Interlaminar shifts of two-layer aggressive-resistant combined plates based on metal and fiberglass

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Abstract. In this work, the stress and deformation states of plates made of metal layers wrapped with composite materials in the inner or outer layers were studied. The main goal of this research is to create durable, long-lasting and aggressive environment-resistant structures that can easily bear external dynamic and static loads. The work used the Navier method to study the effects of interlayer displacement. It was found that the lower the modulus, the greater the effect of joint compliance on the deformation of laminated composite plates. A regularity has been established that the greater the thickness of the bearing fiberglass layer, the less the effect of the shear modulus of the seam on the stresses and deformability of two-layer combined slabs. A regularity has been established, the greater the thickness of the bearing metal layer, the less the effect of the shear modulus of the seam on the stresses and deformation of two-layer combined plates. Keywords: Two-layer plate, slab, shear modulus, shear deformation, tension and deformation conditions, composite layer, epoxy resin, adhesive layer, interlayer shear.

1 Introduction

When operating laminated plates and shells, it is necessary to take into account the operation of the gluing seam, since it allows you to create a reliable structure in adverse production conditions [1,2,3,4], protecting them from heat and external pressures.

In engineering structures, there are two-layer plates and shells, multilayer cylinders, combined structures created on the basis of metal and composite materials.

2 Materials and Methods

Let us continue the study of the stress-strain state of plates and shells, taking into account the compliance of the adhesive joint and various mechanical characteristics of individual layers. The stress-strain state of combined two-layer slabs, taking into account interlayer shifts, built on the basis of metal and fiberglass according to the refined theory [2], makes it possible to estimate the strength and deformability with sufficient high accuracy when solving engineering problems.

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Consider the connection of two-layer orthotropic combined plates (Fig. 1), assuming that the first bearing (fiberglass) layer is significantly different from the second (metal) reinforcing.

3 Results

We believe that in relation to the plates considered in this case, the accepted hypotheses according to the refined theory [2,5,6] are valid: the thicknesses of the first and second layers are constant; the first layer is much more powerful than the second. Therefore, we take approximately $e_{xz} = 0$; $w=w(x, y)$. Here $e_{xz}$ is the relative strain elongation along the $z$ coordinate; $w$-deflection.

Shear deformations of the first layer

$$e_{xz} = 0.5 \left( \frac{h^2}{4} - y^2 \right) \phi_1 + \left( 0.5 - \frac{y}{h} \right) \frac{\tau_1}{G(3)_{13}}$$

Shear deformations of the second layer

$$e'_{xz} = \left( 0.5 + \frac{y_1}{\delta} \right) \frac{\tau_1}{G(2)_{13}}$$

$$e'_{yz} = \left( 0.5 + \frac{y_1}{\delta} \right) \frac{\tau_2}{G(2)_{23}}$$

Where $h, \delta$ is the thickness of the fiberglass and metal layers; $\phi_i = \phi_i(x, y)$ - arbitrary desired functions for shifting coordinates $x, y$; $\tau_i = \tau_i(x, y)$ - desired shear stresses; $G^{(1)}_{ik}, G^{(2)}_{ik}$ - shear moduli of the first and second layers ($i=1,2; k=3$).

The $\gamma$ coordinates have the following change limits:

for the first layer $\frac{h}{2} \leq \gamma_1 \leq \frac{h}{2}$;

for the second layer $\frac{\delta}{2} \leq \gamma_1 \leq \frac{\delta}{2}$.

Fig. 1. Double layer combination board.
We derive the displacements of the fiberglass layer using the relations of the theory of elasticity [7,8,9]:

\[ u^{(1)} = u_0 - \gamma \frac{\partial w}{\partial x} + \left( \frac{\gamma h^2}{8} - \frac{\gamma^3}{6} \right) \phi_1 + \gamma \left( \frac{1}{2} - \frac{\gamma}{2h} \right) \frac{\tau_1}{G^{(1)\_13}} \]

\[ v^{(1)} = v_0 - \gamma \frac{\partial w}{\partial y} + \left( \frac{\gamma h^2}{8} - \frac{\gamma^3}{6} \right) \phi_1 + \gamma \left( \frac{1}{2} - \frac{\gamma}{2h} \right) \frac{\tau_1}{G^{(1)\_13}} \]  

(4)

Similarly for the metal layer:

\[ u^{(2)} = u_0 - \gamma_1 \frac{\partial w}{\partial x} + \gamma_1 \left( \frac{1}{2} - \frac{\gamma_1}{2\delta} \right) \frac{\tau_1}{G^{(2)\_13}} \]

\[ v^{(2)} = v_0 - \gamma_1 \frac{\partial w}{\partial y} + \gamma_1 \left( \frac{1}{2} - \frac{\gamma_1}{2\delta} \right) \frac{\tau_2}{G^{(2)\_23}} \]  

(5)

Where \( u_0 = u_0(x, y) \), \( v_0 = v_0(x, y) \) are the desired tangential displacements of the corresponding point of the middle surface of the first layer.

Tangential displacements \( u^{(1)}, u^{(2)}, v^{(1)}, v^{(0)} \) - any point of the plate, formulas (3), (4) in contrast to from the classical theory depend on \( \gamma \) non-linearly, in the second layer - on \( \gamma_1 \) linearly. This is due to the difference in the thickness of the layers and taking into account the transverse shear in the more powerful first one. Observing the conditions for the continuity of the displacements of the glue line, we find the relationship between the displacements of the first and second layers.

\[ u_{\text{w}} = u_{\text{w}}^\# - \gamma \frac{\tau_1}{G^{(1)\_w13}} \]

\[ v_{\text{w}} = v_{\text{w}}^\# - \gamma \frac{\tau_2}{G^{(1)\_w23}} \]  

(6)

Let us write down the contact conditions of the layers

\[ u_{\text{w}}(\gamma = -\frac{h}{2}) - \tau_1 \varepsilon_{\text{w}13} = u_{\text{w}}(\gamma = \frac{\delta}{2}) \]

\[ v_{\text{w}}(\gamma = -\frac{h}{2}) - \tau_2 \varepsilon_{\text{w}23} = v_{\text{w}}(\gamma = \frac{\delta}{2}) \]  

(7)

Where \( u_{\text{w}}^\# \), \( v_{\text{w}}^\# \) are seam displacements at \( \gamma = -\frac{h}{2} \); \( \varepsilon_{\text{w}ik} = h_{\text{w}}/G_{\text{w}ik} \); \( h_{\text{w}} \), \( G_{\text{w}ik} \) - thickness and shear modulus of the seam.

Observing conditions (6), after the necessary transformations of the displacement of the second layer, we write

\[ u^{(2)} = u_0(\xi_2 - \gamma_1) \frac{\partial w}{\partial x} - \frac{h^3}{24} \phi_1 + \left[ \left( \frac{\gamma_1}{2} + \frac{\gamma_1^2}{2\delta} \right) \frac{1}{G^{(2)\_13}} - \frac{S H^{13}}{13} \right] \tau_1; \]

\[ v^{(2)} = v_0(\xi_2 - \gamma_1) \frac{\partial w}{\partial y} - \frac{h^3}{24} \phi_2 + \left[ \left( \frac{\gamma_1}{2} + \frac{\gamma_1^2}{2\delta} \right) \frac{1}{G^{(2)\_23}} - \frac{S H^{23}}{23} \right] \tau_2. \]  

(8)
Here

$$\xi_2 = 0.5(h + \delta);$$

$$\xi_{413} = \frac{3}{8} \left( \frac{h}{G_{13}^{(1)}} + \frac{\delta}{G_{13}^{(2)}} \right);$$

$$\xi_{423} = \frac{3}{8} \left( \frac{h}{G_{23}^{(1)}} + \frac{\delta}{G_{23}^{(2)}} \right);$$

$$SH_{13} = \xi_{413} + \varepsilon_{m13};$$

$$SH_{23} = \xi_{423} + \varepsilon_{m23}$$

The deformations in the layers are determined by the known Cauchy relations

$$\varepsilon_x^{(i)} = \frac{\partial u^{(i)}}{\partial x};$$

$$\varepsilon_y^{(i)} = \frac{\partial u^{(i)}}{\partial y};$$

$$\varepsilon_y^{(i)} = \frac{\partial v^{(i)}}{\partial y};$$

$$\varepsilon_{xy}^{(i)} = \frac{\partial v^{(i)}}{\partial x} + \frac{\partial u^{(i)}}{\partial y};$$

(9)

For the stresses in the layers, we have

$$\sigma_x^{(i)} = B_{11}^{(i)} \varepsilon_x^{(i)} + B_{12}^{(i)} \varepsilon_y^{(i)};$$

$$\sigma_y^{(i)} = B_{22}^{(i)} \varepsilon_y^{(i)} + B_{12}^{(i)} \varepsilon_x^{(i)};$$

$$\tau_{xy}^{(i)} = G_{11}^{(i)} \varepsilon_{xy}^{(i)};$$

(10)

Where,

$$B_{11}^{(i)} = \frac{E_1^{(i)}}{1 - \mu_1 \mu_2};$$

$$B_{12}^{(i)} = \frac{\mu_1 E_2^{(i)}}{1 - \mu_1 \mu_2};$$

$$B_{22}^{(i)} = \frac{E_2^{(i)}}{1 - \mu_1 \mu_2};$$

$$E_1^{(i)}, E_2^{(i)}$$ - layer elasticity modulus;
\( \mu_1 \) and \( \mu_2 \) - Poisson's ratio for different layers;
\( i = 1.2 \) - for the first layer.

The plate deformation equation is obtained using the variational principle, taking the total energy of the plate as a functional. The functional has the form

\[
u = \frac{1}{2} \iint_S \left( \sigma_{xx}^{(i)} \epsilon_{xx}^{(i)} + \sigma_{yy}^{(i)} \epsilon_{yy}^{(i)} + \sigma_{xy}^{(i)} \epsilon_{xy}^{(i)} + \frac{1}{2} \iint_S \left( \tau_{11}^2 + \tau_{22}^2 + \tau_{12}^2 \right) ds \right) (11)
\]

Using the Euler variational equation, we obtain a system of fourth-order partial differential equations with respect to the unknowns \( w, u_0, v_0, \Phi_1, \Phi_2, \tau_1, \tau_2 \).

Because of the bulkiness, the system of differential equations, coefficients, and boundary conditions are not given. To study the effect of interlaminar shear, we take a slab that is freely sheared along the contour. Applying the Navier method, we assume that the slab carries a uniformly distributed load \( q \). The solution to the system of differential equilibrium equations that satisfies the boundary conditions is a double trigonometric series.

4 Discussion

Example. Consider a rectangular square two-layer slab with dimensions \( a=b=1.2 \) m. The thickness of the first and second layers is \( h=1.5 \times 10^{-2} \) m, respectively. The elastic characteristics of combined slabs are taken from [10-14].

\[
E_1^{(1)} = 3.05 \text{ MPa}; \ E_2^{(1)} = 1.88 \text{ MPa}; \ \mu^{(1)} = 0.18; \n\]

\[
E_1^{(2)} = E_2^{(2)} = 0.21 \times 10^2 \text{ MPa}; \ \mu^{(2)} = 0.26; \n\]

\[
G_{12}^{(1)} = 0.49 \text{ MPa}; \ G_{13}^{(1)} = 0.31 \text{ MPa}; \n\]

\[
G_{23}^{(1)} = 0.35 \text{ MPa}; \ G_{ik}^{(2)} = 81 \text{ MPa}; \ q = 1. \n\]

Numerical examples have shown that the shear modulus and joint thickness have a great influence on the strength and deformability of combined two-layer boards if the shear modulus of the adhesive layer is significantly less than the shear modulus of the layers. If the first thick layer consists of a composite material, then the effect of transverse shear on the stress-strain state of the combined slabs will be greater.

It should be noted that the smaller the modulus, the greater the effect of joint compliance on the deformability of laminated combined slabs.

Assuming for epoxy adhesive (\( G_{u0} = 0.5 \times 10^{-2} \text{ MPa} \)), we see that an increase in the shear modulus of the joint by 10 times reduces the stress in the fiberglass layer, which is 4.45 % (\( \sigma^{(f)} \)), and in the metal layer increases it by 10%. A twofold change in the thickness of the adhesive layer (from \( h_a = 10^{-4} \) to \( 0.5 \times 10^{-4} \) m) changes the stress in fiberglass towards \( x \) by 4.1% (see Fig. 2.) constructed for the point \( x=0.5a, y=0.5b \).
Thus, the greater the shear modulus of the weld, the less its effect on the stress-strain state. The analysis showed that an increase in the thickness of the bonding layer made of K-147 epoxy adhesive ($G_{uilc} = 0.5 \times 10^{-2}$ MPa) by 10 times (from $10^{-4}$ to $10^{-3}$ m) increases the slab deflection by 21%.

**5 Conclusion**

At a large value of $G_{uilc}$ of the order of $0.81 \times 10^2$ MPa, the thickness of the weld affects deflections insignificantly (less than 1 %). A regularity has been established that the greater the thickness of the bearing fiberglass layer, the less the effect of the shear modulus of the
seam on the stresses and deformability of two-layer combined slabs. Deflection of fiberglass slabs with external metal reinforcement according to the theory under consideration, taking into account the interlayer shift at $h=1.5 \times 10^{-2}$ m, $\delta=0.2 \times 10^{-2}$ m, $h_{uw} = 0.5 \times 10^{-3}$ m and $G_{ulk} = 0.5 \times 10^{-2}$ MPa, less by 64.64% compared to the slab deflection without external reinforcing layer. A regularity has been established, the greater the thickness of the bearing metal layer, the less the effect of the shear modulus of the seam on the stresses and deformation of two-layer combined plates. Based on the above calculations, it can be concluded that it is possible to increase the strength of two-layer combined slabs. Taking into account shifts of the non-metallic layer by 18-20% compared to single-layer metal plates.

References