Exploitation of reinforced concrete constructions on a ground base with force and environmental influences

Mikhail Berlinov1*

1Moscow State Construction University (National Research University), Moscow, Yaroslavl highway, 26, Russian Federation

Abstract. A calculation method is proposed, based on the basic starting points of the modern phenomenological theory of a non-linearly deformable elastic-creeping body, taking into account corrosion damage under environmental and force influences in conditions of long-term operation. The influence of reinforcement is investigated based on the compatibility of concrete and reinforcement deformation. Mathematical dependences are obtained that allow us to develop solving equations. These basic phenomenological resolving equations allow us to calculate reinforced concrete elements that work without cracks and with cracks from the point of view of the nonlinearity and rheology of concrete deformation. A method of linearization of the problem based on integral estimates is proposed, which provides fixation of rheological and corrosion processes in the considered period of operation. The implementation of this method will allow us to find a numerical solution to the problem using the method of integral estimates based on the step-by-step method and successive approximations. An example of the calculation of a reinforced concrete flexible foundation on a soil base is given for the considered service life and the presence of corrosion damage. Such an approach in the design of reinforced concrete structures will not only allow for more fully considering the real conditions of their work, identifying additional reserves of load-bearing capacity, but will also open up additional opportunities for analyzing and predicting their functioning at various stages of exploitation.

1 Introduction

The current state of the issue of reliability and durability of buildings and structures is becoming increasingly important in the modern construction industry. This is due to the fact that the vast majority of the operated structures, buildings and infrastructure facilities were erected more than half a century ago and are in a worn-out condition. For constructions of buildings and structures operating in intensive operational mode and adverse environmental influences, degradation processes associated with the development of corrosion processes are manifested, which shorten the expected service life [1-4].

* Corresponding author: berlinov2010@mail.ru

© The Authors, published by EDP Sciences. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (https://creativecommons.org/licenses/by/4.0/).
In conditions of long-term exploitation of reinforced concrete constructions, it is necessary to assess their stress-strain state as a result of corrosion damage, determine reliability, durability and residual service life, taking into account the reduction in the cross-sectional area of concrete and reinforcement [5-10].

Damage to a concrete sample, unloaded by external forces, caused by the unilateral action of a chemical aggressor of constant characteristics (temperature, composition, barometric pressure, intensity, etc.) in conditions of stable humidity of the medium and concrete, are considered to obey the non-linearly generalized Golberg-Waage law, which in the original record was formulated in a linear formulation and as such was first introduced in the entropic assessment of concrete aging:

\[
\frac{d\Delta \delta(t, t_0)}{dt} = -\alpha [\Delta \delta(t, t_0)]^m
\]

(1)

where: \(\Delta \delta(t, t_0) = \frac{\delta_{cr}(t_0) - \delta(t, t_0)}{\delta_{cr}(t_0)} = 1 - \frac{\Delta \delta(t, t_0)}{\delta_{cr}(t_0)}\), - the amount of damage accumulated by time \(t\), \(\delta(t, t_0)\) - the value of the damage; \(\delta_{cr}(t_0)\) - the maximum or critical depth at the colmatation type of damage, depending on the ratio of stresses to the ultimate strength \(\sigma/ R\); \(\Delta \delta(t, t0)\) - the current relative deficit of damage; \(-t, t_0\) - the time of the current observation and the start of the countdown; \(\alpha, m\) are the coefficients of the rate of damage determined empirically for each variant of the combination of concrete and the aggressor (\(\alpha\) characterizes the rate of attenuation of the process, \(m\) reflects the type of corrosion), depending on the level of stress state.

Note, that \(m=0\) corresponds to a non-decreasing damage front advancing linearly in time to the so-called filtration type of corrosion damage, \(m>0\) corresponds to an entropically attenuating, so-called colmatation type of corrosion damage and \(m<0\) corresponds to avalanche-developing damage that can follow, in particular, with intense physical wave aggression. Solution (1) is possible in the form of:

\[
\delta(t, t_0) = f_m(\alpha, m, t)\delta_{cr}(t_0)
\]

(2)

and leads, at \(m=0\), to the filtration type of physical corrosion (1st type of corrosion):

\[
f_0(t, t_0) = [\delta(t_0, t_0) + \alpha(t - t_0)].
\]

(3)

at \(m=1\) to the the colmatation type of corrosion (2nd type of corrosion):

\[
f_1(t, t_0) = 1 - \Delta \delta(t_0, t_0)e^{-\alpha(t-t_0)};
\]

(4)

and at \(m \neq 1\) for \(m\ 2,3,4\)

\[
\ldots \quad f_m = 1 - \{[\Delta \delta(t_0, t_0)]^{|m|+1} + \alpha[(-m) + 1](t - t_0)]^{|m|+1}\}
\]

(5)

Formula (5), however, was used as an empirical and confirmed to be evaluated experimentally. This allows to take the mechanical characteristics of damaged concrete (\(\kappa,\gamma,\varepsilon,\alpha,\varepsilon_0,\varepsilon_1\)) in the form of a series of products of their calculated values by a certain damage function depending on the \(Z\) function, measured from the contact surface of the aggressive medium and concrete surface (Fig.1). Denoting generically any distance characteristic of force loading resistance with the symbol \(L^*\):
\[ L' = K(z)L \]  

(6)

2 Materials and methods

Based on the generally accepted practice, the stresses in the cross section of the reinforced concrete element before the formation of cracks is the sum of the forces perceived by the reinforcement and concrete [12,13], then it is obvious that the following formula can be obtained taking into account corrosion damage in concrete and reinforcement:

\[ \sigma_x(t)A_x = \sigma_{b,x}(t)K_b(z,t)A_{b,x} + \sigma_{s,x}(t)\omega_{s}(t)A_{s,x} \]  

(7)

here: \( \sigma_x(t) \) - the average stress in the section of the reinforced concrete element; \( \sigma_{s,x}(t) \) and \( \sigma_{b,x}(t) \) – are the averaged stresses in reinforcement and concrete; \( A_{b,x} \) and \( A_{s,x} \) – correspond to the values of the area of reinforcement and concrete; \( K_b(z,t) \) – is a coefficient that takes into account the degree of corrosion damage to concrete, changing over time of observation; \( \omega_{s,x}(t) \) - is a similar coefficient for taking into account corrosion damage to reinforcement.

The coefficient that takes into account the degree of corrosion damage to concrete is defined as:

\[ K_b(z,t) = \left\{ 1 - \left[ \frac{P}{\beta(t,t_0)} \right] \right\} + \frac{2P}{\beta^2(t,t_0)}z - \frac{1}{\beta^2(t,t_0)}z^2 \]  

(8)

\( \beta(t,t_0) \) is the value of the damage value (Fig. 1);

\[ P_X K_Z 0 Z \]  

Fig. 1. Corrosion damage of a concrete sample with different resistance of a fiber layer in contact with an aggressive medium; a) from complete preservation \( K=1 \) b) to complete destruction \( K = 0 \) to; b the width of the sample; \( K(z) \) is a graph of the damage function of the force resistance.

As a rule, in parallel with the development of corrosion damage to concrete, corrosion of steel reinforcement can occur. These reinforcement damages are a kind of electrochemical process generated in a humid environment without external electrical sources. For the course of this process, in addition to humidity, as a prerequisite for corrosion, the temperature of the medium and oxygen saturation are of great importance for the process of corrosion of steel.

For reinforcement that have been subjected to corrosion damage, its area must be considered taking into account the decrease over time:
As = \omega_s(t)Aso                                                                                            (9)

Aso – is the area of the intact reinforcement, \( \omega_s(t) \) – is a coefficient that considering the decrease of the area of the reinforcement as a result of corrosion:

\[
\omega_s(t) = \left[ 1 - \frac{2\theta(t,t_0)}{D} + \frac{16\theta^2(t,t_0)}{\pi D^2} \right] \quad (10)
\]

where: \( D \) is the diameter of the reinforcement;

\[
\theta(t,t_0) = \frac{k}{\sqrt{\alpha}}t^n \quad (11)
\]

\( k \) and \( n \) – are empirical coefficients, \( \alpha \) is the value of the protective layer of concrete, mm., \( t \) – is the time.

In case of a crack, you can use the well-known assumption [14]:

\[
\sigma_s = \psi_{s,m} \mu_s \omega_s(t,t_0)\sigma_{s,m}(t,t_0); \quad \varepsilon_s = \frac{\sigma_s}{E_0} = \frac{\psi_{s,m} \mu_s \omega_s(t,t_0)\sigma_{s,m}(t,t_0)}{E_0} \quad (12)
\]

Thus, the calculated values of the cross-sectional area of the working reinforcement, as well as the mechanical characteristics of concrete in conditions of corrosion damage are not constant in the height of the cross-section. In the area located near the zone of maximum corrosion, their values are minimal and gradually increase, reaching maximum values in the zone of stabilization of the degradation process.

When studying the deformation of reinforced concrete constructions, it is necessary to take into account the working conditions of the reinforcement, while consideration should be carried out for 2 cases: before the formation of cracks in the material and after this process. In all models of deformation of constructions, in the absence of cracks, the hypothesis of compatibility of deformation of materials in concrete and reinforcement is taken into account, which makes it as simple as possible to take into account the influence of reinforcement on the stiffness of reinforced concrete during deformation.

The coefficients that considering corrosion damage are variable in cross-sectional height and change over time.

An important value in construction is the improvement of methods for calculating buildings and structures, including their bases and foundations. The solution of this problem is directly dependent on the correct assessment of their real properties.

Most of the currently existing methods for calculating reinforced concrete foundations assume their work to be elastically linear [14], which makes it possible to use solutions of the theory of elasticity, which permits to build a fairly simple, from an engineering point of view, calculation apparatus. However, as evidenced by numerous experimental data, reinforced concrete and foundation soils under the action of an external load on them behave like non-linearly deformed bodies, i.e. they have a nonlinear relationship between stresses and deformations, which is caused by their rheological properties. Moreover, for clay soils, this dependence is more pronounced, for sandy soils to a lesser extent. Therefore, the modulus of total deformation is not a constant value, but depends on the level of stress and corrosion damage of concrete and reinforcement.

As an example, consider the foundation, which is a reinforced concrete slab loaded with a system of concentrated forces, resting on a soil base. The system of external loads corresponds to the arrangement of the grid of columns in the plan. These forces are located at a distance of 3 m in the longitudinal and transverse directions. Taking into account the design features (load symmetry and a relatively small column pitch compared to the size of
the structure), a strip of unit width is taken as the design scheme, conditionally allocated from the plate in the transverse direction.

The ground of the base is considered as a continuous homogeneous nonlinear deformable medium of infinite power, characterized by a deformation modulus and a Poisson's ratio. The calculation is carried out according to the method of I.A. Simvulidi. The strip is divided into separate beams with a length of 3 m to ensure the necessary accuracy of calculation. Each beam is under the action of a given system of external forces and unknown internal forces \( Y_1, \ldots, Y_n \) and \( M_1, \ldots, M_n \), which arise at the cut points instead of discarded bonds (Fig. 2). The resulting calculation scheme makes it possible to calculate each of the sections as a simple beam of finite length on a soil base, and the modulus of deformation of the soil is variable along the entire length of the strip, but within each section it is averagely assumed to be constant and dependent on the stress state.

![Fig. 2. Calculation scheme](image)

Using the method described in [15], the system of resolving differential equations has the form:

\[
B(t) \frac{\partial^4 y_{1}}{\partial x_1^4} = -p_x^{(1)} + \sum \Gamma_{L_1}^{(1)} P_1 + \Gamma_{L_1}^{(2)} Y_1 + \Gamma_{L_1}^{(3)} M_1 \\
B(t) \frac{\partial^4 y_{n+1}}{\partial x_{n+1}^4} = -p_x^{(n+1)} + \sum \Gamma_{L_n}^{(n+1)} P_1 - (1)^{n+1}[\Gamma_{0} Y_n + \Gamma_{L_n}^{(n+1)} Y_{n+1}] - \Gamma_{0} M_n + \Gamma_{L_n}^{(n+1)} M_{n+1}
\]

where: \( B(t) \) is the rigidity of the beam; \( p_x \) is the distributed reaction from the side of the base approximated by a polynomial of the 3rd degree; \( \Gamma_{L_n} \) and \( \Gamma_{L_n}^{(n+1)} \) are, respectively, instantaneous interrupters of the first and second order; \( P_x^{(n)} \) - external load; \( Y_n \) and \( M_n \) are unknown transverse force and bending moment at the places of mental dismemberment of the beam.

Unknown forces \( Y_n \) and \( M_n \) are determined from a system of linear equations obtained on the basis of equality of reactive ground pressures and angular displacements of the beam at the places of mental cutting off one beam from another:

\[
\Omega_2 Y_1 + \Omega_3 Y_2 + \Omega_5 M_1 + \Omega_6 M_2 = \eta_{02} \Phi_1^{(1,2)} - \Phi_1^{(01)}
\]

\[
(-1)^{n+1}(\Omega_0 Y_{n-1} + \Omega_2 Y_n + \Omega_3 Y_{n+1}) + \Omega_4 M_{n-1} + \Omega_5 M_n + \Omega_6 M_{n+1} = \eta_{(n-1)(n+1)} \Phi_n^{(n(n+1))} - \Phi_n^{(n(n-1))}
\]

\[
D_{12} Y_1 + D_{13} Y_2 + D_{15} M_1 + D_{16} M_2 = \lambda_{02} U_1^{(12)} - U_1^{(01)}
\]

\[
(-1)^{n+1}(D_0 Y_{n-1} + D_2 Y_n + D_3 Y_{n+1}) + D_4 M_{n-1} + D_5 M_n + D_6 M_{n+1} = \lambda_{(n-1)(n+1)} U_n^{(n(n+1))} - U_n^{(n(n-1))}
\]

The coefficients for unknown forces and the free terms of equations (14) depend on the ratio of the flexibility indicators \( \lambda \) of neighboring sections, the magnitude of the acting loads and are determined from tabular data:
\[ \lambda_n = \frac{1-\mu}{1-\mu_0} \frac{n[\varepsilon^e(v,t)]^3}{B(t)} \]  

(15)

where: \( \mu \) and \( \mu_0 \) are, respectively, the Poisson coefficients of the strip and base material; \( E^e \) is the modulus of deformation of the base; \( L \) is the length of the cut-off section.

\[ B(t) = E^e(v,t) \left[ \frac{K_e(t)h}{12} x + bx (q_0 - \frac{x}{2})^2 \right] - E^a \left( \frac{A}{\sigma^2} + \frac{A_0}{\sigma_0^2} \right) h_0 - q_0 \]  

(16)

Here: \( E^e(v,t) \) is an integral modulus of deformations, taking into account the nonlinearity of deformation; \( E_0(t) \) is an elastically instantaneous modulus of deformations; \( C(t,t_0) \) is a measure of creep; \( x \) is the height of the compressed concrete zone; \( h_0 \) and \( g_0 \) are, respectively, the distance from the center of gravity of the stretched reinforcement and the center of gravity of the adduced cross-section to the compressed zone; \( E_s,Es' \) is the elastic modulus of the compressed and stretched reinforcement; \( A, As' \) is the areas of the – reinforcement; \( x \) is the height of the compressed zone of the reinforced concrete element; \( q_0 \) is the distance from the center of gravity of the adduced section to the compressed zone.

For sections of the beam working without cracks, the stiffness must be taken in the following form:

\[ B(t) = E^e(v,t) I_{red} \]  

(17)

\( I_{red} \) – is the moment of inertia of the adduced cross-section relative to its center of gravity, determined with the absence of cracks.

First, unknown forces are found from equations (8), then each cut-off part of the strip is considered as a separate beam loaded with an external load and forces \( Y_n \) and \( M_n \), i.e. it is necessary to solve equations (13). In the method proposed in [15], special tables are compiled that significantly facilitate the solution of the problem, making it possible to simply obtain the values of reactive ground pressures, bending moments and transverse forces.

For the soils of the base and the foundation material, the nonlinear dependence between stresses and strains is taken in a form:

\[ \varepsilon(t) = \frac{\sigma(t)}{E^0(t)} \left[ 1 + \eta \left( \frac{\sigma(t)}{K_0(x,t)} \right)^m \right] \]  

(18)

then \( E^e(v,t) \) – the integral modulus of deformations, taking into account the nonlinearity of deformation, will have the following form:

\[ \frac{1}{E^e(v,t)} = \frac{\partial\varepsilon}{\partial\sigma} \left[ \frac{1}{E^0(t)} + C(t,t_0) \right] \left[ 1 + \eta \left( \frac{\sigma(t)}{K_0(x,t)} \right)^m \right] \]  

(19)

\( E^0(t) \) – is the modulus of elastically instantaneous deformations; \( \sigma(t) \) - is the magnitude of the acting stresses; \( R(t) \) is the strength of the material \( n \) and \( m \) are the parameters of the nonlinearity of deformation determined from experimental data obtained on the basis of solving a system of two logarithmic equations composed for two points of the experimental curve (\( \varepsilon-\sigma \)).

When calculating according to the proposed method, the modulus of deformation of the base and the beam material is a variable value depending on the level of stress state, i.e. on the amount of ground resistance arising under the calculated strip and the operating...
stresses, taking into account corrosion damage in the beam section. It follows that equations (13) become nonlinear and the closed integration of equations (14) encounters insurmountable mathematical difficulties. However, the use of the method of successive approximations, which makes it possible to reduce the solution of nonlinear problems to a linear calculation apparatus, makes it possible to obtain the numerical value of the desired force quantities.

3 Results

The essence of this calculation method is as follows:

1. At the first stage, the problem is solved in an elastically linear formulation, when the processes of corrosion and creep are fixed, i.e. the integral modulus of deformation is assumed to be equal to the modulus of elastically instantaneous deformations and is considered constant for all sections of the calculated strip. Consequently, the flexibility index (15) is a constant value over the entire length of the structure. The joint solution of equations (13) and (14), along with the use of tables, gives the first approximation for rebounds and bending moments.

2. In the second approximation, according to the obtained values of bending moments for each section, the characteristics of corrosion and creep, rigidity of the reinforced concrete section are specified according to the method described in [2], and according to the ground stresses plot, the value of the integral modulus of base deformations is assigned for the ground base according to the formula (19), and the flexibility index (15) is calculated and the rebounds and bending moments are specified in the new solution.

3. In the third approximation, according to the second approximation stresses plot and the values of bending moments, the deformation modules are again refined taking into account corrosion damage, the flexibility index, the rebound and bending moment plots are calculated.

4. In a similar sequence, the calculation is carried out until the difference between two consecutive iterations doesn’t exceed the specified degree of accuracy. Figures 3 and 4 show the values of stresses plot and bending moments for a strip of a unit width of the foundation plate of the building with elastic and nonlinear formulation of the problem, taking into account corrosion damage and the duration of operation of 50 years. The base soil has the following physical and mechanical characteristics: $E_0(t) = 20 \text{ MPa}$, $R = 0.25 \text{ MPa}$, $n = 1.29$; $m = 2.48$; $\nu = 0.3$; $c(t,t_0) = 0.00278 \text{ 1/MPa}$; $\gamma = 0.025 \text{ 1/hour}$. The beam material is concrete B30, beam width $b = 1 \text{ m}$, height $h = 0.5 \text{ m}$ reinforcement class A-400, the values of the parameters of the nonlinearity of concrete, creep, corrosion characteristics and other necessary data are given in [2]. The parameters of the external load $P = 750 \text{ KN}$ and corrosion damage to the reinforcement $k = 1.62$, $n = 0.68$, the thickness of the protective layer of concrete $a = 30 \text{ mm}$. Due to the symmetry of the load and design, system (8) consisted of 10 equations, and system (7) consisted of 5 equations. The problem was solved with a computer program. In order to improve the convergence of the solution, the value of the modulus of the total deformation was determined as the arithmetic mean between its values in the last and penultimate iterations.
4 Conclusions

Analysis of the results obtained with elastic and nonlinear formulation of the problem, with corrosion damage and the duration of operation of 50 years, shows that taking into account the nonlinearity of deformation of the ground base of the beam material, corrosion damage during 50 years of operation causes the process of redistribution of forces. Comparing the plots of the reactive pressures of the soil and the values of bending moments in the foundation and a beam in a linear and nonlinear formulation, it can be concluded that the more loaded sections of the base and are unloaded, and the less loaded ones are reloaded. The minimum ordinates of the rebuff plot increased to 15%, and the maximum ones decreased to 11% (Fig. 3,4). This led to the transformation of the bending moments plot (the maximum ordinates decreased to 15%, and the minimum ones increased to 17%). Consequently, the use of non-linear methods for calculating foundations and foundations, taking into account corrosion damage and the duration of exploitation in the practice of modern design, can save materials and lead to a reduction in costs during the construction.

References

1. V.M. Bondarenko, Concrete and reinforced concrete 2, 56-61 (2008)
4. M.V. Berlinov, E3S Web of Conferences 33, 02049 (2018). DOI: 10.1051/e3sconf/20183302049