

Dynamic entropy model of transport flows

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Abstract. In the process of planning transport systems, it is envisaged to determine the movement of vehicles between conditional areas of the city. In the process, a correspondence matrix is compiled. The dynamic entropy transport model makes it possible to determine the flows of movement between individual districts of the city, taking into account temporary changes in consumer preferences of individuals. The dynamic entropy transport model can be used to solve problems of transport planning in an urban environment and allows taking into account many independent as well as random factors taking into account changes in time. The traditional entropy transport model is based on the hypothesis that individuals in the equilibrium state of the system behave independently, however, in our opinion, this hypothesis somewhat simplifies the problem to a certain extent. And although it can be assumed that different individuals behave in the equilibrium state of the system to a certain extent, independently, at the same time individuals are influenced by external factors, and therefore the behavior of these individuals will depend on the influence of these external factors. The dynamic entropy transport model proposed in this paper makes it possible to take into account temporary changes in consumer preferences among buyers of transport services. The entropic dynamic transport model can be applied in forecasting the traffic of urban agglomerations.

1 Introduction

In large megacities, the opportunities for extensive development of transport communications in most cases have already ended. In this regard, optimal forecasting and planning of transport routes is of paramount importance. This, in turn, requires the creation of mathematical models of traffic flows.

According to [1], it is possible to increase the capacity of the road network by using intelligent traffic management systems.

According to [2] intelligent traffic management systems allow (Figure 1).

According to [3], all automated control systems perform the same type of actions, which consist in collecting information about the control object, evaluating the information received, and indirectly and directly influencing the controlled object.

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According to [4], information about the controlled object is collected through related information systems, various detectors and sensors.

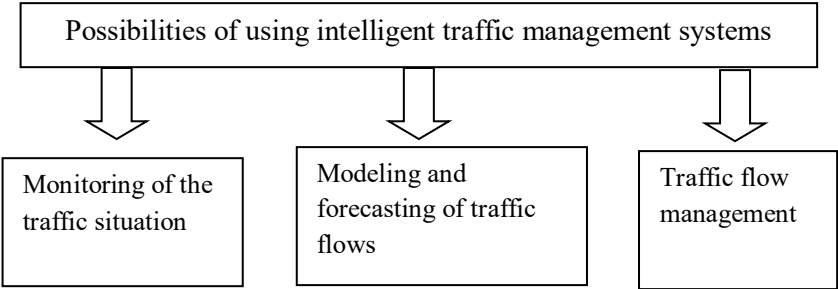


Fig. 1. Possibilities of using intelligent traffic management systems.

The analysis of the information obtained in the process of observation is carried out by means of a mathematical model embedded in the control system. The adequacy of the output data to the real parameters of the traffic flow depends on how accurate this model is. The authors [5] divide mathematical and simulation transport models. Simulation models simulate the movement of traffic flows, the functioning of traffic lights, simulates the behavior of pedestrians. Most of the data on traffic flow correlate with each other, and for a qualitative analysis of traffic flows it is necessary to possess a large amount of information. The authors [6] divide the traffic flow models into (Figure 2):

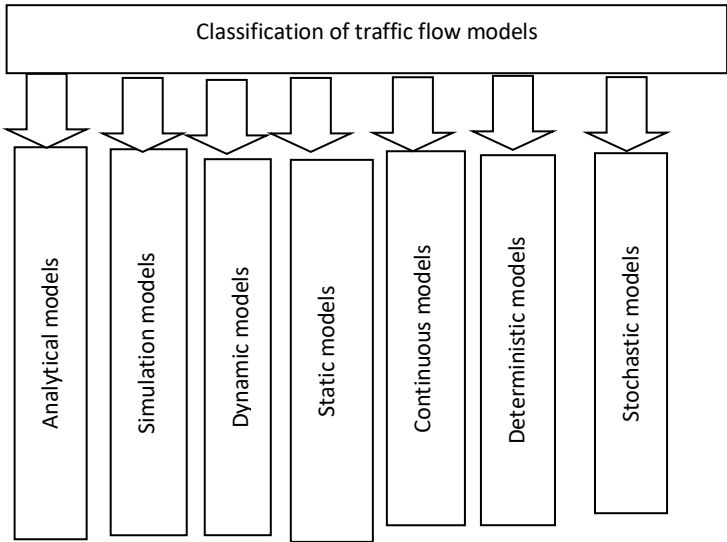


Fig. 2. Classification of traffic flow models.

According to [7], macromodels are of great importance for the description of transport flows. These models study the density, speed and intensity of traffic flow . Transport models allow you to model and predict changes in transport traffic depending on the influence of external factors on traffic flows.

Consequently, there is a need to develop new macromodels of traffic flow in order to improve road safety, as well as increase the capacity of roads.

According to [8], the use of the entropy approach for the analysis of statistical properties in randomly deterministic systems is used for various communication systems. The basis of this method is the classical Boltzmann-Gibbs-Shannon statistics. According to the entropy approach, it is assumed that the system is in a state of equilibrium, provided that the maximum of entropy is achieved in the system while some restrictions are fulfilled.

The classical formula for the Boltzmann-Gibbs-Shannon entropy looks like this

$$S = -k \sum_{i=1}^W p_i \ln p_i$$

$k > 0$ is the Boltzmann constant ; W – the statistical weight that determines the number of discrete states i

$$\left(\sum_{i=1}^W p_i = 1 \right)$$

According to Tsallis [9], the entropy equation can be written as follows

$$S_q = \frac{k}{q-1} \sum_{i=1}^W p_i (1 - p_i^{q-1}); \quad S_1 = \lim_{q \rightarrow 1} \frac{k}{q-1} \sum_{i=1}^W p_i (1 - p_i^{q-1}) = -k \sum_{i=1}^W p_i \ln p_i$$

2 Methods

In this study, we used an analytical method by which the problems under consideration were studied in their development and unity. Taking into account the goals and objectives of the conducted research, a functional-structural method of scientific research was used. This allowed us to consider some problems concerning the use of a dynamic entropy model of transport flows.

3 Results

Despite the fact that various physical models have found their successful application in other sciences, often traditional physical models in relation to non-physical phenomena require clarification.

Consider a hypothetical system that has the following properties:

The system exchanges energy with the external environment

$$E_q = \langle \varepsilon_i \rangle = \sum_{i=1}^W \varepsilon_i \cdot p_i^q \neq \text{const} \quad (1)$$

Where ε_i – is the energy of the i -th state , p_i is the probability of the i –th state.
 p_i - the probability of the i –th state depends on time

$$p_i = p_i(t) \quad (2)$$

However, there is a constant normalization of the probability distribution

$$\sum_{i=1}^W p_i = 1 \quad (3)$$

The entropy of our hypothetical system will look like this

$$S(t) = -k \sum_{i=1}^W p_i(t) * \ln p(t)_i \quad (4)$$

In our case, the Tsallis entropy equation [9] can be written as follows

$$S_q(t) = \frac{k}{q-1} \sum_{i=1}^W p_i(t) (1 - p(t)_i^{q-1}), \quad (5)$$

We find the extremum of our hypothetical system, we will use the Lagrange multiplier method for calculations. To do this, we use the Lagrangian

$$L_q(p_i, \alpha) = \frac{k}{q-1} \sum_{i=1}^W p_i(t) (1 - p(t)_i^{q-1}) - \alpha (\sum_{i=1}^W p(t)_i - 1) \quad (6)$$

Where α – is the Lagrange multiplier corresponding to constraints (3).

Using Lagrange's theorem, it is possible to determine the probability distribution p_i of the Tsallis entropy of our hypothetical system, for this it is necessary to solve the following system of differential equations:

$$\begin{aligned} \frac{\partial L_q}{\partial p_i}(p_i, t, \alpha) &= \frac{q \cdot k}{1-q} p_i^{q-1} - \frac{k}{1-q} + \alpha = 0 \\ \frac{\partial L_q}{\partial t}(p_i, t, \alpha) &= \frac{k - \alpha}{q - 1} * \frac{\partial p(t)_i}{\partial t} - \frac{k}{q - 1} * p(t)_i^{q-1} * \frac{\partial p(t)_i}{\partial t} = 0 \end{aligned} \quad (7)$$

By solving a system of partial differential equations (7) and substituting the obtained value of p_i into equation (4), it is possible to determine the entropy value at the extremum point.

According to the authors, the dynamic entropy model proposed in this paper can be used to model traffic flows.

According to [10], the traditional transport entropy model consists of the following equations:

$$\begin{aligned} \max_{T_{ij}} \sum_{i=1}^N \sum_{j=1}^M T_{ij} \ln \frac{\alpha_{ij}}{T_{ij}} \\ \sum_{j=1}^M T_{ij} &= Q_i, \dots, \forall i = 1 \dots N \\ \sum_{i=1}^N T_{ij} &= D_j, \dots, \forall j = 1 \dots M \\ T_{ij} &\geq 0, \forall i = 1 \dots N, \forall j = 1 \dots M \\ \alpha_{ij} &= e^{-\gamma c_{ij}}, \forall i = 1 \dots N, \forall j = 1 \dots M \end{aligned} \quad (8)$$

Where T_{ij} is correspondence from district i to district j ; Q_i is the number of trips from district i , D_j is the number of trips to district j , γ is the settlement parameter, c_{ij} is the average cost of movement.

The traditional entropy model (8) assumes the availability of information about consumers' preferences regarding their use of transport routes. The information about these preferences is contained in the matrix $\|\alpha_{ij}\|$.

In a real situation, consumer preferences may vary depending on the time of day, weather conditions, in addition, consumers can use different transport routes depending on the purpose of the trip. For example, after work, you can go straight home or pre-go to some store that is away from traditional travel destinations.

Thus, though

$$\sum_{j=1}^M T_{ij} = Q_i, \dots, \forall i = 1 \dots N$$

$$\sum_{j=1}^N T_{ij} = D_j, \dots, \forall j = 1 \dots M$$

Correspondence between districts T_{ij} , in general, depends on time. The dynamic entropy model of transport flows proposed by us in this paper will allow this time dependence to be taken into account.

4 Discussion

The process of planning transport systems involves determining the movement of vehicles between conditional areas of the city. In the process, a correspondence matrix is compiled. The dynamic entropy transport model makes it possible to determine the flows of movement between individual districts of the city, taking into account temporary changes in consumer preferences of individuals.

The dynamic entropy transport model can be applied to solve transport planning issues in an urban environment and allows taking into account many independent and random factors, taking into account changes in time.

The traditional entropy transport model is based on the hypothesis that individuals in the equilibrium state of the system behave independently, however, according to the authors, this hypothesis somewhat simplifies the problem.

And although it can be assumed that individuals behave to a certain extent, independently in the equilibrium state of the system, at the same time individuals are under the influence of external factors, and therefore the behavior of these individuals will depend on the influence of these external factors.

5 Conclusions

The dynamic entropy transport model proposed in the paper allows taking into account temporary changes in consumer preferences of buyers of transport services.

It can be used to predict the transport traffic of urban agglomerations.

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