freedom.

At the present, a lot of experience has been accumulated in testing automotive equipment, methods have been developed for assessing the performance properties of a car as a whole, as well as individual systems, components and assemblies. Nevertheless, it is relevant to conduct full-scale tests to check the operability of the structure, compliance with the requirements stated in the design specification, as well as to solve other related problems in the works [1-3].

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In the work [4], mathematical models of the movement of a small truck “Labo” were developed on the basis of the obtained equations of motion. The values of vertical and horizontal vibrations of the car during the process of moving over irregularities are determined. According to the given masses, the parameters of the spring are determined and the model of the vehicle movement is solved using the Runge-Kutta numerical method.

In the work [5], a mathematical model of car movement was studied. The motion model includes mathematical models of the engine, steering, transmission and wheels. The mathematical model is of interest for further improvement of the design parameters of multi-axle vehicles.

In the work [6], a model of car motion is considered, and models of car motion with all steerable wheels are developed.

Mathematical modeling of nonlinear mechanical systems, studying their dynamics, exploring the stability of their vibrations and reducing harmful vibrations at low and high frequencies were solved in the works [7-13]. The Stability behavior of the system was checked at different values of the parameters, conclusions were obtained as a result of numerical calculations.

In the works [14-17], the problem of mathematical modeling and exploring dynamics of curvilinear motion of the car is considered according to properties of the elastic and deformability.

The use of dynamic models makes it possible to assess the impact design parameters on the motion of the car, to develop effective algorithms for driving cars, to implement them in the form of so-called active safety means.

2 Material and methods

The problem of developing a mathematical model of the rectilinear motion of a car, taking into account the elasticity and deformability of tires, and also, for the same radius of the wheels of the car, plane-parallel motion at a constant speed. The kinetic energy of the rectilinear motion of the car in the case of the same rotation of the front wheels around the axis of the pivots. This problem is considered as a special case of curvilinear motion of the car.

Let us assume that for small deviations of the vehicle from rectilinear motion at a constant speed V along the axis Oy, there is no tire slip along the road, and the values \( \theta_i, \chi_i, \xi_i, \phi_i \) are sufficiently small.

Fig. 1. Frame view of the vehicle.
We can express the kinematic relations of the system as following:

Denote the product of matrices $A_1 A_2 A_3 A_4$ through the matrix $A$

$$A = A_1 A_2 A_3 A_4 = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix},$$  

(1)

where the elements $A_{ij}$ ($i,j=1,2,3$) of this matrix in the linear approximation will have the following forms:

$$A_{11}^{(i)} = 1; \quad A_{12}^{(i)} = -\theta_1 + \gamma_0 \psi; \quad A_{13}^{(i)} = \psi; \quad A_{21}^{(i)} = \theta_1; \quad A_{22}^{(i)} = 1; \quad A_{23}^{(i)} = \beta_0 \theta_1 - \gamma_0; \quad A_{31}^{(i)} = -\psi + \gamma_0 \theta_1; \quad A_{32}^{(i)} = \gamma_0 + \beta_0 \theta_1; \quad A_{33}^{(i)} = 1;$$  

(2)

$$a_{11}^{(i)} = 1 - \beta_0 \psi; \quad a_{12}^{(i)} = -\theta_1 + \gamma_0 \psi; \quad a_{13}^{(i)} = \beta_0 + \psi; \quad a_{21}^{(i)} = \theta_1; \quad a_{22}^{(i)} = 1; \quad a_{23}^{(i)} = -\gamma_0; \quad a_{31}^{(i)} = \psi - \beta_0 - \gamma_0 \theta_1; \quad a_{32}^{(i)} = \gamma_0 + \beta_0 \theta_1; \quad a_{33}^{(i)} = -\beta_0 \psi + 1,$$

$\theta_1$ is the angle of the rotation of the front wheel around the pivot, counted from the direction of the longitudinal axis of the vehicle. $\psi$ is the angle of the rotation of the front suspension axle together with the wheels around the longitudinal axis of the vehicle. $\gamma_0$ is the angle of longitudinal inclination of the pin. It is positive when the upper end of the pin is moved back; $\beta_0$ is the angle of the transverse inclination of the pin.

For the rear axle and rear wheels, the relations take place $A_{ij}^{(i)}(\beta_0, \theta_1) = B_{ij}^{(2)}(-\beta_0, \theta_2)$, $a_{ij}^{(i)}(\beta_0, \theta_1) = b_{ij}^{(2)}(-\beta_0, \theta_2)$, therefore, we will have:

$$B_{11}^{(2)} = 1; \quad B_{12}^{(2)} = -\theta_2 + \gamma_0 \psi; \quad B_{13}^{(2)} = \psi; \quad B_{21}^{(2)} = \theta_2; \quad B_{22}^{(2)} = 1; \quad B_{23}^{(2)} = \beta_0 \theta_2 - \gamma_0; \quad B_{31}^{(2)} = \psi + \gamma_0 \theta_2; \quad B_{32}^{(2)} = \gamma_0 - \beta_0 \theta_2; \quad B_{33}^{(2)} = 1;$$  

(3)

$$b_{11}^{(2)} = -1 + \beta_0 \psi; \quad b_{12}^{(2)} = -\theta_2 + \gamma_0 \psi; \quad b_{13}^{(2)} = -\beta_0 + \psi; \quad b_{21}^{(2)} = \theta_2; \quad b_{22}^{(2)} = 1; \quad b_{23}^{(2)} = -\gamma_0; \quad b_{31}^{(2)} = \psi + \beta_0 - \gamma_0 \theta_2; \quad b_{32}^{(2)} = \gamma_0 - \beta_0 \theta_2; \quad b_{33}^{(2)} = \beta_0 \psi + 1,$$

$\theta_2$ is the angle of the rotation of the front wheel around the pivot, counted from the direction of the longitudinal axis of the vehicle.

Using relations (2) - (3), we find:

$O(x_1, y_1, z_1)$ is center of mass of the front suspension

$$x_{01} = x - l_1 \theta, \quad y_{01} = y + l_1, \quad z_{01} = z,$$

(4)

$x$, $y$, $z$ are coordinates of the center of mass of the car; $l_1$ is distance from the center of mass of the car to its front axle; $\theta$ is the angle of rotation of the car around the vertical axis.


\( B(x_2, y_2, z_2) \) is center of mass of the rear axle

\[
\begin{align*}
x_{02} &= x + l_2 \theta, \quad y_{02} = y - l_2, \quad z_{02} = z, \\
\end{align*}
\]

\( l_2 \) is distance from the center of mass of the car to its rear axle.

\( C_i(x_3, y_3, z_3) \) is center of mass of the front left wheel

\[
\begin{align*}
x_{03} &= x - l_1 \theta - L_1 \theta, \quad y_{03} = y + l_1 + L_1 \psi, \quad z_{03} = z + L_1 \psi - \gamma_0 l_3 \theta, \\
\end{align*}
\]

\( l_3 \) is distance from the center of the pivot to the center of the wheel; \( L_1 = l_1 + l_1 \) is distance from the center of mass of the front suspension to the center of the vehicle wheel (half-track).

\( C(x_4, y_4, z_4) \) is center of mass of front right wheel

\[
\begin{align*}
x_{04} &= x - l_1 \theta + L_1 \theta, \quad y_{04} = y + l_1 + L_1 \psi, \quad z_{04} = z - L_1 \psi + \gamma_0 l_3 \theta, \\
\end{align*}
\]

\( D_i(x_5, y_5, z_5) \) is center of mass of the rear left wheel

\[
\begin{align*}
x_{05} &= x + l_2 \theta - L_1, \quad y_{05} = y - l_2 - L_1 \theta, \quad z_{05} = z, \\
\end{align*}
\]

\( D(x_6, y_6, z_6) \) is center of mass of rear right wheel

\[
\begin{align*}
x_{06} &= x + l_2 \theta + L_1, \quad y_{06} = y - l_2 + L_1 \theta, \quad z_{06} = z, \\
\end{align*}
\]

coordinates of the center of the left pivot

\[
x_0 = -l, \quad y_0 = 0, \quad z_0 = l \psi, \\
\]

\( l \) is distance from the center of mass of the front suspension to the center of the pivot.

Using formulas (4) - (10), we find the squares of the speed of the center of mass of the front suspension \( V_i \), the rear axle \( V_2 \) and the four wheels of the car \( V_3, V_4, V_5, V_6 \).

\[
\begin{align*}
V_1^2 &= \dot{x}^2 + \dot{y}^2 + \dot{z}^2 + 2l_3 \dot{\theta} \dot{\psi} - 2l_3 \dot{x} \dot{\theta} \\
V_2^2 &= \dot{x}^2 + \dot{y}^2 + \dot{z}^2 + 2l_3 \dot{\theta} \dot{\psi} + 2l_3 \dot{x} \dot{\theta} \\
V_3^2 &= \dot{x}^2 + \dot{y}^2 + \dot{z}^2 + (\ell_1 + L_1)^2 \dot{\theta}^2 - 2(\ell_1 + L_1) \dot{x} \dot{\theta} + \ell_3 \dot{\theta}^2 + L_1 \dot{\psi}^2 - 2l_3 \dot{y} \dot{\theta} - 2l_3 \dot{y} \dot{\psi} - 2\gamma_0 l_3 \dot{x} \dot{\theta} - 2\gamma_0 l_3 \dot{y} \dot{\psi} \\
V_4^2 &= \dot{x}^2 + \dot{y}^2 + \dot{z}^2 + (\ell_1 - L_1)^2 \dot{\theta}^2 - 2(\ell_1 - L_1) \dot{x} \dot{\theta} + \ell_3 \dot{\theta}^2 + L_1 \dot{\psi}^2 + 2l_3 \dot{y} \dot{\theta} + 2l_3 \dot{y} \dot{\psi} + 2\gamma_0 l_3 \dot{x} \dot{\theta} + 2\gamma_0 l_3 \dot{y} \dot{\psi} \\
V_5^2 &= \dot{x}^2 + \dot{y}^2 + \dot{z}^2 + (\ell_2 + L_1)^2 \dot{\theta}^2 + 2l_2 \dot{x} \dot{\theta} - 2L_1 \dot{y} \dot{\theta} \\
V_6^2 &= \dot{x}^2 + \dot{y}^2 + \dot{z}^2 + (\ell_2 + L_1)^2 \dot{\theta}^2 + 2l_2 \dot{x} \dot{\theta} + 2L_1 \dot{y} \dot{\theta} .
\end{align*}
\]

Let us now express the angles \( \chi_i, \theta_i \) of the wheels and the coordinates \( x_i, y_i \) \( (i = 1, 4) \) through the generalized coordinates of the system. Here \( \chi_i \) is the angle between the \( O_i \) axis
and the median plane of the wheel, $\theta_i$ is the angle between the $Oy$ axis and the trace of the median plane of the wheel on the road, $x_i, y_i$ are the coordinates of the meeting point of the straight line of greatest inclination passing in the median plane of the wheel through its center with the plane $XOY$ roads. According to matrix (1)

$$
\cos\left(\frac{\pi}{2} + \chi_1\right) = A^{(1)}_{31}, \quad \cos\left(\frac{\pi}{2} + \chi_2\right) = B^{(2)}_{31}, \quad \cos\left(\frac{\pi}{2} + \chi_3\right) = \sin \beta, \quad \cos\left(\frac{\pi}{2} + \chi_4\right) = \sin \beta,
$$

$$
\cos\left(\frac{\pi}{2} - \theta_1\right) = A^{(1)}_{21}, \quad \cos\left(\frac{\pi}{2} - \theta_2\right) = B^{(2)}_{21}, \quad \cos\left(\frac{\pi}{2} - \theta_3\right) = \sin \theta, \quad \cos\left(\frac{\pi}{2} - \theta_4\right) = \sin \theta,
$$

$\beta$ is the angle of the transverse inclination of the rear wheels.

From these relations we find (in a linear approximation):

$$
\theta_1 = \theta + \theta_1, \quad \theta_2 = \theta + \theta_2, \quad \theta_3 = \theta, \quad \theta_4 = \theta,
$$

$$
\chi_1 = \psi - \gamma_0 \theta_1, \quad \chi_2 = \psi - \gamma_0 \theta_2, \quad \chi_3 = \beta, \quad \chi_4 = -\beta.
$$

(12)

The quantities $x_i, y_i$ are related to the generalized coordinates as follows:

$$
x_1 = x_{03} - r_i A^{(1)}_{31} = x - l_3 \theta - l_1 \theta - r_1 \psi + \gamma_0 r_1 \theta_1,
$$

$$
y_1 = y_{03} - r_i A^{(1)}_{32} = y + l_3 \theta - l_1 \theta - \gamma_0 r_1 \beta_0 r_1 \theta_1,
$$

$$
x_2 = x_{04} - r_i A^{(2)}_{31} = x - l_3 \theta + l_1 \theta - r_2 \psi + \gamma_0 r_2 \theta_2,
$$

$$
y_2 = y_{04} - r_i A^{(2)}_{32} = y + l_3 \theta - l_1 \theta - \gamma_0 r_2 + \beta_0 r_2 \theta_2,
$$

(12')

$$
x_3 = x_{05} - r_i \sin \beta \sin \theta = x + l_2 \theta - l_1 \beta_3 \theta,
$$

$$
y_3 = y_{05} \cos \theta = y - l_2 - l_1 \theta,
$$

$$
x_4 = x_{06} + r_i \sin \beta \sin \theta = x + l_2 \theta + l_1 + \beta r_4 \theta,
$$

$$
y_4 = y_{06} \cos \theta = y - l_2 + l_1 \theta,
$$

$r_i$ is rolling radius of the $i$-th wheel.

### 3 Results and discussion

The dynamic system under consideration consists of 6 interconnected bodies: a front dependent suspension, a rear part of the car without a front wheel suspension, and four wheels. Therefore, the kinetic energy of the system is

$$
T = \sum_{i=1}^{6} T_i,
$$

(13)
where \( T_i \) is the kinetic energy of the front suspension, \( T_2 \) is the kinetic energy of the rear of the vehicle without front suspension and wheels, \( T_3 \) is the kinetic energy of the front left wheel, \( T_4 \) is the kinetic energy of the front right wheel, \( T_5 \) is the kinetic energy of the rear left wheel, \( T_6 \) is the kinetic energy of the rear right wheel.

Expressions \( T_i \) \((i = 1, 6)\) are defined as follows:

\[
T_1 = 0.5m_1 V_1^2 + 0.5B_i \psi^2 + 0.5B_i \dot{\Theta}^2, \quad T_2 = 0.5m_2 V_2^2 + 0.5D \Theta^2,
\]

\[
T_3 = 0.5m_{21} V_3^2 + 0.5C_i \Omega_{1x}^2 + 0.5A_i (\Omega_{1y}^2 + \Omega_{1z}^2), \quad T_4 = 0.5m_{22} V_4^2 + 0.5C_i \Omega_{2x}^2 + 0.5A_i (\Omega_{2y}^2 + \Omega_{2z}^2), \quad (14)
\]

\[
T_5 = 0.5m_{23} V_5^2 + 0.5m_{23} \Omega_{3x}^2 + 0.5A_i (\Omega_{3y}^2 + \Omega_{3z}^2), \quad T_6 = 0.5m_{2a} V_6^2 + 0.5m_{2a} \Omega_{4x}^2 + 0.5A_i (\Omega_{4y}^2 + \Omega_{4z}^2),
\]

\( m_i \) is the mass of the rear of the car without wheels and front suspension; \( m_{2i} \) is mass of the \( i \)-th wheel; \( D \) is the moment of inertia of the rear of the car without front suspension and wheels relative to the axis passing through its center of mass; \( A_i \) is the moment of inertia of the \( i \)-th wheel with the hub and a brake drum relative to its diameter; \( B \) is the moment of inertia of the front suspension about an axis perpendicular to it and passing through the center of mass (central moment of inertia of the front suspension); \( C_i \) is axial moment of inertia of the \( i \)-th wheel; \( \Omega_i \) instantaneous angular velocity of the system relative to its center of mass.

In accordance with König's theorem, we have \( T_i = T_i' + T_i'' \), where \( T_i' \) is the kinetic energy of translational motion with the velocity \( \vec{V}_i \) of the center of mass of the system, and \( T_i'' \) is the kinetic energy of rotational motion with the instantaneous angular velocity \( \vec{\Omega}_i \) of the system relative to its center of mass. These projections of instantaneous angular velocities relative to the center of mass are determined by:

\[
\Omega_{1x} = -\dot{\Lambda}_1 + \beta_0 \dot{\Theta}_1 + \psi \dot{\Theta}_1 + \psi \dot{\Theta} - \gamma_0 \dot{\Theta} \dot{\Theta}_1; \quad \Omega_{1y} = \psi + \gamma_0 \dot{\Theta} + \beta_0 \dot{\Theta} \dot{\Theta}_1;
\]

\[
\Omega_{1z} = -\dot{\Lambda}_1 + \beta_0 \dot{\Theta}_1 + \psi \dot{\Theta}_1 + \psi \dot{\Theta} - \gamma_0 \dot{\Theta} \dot{\Theta}_1;
\]

\[
\Omega_{2y} = \psi + \gamma_0 \dot{\Theta}; \quad \Omega_{2x} = -\dot{\Lambda}_2 + \beta_0 \dot{\Theta}_2 + \psi \dot{\Theta}_2 + \psi \dot{\Theta} - \gamma_0 \dot{\Theta} \dot{\Theta}_2;
\]

\[
\Omega_{3x} = -\dot{\Lambda}_4 + \beta_0 \dot{\Theta}; \quad \Omega_{3y} = 0; \quad \Omega_{3z} = \dot{\Theta} ; \quad (15)
\]

\[
\Delta_i \text{ is angle of rotation around its axis of the } i \text{-th wheel}; \quad \rho_0 = \text{const.}
\]

The squares of the projections of the instantaneous angular velocities are equal in the linear approximation and have the form:

\[
\Omega_{1x}^2 = (-\dot{\Lambda}_1 + \beta_0 \dot{\Theta}_1)^2 - \Lambda_1^2 - 2\beta_0 \Lambda_1 \dot{\Theta}_1 - 2\Delta_i \dot{\Theta} \dot{\Theta}_1 - \dot{\Lambda}_1 \dot{\Theta} \psi - 2\gamma_0 \dot{\Theta} \dot{\Theta}_1;
\]

\[
\Omega_{1y}^2 = (\psi + \gamma_0 \dot{\Theta})^2 = \psi^2 + 2\gamma_0 \psi \dot{\Theta};
\]
\[
\Omega_{1z}^2 = (-\gamma_0 \dot{\psi} + \dot{\phi} + \dot{\phi}_1)^2 = \dot{\phi}^2 + \dot{\phi}_1^2 - 2\gamma_0 \dot{\psi} \dot{\phi} - 2\gamma_0 \dot{\psi} \dot{\phi}_1 + 2\dot{\phi} \dot{\phi}_1;
\]
\[
\Omega_{2x}^2 = (-\lambda_2 - \beta_0 \dot{\phi}_2)^2 - \lambda_2^2 + 2\beta_0 \lambda_2 \dot{\phi}_2 - 2\dot{\lambda}_2 \dot{\psi} \dot{\phi}_2 - 2\gamma_0 \dot{\psi} \dot{\phi}_2 + 2\gamma_0 \dot{\lambda}_2 \dot{\psi} \dot{\phi}_2;
\]
\[
\Omega_{2y}^2 = (\dot{\psi} + \gamma_0 \dot{\phi})^2 = \dot{\phi}^2 + 2\gamma_0 \dot{\psi} \dot{\phi};
\]
\[
\Omega_{2z}^2 = (-\gamma_0 \dot{\psi} + \dot{\phi} + \dot{\phi}_2)^2 = \dot{\phi}^2 + \dot{\phi}_2^2 - 2\gamma_0 \dot{\psi} \dot{\phi} - 2\gamma_0 \dot{\psi} \dot{\phi}_2 + 2\dot{\phi} \dot{\phi}_2;
\]
\[
\Omega_{3x}^2 = (-\lambda_3 + \beta \dot{\phi})^2 = \dot{\phi}^2 - 2\beta \dot{\lambda}_3 \dot{\phi} + \Omega_{3y}^2 = 0;
\]
\[
\Omega_{3z}^2 = \dot{\phi}^2; \quad \Omega_{4x}^2 = (-\lambda_4 - \beta \dot{\phi})^2 = \dot{\phi}^2 + 2\beta \dot{\lambda}_4 \dot{\phi}; \quad \Omega_{4y}^2 = 0; \quad \Omega_{4z}^2 = \dot{\phi}^2.
\]
Substituting the values \(\nu_i^2 (i=1,6)\) and \(\Omega_{jx}^2, \Omega_{jy}^2, \Omega_{jz}^2\) respectively from (11) and (16) into (14) we find the kinetic energies of the center of mass of the front and rear axles, as well as the left and right wheels of the front and rear axles:

\[
T_1 = 0.5m_1 (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5(m_1 \ell_1^2 + B_1) \dot{\phi}^2 + 0.5B \dot{\psi}^2 - m_3 \dot{\xi}_1 \dot{\theta} + \dot{\theta}^2;
\]
\[
T_2 = 0.5m_1 (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5(m_2 \ell_2^2 + D) \dot{\phi}^2 + m_1 \dot{\xi}_2 \dot{\theta};
\]
\[
T_3 = 0.5m_2 (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5C_1 \dot{\lambda}_1^2 + 0.5[ \ell_1 \dot{\lambda}_1 + (\dot{\ell}_1 + \dot{L}_1) \dot{\lambda}_1 \dot{\phi} + 0.5(\dot{\ell}_1 + \dot{L}_1) \dot{\lambda}_1 \dot{\phi} + \dot{\lambda}_1 \dot{\phi} - m_2 \ell_2 \dot{\xi}_3 \dot{\theta} - m_2 \ell_3 \dot{\xi}_2 \dot{\theta} + m_2 \dot{L}_2 \dot{\xi}_3 \dot{\theta} - \gamma_0 \dot{\ell}_1 \dot{\psi} \dot{\phi} - \gamma_0 \dot{\ell}_2 \dot{\psi} \dot{\phi} - \gamma_0 \dot{\ell}_3 \dot{\psi} \dot{\phi} - \gamma_0 \dot{L}_1 \dot{\psi} \dot{\phi} - \gamma_0 \dot{L}_2 \dot{\psi} \dot{\phi} - \gamma_0 \dot{L}_3 \dot{\psi} \dot{\phi} - \gamma_0 \dot{\psi} \dot{\phi} - \gamma_0 \dot{\psi} \dot{\phi}_1 - \gamma_0 \dot{\psi} \dot{\phi}_2 - \gamma_0 \dot{\psi} \dot{\phi}_3] \dot{\phi}^2 + (17)
\]
\[
T_4 = 0.5m_2 (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5C_2 \dot{\lambda}_2^2 + 0.5[ \ell_2 \dot{\lambda}_2 + (\dot{\ell}_2 + \dot{L}_2) \dot{\lambda}_2 \dot{\phi} + 0.5(\dot{\ell}_2 + \dot{L}_2) \dot{\lambda}_2 \dot{\phi} + \dot{\lambda}_2 \dot{\phi} - m_2 \ell_2 \dot{\xi}_3 \dot{\theta} - m_2 \ell_3 \dot{\xi}_2 \dot{\theta} + m_2 \dot{L}_2 \dot{\xi}_3 \dot{\theta} + \beta_0 C_2 \dot{\lambda}_2 \dot{\phi} - \beta_0 C_2 \dot{\lambda}_2 \dot{\phi} - \beta_0 C_2 \dot{\lambda}_2 \dot{\phi} - \beta_0 C_2 \dot{\lambda}_2 \dot{\phi} - \gamma_0 \dot{L}_2 \dot{\psi} \dot{\phi} - \gamma_0 \dot{L}_3 \dot{\psi} \dot{\phi} - \gamma_0 \dot{L}_3 \dot{\psi} \dot{\phi} - \gamma_0 \dot{L}_3 \dot{\psi} \dot{\phi} - \gamma_0 \dot{\psi} \dot{\phi} - \gamma_0 \dot{\psi} \dot{\phi}_1 - \gamma_0 \dot{\psi} \dot{\phi}_2 - \gamma_0 \dot{\psi} \dot{\phi}_3];
\]
\[
T_5 = 0.5m_3 (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5C_3 \dot{\lambda}_3^2 + 0.5[ \ell_3 \dot{\lambda}_3 + (\dot{\ell}_3 + \dot{L}_3) \dot{\lambda}_3 \dot{\phi} + +m_3 \dot{L}_3 \dot{\xi}_2 \dot{\theta} - m_3 \dot{L}_4 \dot{\xi}_2 \dot{\theta} - \beta C_3 \dot{\lambda}_3 \dot{\phi}];
\]
\[
T_6 = 0.5m_4 (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5C_4 \dot{\lambda}_4^2 + 0.5[ \ell_4 \dot{\lambda}_4 + (\dot{\ell}_4 + \dot{L}_4) \dot{\lambda}_4 \dot{\phi} + +m_4 \dot{L}_4 \dot{\xi}_2 \dot{\theta} + m_4 \dot{L}_4 \dot{\xi}_2 \dot{\theta} + \beta C_4 \dot{\lambda}_4 \dot{\phi}].
\]
Substituting the values \(T_1 (i=1,6)\) from (17) into (13), we find the kinetic energy of the car:
\[ T = 0.5(m_1 + m_i + m_{21} + m_{22} + m_{23} + m_{24}) \left( x^2 + y^2 + z^2 \right) + 0.5\left[ (m_{1} \ell_{1}^2 + B_i) + 
\right.
\]
\[ +m_{1} \ell_{1}^2 + D + A_i + m_{21} (\ell_{1} + L_1)^2 + A_2 + m_{22} (\ell_{1} - L_1)^2 + A_3 + m_{23} (\ell_{2} + L_2)^2 + A_4 + 
\]
\[ +m_{24} (\ell_{2} + L_1)^2 \right) \dot{\theta}_1^2 + 0.5\left[ (A_i + m_{21} \ell_{1}^2) \dot{\theta}_2^2 + (A_2 + m_{22} \ell_{1}^2) \dot{\theta}_3^2 \right] + 0.5\left[ A_i + m_{21} L_1^2 + A_2 + 
\]
\[ +m_{22} L_1^2 + B \right) \psi^2 + 0.5(C_i \dot{\Delta}_1^2 + C_i \dot{\Delta}_2^2 + C_i \dot{\Delta}_3^2 + C_i \dot{\Delta}_4^2) \right] + [m_{1} \ell_{1} - m_{1} \ell_{1} - m_{1} (\ell_{1} + L_1) - 
\]
\[ -m_{22} (\ell_{1} - L_1) + m_{23} \ell_{2} + m_{24} \ell_{2}] \dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_3 + \gamma_0 [m_{22} \ell_{3} \dot{\theta}_3 - 
\]
\[ -m_{21} \dot{\ell}_{1} \dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_3 + (m_{21} L_1 - m_{22} L_2) \dot{\psi} \dot{\theta}_1 + (m_{24} L_1 - m_{23} L_2) \dot{\psi} \dot{\theta}_2 + A_i \dot{\theta}_1 + A_i \dot{\theta}_2 - 
\]
\[ -\gamma_0 (A_2 + m_{22} \ell_{3} L_1) \dot{\psi} \dot{\theta}_1 - \gamma_0 (A_i + m_{21} \ell_{1} L_1) \dot{\psi} \dot{\theta}_2 - \beta C_i \dot{\Delta}_1 \dot{\theta}_1 + 
\]
\[ + \beta C_i \dot{\Delta}_2 \dot{\theta}_2 - \beta C_i \dot{\Delta}_3 \dot{\theta}_3 - \beta C_i \dot{\Delta}_4 \dot{\theta}_4 - C_i \dot{\Delta}_1 (\psi \dot{\theta}_1 + \dot{\theta} \psi - \gamma_0 \dot{\theta} \psi) - C_i \dot{\Delta}_2 (\psi \dot{\theta}_2 + \dot{\theta} \psi - \gamma_0 \dot{\theta} \psi). \] 

Introducing the following notation:

\[ m = m_{1} + m_{3} + m_{21} + m_{22} + m_{23} + m_{24}; \]

\[ J_1 = B_i + m_{1} \ell_{1}^2 + D + m_{1} \ell_{1}^2 + A_i + m_{21} (\ell_{1} + L_1)^2 + A_2 + m_{22} (\ell_{1} - L_1)^2 + 
\]
\[ + A_3 + m_{23} (\ell_{2} + L_2)^2 + A_4 + m_{24} (\ell_{2} + L_1)^2; \]

\[ J_2 = J_{21} + J_{22}; J_{21} = A_1 + m_{21} \ell_{1}^2; J_{22} = A_2 + m_{22} \ell_{3}^2; \]

\[ J_3 = A_1 + m_{21} L_1^2 + A_2 + m_{22} L_1^2 + B; \]

\[ J_{x\theta} = m_{1} \ell_{2} - m_{1} \ell_{1} - m_{22} (\ell_{1} + L_1) - m_{23} \ell_{2} + m_{24} \ell_{2}; \]

\[ J_{y\theta} = m_{22} \ell_{3}^2; J_{z\theta} = -m_{21} \ell_{3}^2; J_{x\dot{\theta}_1} = m_{22} \ell_{3}^2; J_{z\dot{\theta}_1} = m_{22} \ell_{3}^2; J_{y\dot{\theta}_3} = m_{22} L_1 - m_{23} L_1; \]

\[ J_{y\dot{\theta}_2} = A_4 + m_{21} \ell_{3} L_1; J_{y\dot{\theta}_2} = A_2 + m_{22} \ell_{3} L_1. \]

Taking into account (18'), we rewrite the kinetic energy of the system:

\[ T = 0.5m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5(J_{x\dot{\theta}}^2 + J_{y\dot{\theta}_1}^2 + J_{z\dot{\theta}_1}^2 + J_{y\dot{\theta}_2}^2 + J_{z\dot{\theta}_2}^2 + C_i \dot{\Delta}_1^2 + C_i \dot{\Delta}_2^2 + 
\]
\[ + C_i \dot{\Delta}_3^2 + C_i \dot{\Delta}_4^2) + J_{x\theta} \dot{x} \dot{\theta}_1 + J_{y\theta} \dot{y} \dot{\theta}_1 + J_{z\theta} \dot{z} \dot{\theta}_1 + \gamma_0 (J_{x\dot{\theta}_1} z \dot{\theta}_2 + J_{y\dot{\theta}_2} \dot{\theta}_2) + 
\]
\[ + J_{z\dot{\theta}_1} \dot{z} \dot{\theta}_2 + A_i \dot{\theta}_1 + A_i \dot{\theta}_2 - \gamma_0 J_{y\dot{\theta}_2} \dot{\theta}_1 - \gamma_0 J_{z\dot{\theta}_2} \dot{\theta}_2 - 
\]
\[ - \beta C_i \dot{\Delta}_1 \dot{\theta}_1 + \beta C_i \dot{\Delta}_2 \dot{\theta}_2 - \beta C_i \dot{\Delta}_3 \dot{\theta}_3 - \beta C_i \dot{\Delta}_4 \dot{\theta}_4 - C_i \dot{\Delta}_1 (\psi \dot{\theta}_1 + \dot{\theta} \psi - \gamma_0 \dot{\theta} \psi) - 
\]
\[ - C_i \dot{\Delta}_2 (\psi \dot{\theta}_2 + \dot{\theta} \psi - \gamma_0 \dot{\theta} \psi). \] 

The first case. Let us assume that the wheel radii \( r = r_1 = r_2 = r_3 = r_4 \) are equal and the car moves in a plane-parallel manner with a constant speed \( V = \omega \), where \( \omega = \Delta_1 = \ldots = \Delta_4 \), \( A = A_1 = A_2 = A_3 = A_4 \), \( C = C_1 = C_2 = C_3 = C_4 \), \( m_{21} = m_{22} = m_{23} = m_{24} \).

Then the kinetic energy of the rectilinear motion of the car (19) will take the form:
\[ T = 0.5m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5(J_1 \dot{\theta}^2 + J_{21}(\dot{\theta}_1^2 + \dot{\theta}_2^2) + J_3 \dot{\psi}^2 + 4C_0 \omega^2) + J_{x\omega} \dot{x} \dot{\theta} + \\
+J_{y\theta}(\dot{y} \dot{\theta}_1 - \dot{y} \dot{\theta}_2) + \gamma_0 [J_{x\psi}(\dot{z} \dot{\theta}_2 - \dot{z} \dot{\theta}_1) + A\ddot{\theta}_2 + \dot{A}\dot{\theta}_2 - \gamma_0 \dot{\psi}_0 \dot{\psi} + \dot{\psi}_0 \dot{\psi}_1] - \\
-\beta_0 C\omega (\dot{\theta}_2 - \dot{\theta}_1) - C\alpha \ddot{\psi}_0 (\dot{\theta}_1 + \dot{\theta}_2) + 2\dot{\psi}_0 \dot{\psi} - 2\gamma_0 \dot{\psi} (\dot{\theta}_1 + \dot{\theta}_2)] \tag{20} \]

where

\[ J_1 = B_1 + m_1 \ell_1^2 + D + m_1 \ell_2^2 + 4A + 2m_2 \ell_2^2 + 4m_3 \ell_3^2 + 2m_4 \ell_2^2; \]
\[ J_2 = 2J_{21} = 2(A + m_2 \ell_2^2); \quad J_{21} = A + m_2 \ell_2^2 = J_{22}; \quad J_3 = B + 2A + 2m_3 \ell_1^2 = B + 2(A + m_2 \ell_2^2); \]
\[ J_{x\theta} = (m_1 + 2m_2) \ell_2 - (m_3 + 2m_2) \ell_4; \quad J_{y\theta} = -m_2 \ell_3; \quad J_{\psi \theta} = m_2 \ell_3; \quad J_{\psi \phi} = 0; \tag{21} \]
\[ J_{x\phi} = -m_2 \ell_3; \quad J_{y\phi} = m_2 \ell_3; \quad J_{\psi \phi} = 0; \quad J_{\phi \phi} = A + m_2 \ell_3 \ell_4; \quad J_{\phi \psi} = A + m_2 \ell_3 L_1. \]

The second case. The kinetic energy of the rectilinear motion of the car in the case of the same rotation of the front wheels around the axis of the pivots. Let us assume that all conditions of the first case are satisfied and \( \phi = \phi_1 = \phi_2, \quad \dot{\phi} = \dot{\phi}_1 = \dot{\phi}_2 = \dot{\phi}. \)

Then the kinetic energy of the rectilinear motion of the car in the case of the same rotation of the front wheels around the axis of the pivots will take the form:

\[ T = 0.5m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5[J_1 \dot{\theta}^2 + J_2 \dot{\theta}_2^2 + J_3 \dot{\psi}^2 + 4C_0 \omega^2] + \\
+J_{x\theta} \dot{x} \dot{\theta} + J_{x\phi} \dot{x} \dot{\phi} - 2\gamma_0 J_{x\psi} \dot{\psi} \dot{\theta} - 2C_0 \dot{\psi} \dot{\phi} - 2C \dot{\psi} \dot{\theta} + 2 \gamma_0 C_0 \dot{\theta} \dot{\phi}. \tag{22} \]

When compiling the equations of vehicle motion, taking into account the deformability of tires, one should take into account the kinematic relationships imposed on the rolling of the wheels [18]. We assume that the wheel is dynamically and geometrically symmetrical about its median plane. Consider the line obtained by the intersection of the middle plane of the undeformed tire. This line is called the center line of the wheel. When a wheel rolls on a horizontal plane with a deformable tire, the deformation of the center line is associated with the deformation of the entire tire. We will assume that tire deformations are sufficiently small. According to the theory of rolling of an elastic tire [19], due to the smallness of the tire deformation, we will consider the transverse, longitudinal, angular and radial deformations of the tire.

When the tire is deformed, a contact patch will be obtained, in which there is a whole segment of the center line. Further, from the assumption that the center of the contact area does not slip, the condition that the velocity is equal to zero for any point of this segment follows. In the case of rectilinear motion, the kinematic equations, which mean the conditions for rolling the wheel without sliding, with transverse and longitudinal deformation of the tires have the form

\[ \begin{align*}
\dot{x}_i + \dot{\xi}_i + V \theta_i + V \phi_i &= 0, \\
\dot{\theta}_i + \dot{\phi}_i - \alpha_i V \dot{\xi}_i + \beta_i V \dot{\phi}_i + \gamma_i V \chi_i &= 0, \\
\dot{y}_i + r_i \dot{\Lambda}_i + \dot{\eta}_i + \lambda_i \dot{y}_i \eta_i - \nu_i \dot{y}_i (r_i - r_i) &= 0. \tag{23}
\end{align*} \]
As is known from the kinematics of the system, the quantities \( \chi_i, x_i, y_i, \theta_i \) are expressed in terms of generalized coordinates \( x, y, z, ..., \Delta_4 \) using formulas (12) and (12'). Substituting the values of these quantities in (23), we obtain the equations of kinematic constraints in the case under consideration:

\[
\begin{align*}
\dot{x} - (\ell_1 + L_1) \dot{\theta} - r_1 \dot{\psi} + \gamma_0 r_1 \dot{\phi}_1 + \dot{\xi}_1 + V \theta + V \phi_1 &= 0, \\
\dot{\theta} + \dot{\phi}_1 - \alpha_i \dot{V} \xi_1 + \alpha \dot{V} \phi_1 + \gamma_1 \dot{V} \psi - \gamma_0 \psi V \theta_1 &= 0, \\
r_1 \ddot{\phi}_1 + \dot{\phi}_1 - [1 + \lambda_2 \eta_2 - v_2 (r_{10} - r_1)] (\dot{y} - (\ell_2 + \beta_0 r_2) \dot{\phi}_2) &= 0, \\
\dot{x} - (\ell_1 - L_1) \theta - r_1 \dot{\psi} + \gamma_0 r_1 \dot{\phi}_2 + \dot{\xi}_2 + V \theta + V \phi_2 &= 0, \\
\dot{\theta} + \dot{\phi}_2 - \alpha_2 \dot{V} \xi_2 + \beta_2 \dot{V} \phi_2 + \gamma_2 \dot{V} \psi - \gamma_0 \dot{V} \phi_2 &= 0, \\
r_2 \ddot{\phi}_2 + \dot{\phi}_2 - [1 + \lambda_2 \eta_2 - v_2 (r_{20} - r_2)] (\dot{y} + (\ell_2 + \beta_0 r_2) \dot{\phi}_2) &= 0, \\
\dot{x} + [\lambda_2 \eta_2 - v_2 (r_{20} - r_2)] (\dot{y} - L_2 \dot{\theta}) &= 0.
\end{align*}
\]

The system of equations (24) is the equation of the kinematic relations of the vehicle along a straight path.

In the case of identical wheel radii \( r = r_1 = r_2 = r_3 = r_4 \), the kinematic equations will have the form:

\[
\begin{align*}
\dot{x} - (\ell_1 + L_1) \dot{\theta} - r_1 \dot{\psi} + \gamma_0 r_1 \dot{\phi}_1 + \dot{\xi}_1 + V \theta + V \phi_1 &= 0, \\
\dot{\theta} + \dot{\phi}_1 - \alpha_i \dot{V} \xi_1 + \alpha \dot{V} \phi_1 + \gamma_1 \dot{V} \psi - \gamma_0 \psi V \theta_1 &= 0, \\
r_1 \omega + \dot{\phi}_1 - [1 + \lambda_2 \eta_2 - v_2 (r_20 - r)] (\dot{y} - (\ell_2 + \beta_0 r) \dot{\phi}) &= 0, \\
\dot{x} - (\ell_1 - L_1) \theta - r_1 \dot{\psi} + \gamma_0 r_1 \dot{\phi}_2 + \dot{\xi}_2 + V \theta + V \phi_2 &= 0, \\
\dot{\theta} + \dot{\phi}_2 - \alpha_2 \dot{V} \xi_2 + \beta_2 \dot{V} \phi_2 + \gamma_2 \dot{V} \psi - \gamma_0 \dot{V} \phi_2 &= 0, \\
\omega + \dot{\phi}_2 - [1 + \lambda_2 \eta_2 - v_2 (r_{20} - r_2)] (\dot{y} + (\ell_2 + \beta_0 r) \dot{\phi}_2) &= 0, \\
\dot{x} + [\lambda_2 \eta_2 - v_2 (r_{20} - r_2)] (\dot{y} - L_2 \dot{\theta}) &= 0, \\
\dot{x} + [\lambda_2 \eta_2 - v_2 (r_{20} - r_2)] (\dot{y} + (\ell_2 + \beta_0 r) \dot{\phi}_2) &= 0, \\
\dot{x} + (\ell_2 + \beta r) \dot{\theta} + \dot{\xi}_3 + V \theta + V \phi_3 &= 0, \\
\dot{\theta} + \dot{\phi}_3 - \alpha_3 \dot{V} \xi_3 + \beta_3 \dot{V} \phi_3 + \gamma_3 \dot{V} \psi - \gamma_0 \dot{V} \phi_3 &= 0, \\
\dot{x} + (\ell_2 + \beta r) \dot{\theta} + \dot{\xi}_4 + V \theta + V \phi_4 &= 0, \\
\dot{\theta} + \dot{\phi}_4 - \alpha_4 \dot{V} \xi_4 + \beta_4 \dot{V} \phi_4 - \gamma_4 \dot{V} \psi &= 0, \\
\dot{x} + (\ell_2 + \beta r) \dot{\theta} + \dot{\xi}_4 + V \theta + V \phi_4 &= 0, \\
\dot{\theta} + \dot{\phi}_4 - \alpha_4 \dot{V} \xi_4 + \beta_4 \dot{V} \phi_4 - \gamma_4 \dot{V} \psi &= 0.
\end{align*}
\]
\[ \dot{\theta} + \dot{\phi}_4 - \alpha_4 V \xi_4 + \beta_4 V \phi_4 - \gamma_4 V \beta = 0, \]
\[ \omega r + \dot{\eta}_4 + [1 + \lambda \eta_4 - \nu_4 (r_0 - r)] (\dot{y} + L \dot{\theta}) = 0. \]

Let's assume that takes place:
\[ 2\xi_n = \xi_1 + \xi_2, \quad 2\xi_2 = \xi_3 + \xi_4, \quad 2\phi_n = \phi_1 + \phi_2, \quad 2\phi_2 = \phi_3 + \phi_4, \quad 2\eta_n = \eta_1 + \eta_2, \quad 2\eta_2 = \eta_3 + \eta_4, \]
\[ \begin{align*}
  \varphi_1 &= \varphi_2 = \varphi, \\
  \alpha_1 &= \alpha_2 = \alpha_n, \\
  \beta_1 &= \beta_2, \\
  \gamma_1 &= \gamma_2, \\
  \alpha_3 &= \alpha_4, \\
  \beta_3 &= \beta_4, \\
  \gamma_3 &= \gamma_4, \\
  \alpha_5 &= \alpha_6 = \alpha_7, \\
  \beta_5 &= \beta_6, \\
  \gamma_5 &= \gamma_6 = \gamma_7, \\
  \alpha_8 &= \alpha_9 = \alpha_{10}, \\
  \beta_8 &= \beta_9, \\
  \gamma_8 &= \gamma_9 = \gamma_{10}, \\
  \alpha_{11} &= \alpha_{12}, \\
  \beta_{11} &= \beta_{12}, \\
  \gamma_{11} &= \gamma_{12}.
\end{align*} \]

Then the kinematic equations (25) will take the form:
\[ \begin{align*}
  \dot{\xi} - \xi_1 \dot{\theta} - \xi_2 \dot{\phi}_4 + \xi_n \dot{\phi}_n + V \theta + V \phi_4 + V \phi_n &= 0, \\
  \dot{\phi}_n + \dot{\phi}_4 - \alpha_n V \xi_n + \beta_n V \phi_n + \gamma_n V \psi - \gamma_0 \nu_n V \varphi &= 0, \\
  r \omega + \eta_n + [1 + \lambda \eta_4 - \nu_4 (r_0 - r)] \dot{\varphi} &= 0, \\
  \dot{\theta} + \xi_2 \dot{\phi}_4 + V \theta + V \phi_4 &= 0, \\
  \dot{\phi}_4 - \alpha_4 V \xi_4 + \beta_4 V \phi_4 &= 0, \\
  \omega r + \eta_4 + [1 + \lambda \eta_4 - \nu_4 (r_0 - r)] \dot{\varphi} &= 0.
\end{align*} \]

The equations of motion of a vehicle, according to the theory of motion of rolling systems, are written as
\[ \frac{d}{dt} \frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} = Q_j + R_j \quad (j = 1, 11) \]

where \( Q \) are the generalized forces acting on the system, \( R \) are the generalized forces due to the deformation of the pneumatics, \( T \) is the kinetic energy of the system under consideration, \( q_1 = x, q_2 = y, q_3 = z, q_4 = \theta, q_5 = \psi, q_6 = \varphi, q_7 = \xi, q_8 = \eta, q_9 = \Delta, q_10 = \Delta, q_{11} = \Delta \) are generalized coordinates.

The generalized forces are calculated by the formula
\[ R_j = \sum_{i=1}^{m} \left( F_i \frac{\partial x_i}{\partial q_j} + M_{i\theta} \frac{\partial \theta}{\partial q_j} + M_{i\psi} \frac{\partial \psi}{\partial q_j} + M_{i\varphi} \frac{\partial \varphi}{\partial q_j} + P_i \frac{\partial y_i}{\partial q_j} \right), \]

or in our case
\[ R_j = \sum_{i=1}^{11} \left[ (F_i \sin \theta_i - P_i \cos \theta_i) \frac{\partial y_i}{\partial q_j} + (P_i \sin \theta_i - F_i \cos \theta_i) \frac{\partial x_i}{\partial q_j} + \right. \\
\left. + (N_i + F_{i\nu}) \frac{\partial z_i}{\partial q_j} + M_{\nu\theta} \frac{\partial \theta}{\partial q_j} + M_{\nu\psi} \frac{\partial \psi}{\partial q_j} + (rP_i + M_{i\nu}) \frac{\partial \Delta_i}{\partial q_j} \right], \]
where \( F_i \) is the transverse force \((F_i = F_{x_i})\), \( P_i \) is longitudinal force \((P_i = P_{y_i})\), \( M_{\theta_i} \) is moment about the vertical axis \( M_{\theta_i} = M_{x_i}, \)

\( M_{z_i} \) is moment relative to the longitudinal axis \( M_{z_i} = M_{y_i}, \)

\( M_i \) is moment relative to the transverse axis \( M_i = M_{z_i}. \)

The projection of the generalized forces on the \( x, y, z, \theta, \psi, 91, 92, \Delta 1, \Delta 2, \Delta 3, \Delta 4 \) axes have the form:

\[
R_x = F_1 \cos \theta_1 + F_2 \cos \theta_2 + (F_3 + F_4) \cos \theta - P_1 \sin \theta_1 - P_2 \sin \theta_2 - (P_3 + P_4) \sin \theta,
\]

\[
R_y = F_1 \sin \theta_1 + F_2 \sin \theta_2 + (F_3 + F_4) \sin \theta + P_1 \cos \theta_1 + P_2 \cos \theta_2 + (P_3 + P_4) \cos \theta,
\]

\[
R_z = \sum_{i=1}^{4} N_i + \sum_{i=1}^{4} F_{x_i},
\]

\[
R_\theta = \sum_{i=1}^{4} M_{z_i} - (F_1 \cos \theta_1 + F_2 \cos \theta_2) \ell_1 + (F_3 + F_4) \ell_2 + (F_1 \sin \theta_1) \ell_1 + (P_2 \sin \theta_1 - P_1 \sin \theta_1) \ell_1 + (P_4 - P_3) L,
\]

\[
R_\psi = M_{z_1} \cos \beta_0 \cos \gamma_0 - M_{x_1} \sin \beta_0 + P_1 e_1 \cos \beta_0 \sin \gamma_0,
\]

\[
R_\psi = M_{z_2} \cos \beta_0 \cos \gamma_0 + M_{x_2} \sin \beta_0 + P_2 e_2 \cos \beta_0 \sin \gamma_0,
\]

\[
R_\lambda = M_{n_1} = -r_1 P_1 + M_{z_1}, R_\lambda = M_{n_2} = -r_1 P_2 + M_{z_2},
\]

\[
R_\lambda = M_{n_3} = -r_3 P_3 \cos \beta + M_{z_3} \cos \beta, R_\lambda = M_{n_4} = -r_4 P_4 \cos \beta + M_{z_4} \cos \beta,
\]

where \( e_1, e_2 \) are the running-in arm of the left and right front wheels, respectively,

\[
\theta_1 = \theta + \delta_1, \theta_2 = \theta + \delta_2, F_i = F_{x_i}, P_i = P_{y_i}, M_{\theta_i} = M_{x_i}, M_{z_i} = M_{y_i}, M_i = M_{z_i}.
\]

Substituting the values \( \theta_1 = \theta + \delta_1, \theta_2 = \theta + \delta_2 \) and \( F_i = F_{x_i}, P_i = P_{y_i}, M_{\theta_i} = M_{x_i}, M_{z_i} = M_{y_i}, M_i = M_{z_i} \) into (29) we obtain expressions for the generalized reaction forces acting on the wheels from the bearing surface.

The projection of the generalized forces \( R_j \) on the \( x, y, z, \theta, \psi, \theta_1, \theta_2, \Delta 1, \Delta 2, \Delta 3, \Delta 4 \) axes have the form:

\[
R_x = a_1 \xi_1 + a_2 \xi_2 + a_3 \xi_3 + a_4 \xi_4 + h_1 \hat{\xi}_1 + h_2 \hat{\xi}_2 + h_3 \hat{\xi}_3 + h_4 \hat{\xi}_4 + \sigma_1 N\psi + \sigma_2 N\psi + \sigma_3 N\psi + \sigma_4 N\psi - \gamma_0 N\psi - \gamma_0 N\psi - \gamma_0 N\psi - \gamma_0 N\psi + \gamma_0 h_1 \hat{\psi} + h_2 \hat{\psi} + h_3 \hat{\psi} + h_4 \hat{\psi} + \gamma_0 h_1 \hat{\psi} + h_2 \hat{\psi} + h_3 \hat{\psi} + h_4 \hat{\psi} + \gamma_0 h_1 \hat{\psi} + h_2 \hat{\psi} + h_3 \hat{\psi} + h_4 \hat{\psi};
\]

\[
R_y = P_1 + P_2 + P_3 + P_4 = k_{\psi} \eta_1 + k_{\psi} \eta_2 + k_{\psi} \eta_3 + k_{\psi} \eta_4; R_z = -4c_a z;
\]
\[ R_\varphi = h_1 \phi_1 + h_2 \phi_2 + h_3 \phi_3 + h_4 \phi_4 + h_5 \phi_5 + h_6 \phi_6 + h_7 \phi_7 + h_8 \phi_8 - (\ell_1 + L_1) (a_1 \xi_1 + h_{11} \xi_1 + \sigma_1 N_1 \psi - \gamma_0 \sigma_1 N_1 \theta + h_{22} \psi - \gamma_0 h_{22} \hat{\psi} - (\ell_2 + L_2) a_2 \xi_2 + h_{12} \xi_2 + \sigma_2 N_2 \psi - \gamma_0 \sigma_2 N_2 \theta + h_{32} \psi - \gamma_0 h_{32} \hat{\psi} - (\ell_3 + L_3) a_3 \xi_3 + h_{13} \xi_3 + \sigma_3 N_3 \psi - \gamma_0 \sigma_3 N_3 \theta + h_{42} \psi - \gamma_0 h_{42} \hat{\psi} + (\ell_4 + L_4) a_4 \xi_4 + h_{14} \xi_4 - \sigma_4 N_4 \beta - L_1 k_1 \eta_1 + L_4 k_4 \eta_4 ; \]

\[ R_\psi = M_{x_1} + M_{x_2} - r_1 F_1 - r_2 F_2 = -\sigma_1 N_1 \xi_1 - h_{11} \xi_1 - \rho_1 N_1 \psi + \gamma_0 \rho_1 N_1 \theta - h_{21} \psi + \gamma_0 h_{21} \hat{\psi} - (\ell_3 + \beta_3 r_3) a_3 \xi_3 + h_{13} \xi_3 + \sigma_3 N_3 \psi - \gamma_0 \sigma_3 N_3 \theta + h_{41} \psi - \gamma_0 h_{41} \hat{\psi} + (\ell_4 + \beta_4 r_4) a_4 \xi_4 + h_{14} \xi_4 - \sigma_4 N_4 \beta - L_1 k_1 \eta_1 + L_4 k_4 \eta_4 ; \]

\[ R_\beta = b_1 \phi_1 + h_3 \phi_3 + \gamma_0 \sigma_1 N_1 \xi_1 + h_{11} \xi_1 + \rho_1 N_1 \psi + h_{41} \psi + (\ell_3 + \beta_3 r_3) k_3 \eta_3 = b_1 \phi_1 + h_3 \phi_3 + \gamma_0 \sigma_1 N_1 \xi_1 + h_{11} \xi_1 + \rho_1 N_1 \psi + h_{41} \psi + (\ell_3 + \beta_3 r_3) k_3 \eta_3 ; \]

\[ R_\beta = b_2 \phi_2 + h_{22} \phi_2 + (\ell_3 + \beta_3 r_3) k_3 \eta_3 + \gamma_0 \sigma_2 N_2 \xi_2 + h_{32} \xi_2 + \rho_2 N_2 \psi + h_{52} \psi + (\ell_4 + \beta_4 r_4) k_4 \eta_4 = b_2 \phi_2 + h_{22} \phi_2 + (\ell_3 + \beta_3 r_3) k_3 \eta_3 + \gamma_0 \sigma_2 N_2 \xi_2 + h_{32} \xi_2 + \rho_2 N_2 \psi + h_{52} \psi + (\ell_4 + \beta_4 r_4) k_4 \eta_4 ; \]

\[ R_\alpha = -r_1 P_1 + M_1 = -r_1 k_1 \eta_1 + \mu_1 N_1 \eta_1 = (-r_1 k_1 + \mu_1 N_1) \eta_1 ; \]

\[ R_\alpha = -r_2 P_2 + M_2 = (-r_2 k_2 + \mu_2 N_2) \eta_1 ; \]

\[ R_\alpha = r_3 P_3 + M_3 = (-r_3 k_3 + \mu_3 N_3) \eta_1 ; \]

The generalized forces \( Q_j = (q_j, \dot{q}_j, t), (j = 1, 11) \) acting on the system under consideration, in the calculation of which all forces are taken into account, except for the tire deformation forces associated with angles \( \chi_j, \phi_j \) and displacements \( \xi_j, \eta_j \), have the form:

\[ Q_\varphi = -k_1 \psi - h_1 \ddot{\psi}, Q_{\alpha_1} = -k_2 \ddot{\theta}_1 - h_2 \ddot{\theta}_1, Q_{\beta} = -k_3 z - h_3 \dot{z}, Q_\alpha = -k_4 \ddot{\theta}_2 - h_4 \ddot{\theta}_2 , \]

\[ Q_0 = Q_{\alpha_1} = Q_{\alpha_2} = Q_{\alpha_3} = Q_{\theta} = Q_\beta = 0 , \]

where \( k_1 = k_1 + 2C_{wz} l_{wz}^2 + 2C_{wz} l_{wz}^2 \) is angular rigidity of the system along the coordinate \( \psi, k^1_2, k^2_2, k_3 \) is angular rigidity along the coordinates \( \theta_1, \theta_2 \) and \( z \).
coefficient of viscous friction along the coordinate \( \psi \), \( h_1^1 \), \( h_2^2 \), \( h_3 \) are coefficients of viscous friction along the coordinates \( \partial_1 \), \( \partial_2 \), \( z \). Here, \( k_1^1 \) is the angular stiffness of the rod device, \( C_{pc} \) is the coefficient of elasticity of the spring, \( C_{un} \) is the radial stiffness of the tire, \( L_{pc} \) is the distance from the center of mass of the system to the spring, \( h_{un} \), \( h_{pc} \), \( h_a \) are the internal resistances of the tire, springs and shock absorber, \( h_s \) is the coefficient of viscous friction of the rod device. We assume that, within the limits of the change in generalized coordinates, the damping in the system has linear characteristics.

Substituting the values \( R_j, Q_j \), and \( T \) respectively from (30), (31) and (19) into the dynamic equation of vehicle motion (27), we find:

\[
m\ddot{x} + J_x\ddot{\theta} + a_x\dot{\xi}_1 + a_x\xi_2 + a_x\xi_3 + a_x\xi_4 + h_1\dot{\xi}_1 + h_2\dot{\xi}_2 + h_3\dot{\xi}_3 + \sigma_1 N_1 \psi + \sigma_2 N_2 \psi + + \sigma_3 N_3 \beta - \sigma_4 N_1 \beta - \gamma_0 (\sigma_1 N_1 \dot{\theta}_1 + \gamma_0 \sigma_2 N_2 \dot{\theta}_2 + h_3 \psi - \gamma_0 h_2 \dot{\theta}_1 + h_2 \psi - \gamma_0 h_1 \dot{\theta}_1 + h_1 \psi - \gamma_0 h_0 \dot{\theta}_2 = 0; \]

\[
m\ddot{y} + J_y\ddot{\theta} + J_{\theta y}\ddot{\theta}_1 + J_{\theta y}\ddot{\theta}_2 - k_1 \eta_1 - k_2 \eta_2 + k_3 \eta_3 = k_4 \eta_4 = 0; \]

\[
m\ddot{z} + h_{z}\ddot{z} + (k_4 + 4c_a)z + \gamma_0 [J_{z\delta} m_{21} \ddot{\theta}_1 + J_{z\delta} m_{22} \ddot{\theta}_2] + J_{z\psi} \ddot{\psi}_0 = 0; \]

\[
J_1 \ddot{\theta} + J_{x\theta}\ddot{x} + J_{y\theta}\ddot{y} + J_{z\theta}\ddot{z} - \beta C_x \dot{\lambda}_1 + \beta C_y \dot{\lambda}_2 + \beta C_z \dot{\lambda}_3 + (((\ell_1 + L_1) h_{21} + (\ell_1 + L_1) h_{22}) \psi + + [((\ell_1 + L_1) \sigma_1 N_1 + (\ell_1 - L_1) \sigma_2 N_2) \psi - \gamma_0 ((\ell_1 + L_1) \sigma_1 N_1 - \gamma_0 ((\ell_1 + L_1) \sigma_2 N_2) \psi - - \gamma_0 (\ell_1 - L_1) h_{21} \dot{\theta}_1 - \gamma_0 (\ell_1 - L_1) h_{22} \dot{\theta}_2 + h_3 \dot{\psi} - (\ell_1 - L_1)(a_x \xi_2 + h_1 \xi_1) - (\ell_1 - L_1)(a_x \xi_3 + h_1 \xi_2) - (\ell_1 - L_1)(a_x \xi_4 + h_1 \xi_3) - - (\ell_1 - L_1) \sigma_1 N_1 + (\ell_1 + L_1) \sigma_2 N_2) \beta = 0; \]

\[
J_2 \ddot{\psi} + [h_1 + (h_4 - r_1 h_2)] \psi + [k_1 + (\alpha_1 + \sigma_1 r_1) N_1 + (\alpha_2 + \sigma_2 r_2) N_2] \psi + + J_{z\psi} + \gamma_0 J_{z\psi} \ddot{\theta}_1 + (a_x \dot{r}_1 + \sigma_1 N_1) \ddot{\theta}_1 + (a_x \dot{r}_2 + \sigma_2 N_2) \ddot{\theta}_2 + (h_{z2} + + r_1 h_{z2}) \ddot{\theta}_2 - \gamma_0 (\alpha_1 + \sigma_1 r_1) \dot{\theta}_1 + \gamma_0 (h_{z1} + r_1 h_{z1}) \dot{\theta}_1 + (h_{z2} + r_1 h_{z2}) \ddot{\theta}_2 = 0; \]

\[
J_{21} \ddot{\lambda}_1 + h_{1} \dot{\lambda}_1 - h_{1} \dot{\lambda}_2 + k_1 \dot{\lambda}_2 + J_{\psi2} \ddot{\theta}_1 + J_{\psi2} \ddot{\theta}_2 - \beta_0 C_1 \dot{\lambda}_1 + h_3 \dot{\lambda}_2 - \gamma_0 (h_{z1} + r_1 h_{z1}) \dot{\lambda}_1 - \gamma_0 (a_x \dot{r}_1 + \sigma_1 N_1) \dot{\lambda}_1 + (\ell_1 + L_1) \psi = 0; \]

\[
J_{22} \ddot{\dot{\lambda}}_2 + h_{1} \ddot{\lambda}_2 + h_{2} \ddot{\lambda}_2 + J_{\psi2} \ddot{\lambda}_2 - \beta_0 C_2 \ddot{\lambda}_2 - (h_{z2} + r_2 h_{z2}) \ddot{\theta}_2 = 0; \]

\[
C_1 \dot{\lambda}_1 - \beta_0 C_1 \dot{\lambda}_1 - (\mu_1 N_1 - r_1 k_1) \eta_1 = 0; C_2 \dot{\lambda}_2 + \beta_0 C_2 \dot{\lambda}_2 - (\mu_2 N_2 - r_2 k_2) \eta_2 = 0; \]

\[
C_3 \dot{\lambda}_3 - \beta_0 C_3 \dot{\lambda}_3 - (\mu_3 N_3 - r_3 k_3) \eta_3 = 0; C_4 \dot{\lambda}_4 + \beta_0 C_4 \dot{\lambda}_4 - (\mu_4 N_4 - r_4 k_4) \eta_4 = 0. \]
(24) and (32) are a mathematical model of the rectilinear motion of the car, taking into account the elasticity and deformability of tires, and also, for the same radii of the wheels of the car, plane-parallel movement at a constant speed. The kinetic energy of the rectilinear motion of the car in the case of the same rotation of the front wheels around the axis of the pivots.

4 Conclusion

Further refinement of the right parts of equations (32) depends on certain assumptions about the nature of the deformation by pneumatics, which is determined by carrying out computational experiments.

This obtained system of differential equations allows to estimate the dynamics and stability of the considered system at different values of structural parameters and elastic dissipative properties.

References

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