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Development of a mathematical model of sliding cutting of food raw materials and semi-finished products

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Abstract. The review of the main methods of intensifying the processes of cutting finished and half-finished products in various sectors of the processing complex shows that today the improvement of technological equipment is carried out mainly by optimizing cutting conditions and by improving the geometric characteristics of knives. At the same time, increasing the intensity of the cutting process and, accordingly, equipment performance, it is impossible not to take into account the technological requirements and conditions associated with improving the quality of the finished product. The article covers a mathematical model for a generalized criterion of the process, i.e. the total cutting force and its components in the directions of movement of the knife and the supply of material.

1 Introduction

The physical matter of all types of cutting is the organized destruction of the processing object due to the concentration of stress on the cutting edge of the tool when imposing restrictions on the quality indicators of the resulting half-finished or finished product (shape or size accuracy, cut surface evenness, amount of waste). Therefore, when analyzing the phenomena occurring in the zone of contact of the tool with the material being cut, it is advisable to consider the results of studies in the field of the theory of elasticity and plasticity, as well as mechanics of destruction of materials [1-4].

Modern ideas about the laws of sliding cutting are largely due to the success of experimental studies. Most of the scientific works are devoted to research of the characteristics of the processing of materials by cutting the force intensity of the process [5-11]. The factors influencing the magnitude and direction of cutting forces have been studied, and theoretical and experimental dependences of cutting forces on many parameters have been obtained [6, 12-14]. An analysis of the work performed in this direction shows that the qualitative, power, and energy aspects of cutting have been studied mainly at the macrolevel.

Nowadays, significant scientific and practical material on the methods of processing various types of food raw materials and half-finished products by cutting, the development of designs and operating experience of knife grinders, cutting machines and tools has been accumulated. Nevertheless, relatively few studies have been devoted to the determination of

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the main patterns and features of the mechanism of sliding cutting of food materials, which have so far been studied mainly at the macrolevel from the standpoint of establishing empirical dependences of output parameters (cut quality, amount of waste, productivity, energy consumption) on factors conditioned with processing modes, characteristics of the cutting tool and properties of the material being cut.

Mentioned works are of great importance, since in frame of the studied area of the factor space, they allow objectively choosing rational cutting modes, geometric characteristics of cutting tools, as well as design parameters of cutting machines. As for the radical improvement of the process of cutting food materials, the obtained dependences do not always give a satisfactory solution, since they do not sufficiently reveal the mechanism of material destruction and related phenomena. Therefore, theoretical and practical generalizations aimed at creating scientific foundations for improving the sliding cutting of food materials and its implementation in highly efficient designs of cutting machines and knife grinders are a major and urgent problem of great national economic importance.

The purpose of this article is to create scientific and mathematical foundations for improving the process of sliding cutting of food materials.

2 Methods

Based on the developed physical model of sliding cutting, it is possible to consider the interaction of macro- and micromechanisms for the formation of new surfaces during sliding cutting and obtain quantitative relationships between the initial factors and the output indicators of the process under study. From the sliding cutting scheme, a stable relationship between cutting forces and such indicators as cut quality and waste, energy costs, tool life and its technological rigidity can be seen.

3 Results

When considering the penetration of a knife in the chopping cutting mode (Figure 1), we introduce the following assumptions: the knife has a unit length along the blade and is penetrated into the material at a constant speed; the components of the total cutting force do not mutually influence.

The total cutting force can be represented as the sum of:

$$R = R_0 + 2R_v + 2F_{ff}^{ch} + 2F_{ff}^{c}, \tag{1}$$

where: R_0 is the force that provides the formation of a new surface and is applied to the cutting edge;

R_v is the vertical component of the elastic force of the half-finished product, which occurs because of its compression by the chamfer of the knife;

F_{ff}^{ch} is the projection of the friction force of the half-finished product on the knife facet on the direction of cutting;

F_{ff}^{c} is the friction force of the half-finished product on the side edge of the knife.

When determining the value of R_0, there is no need to consider the parameters of the microrelief of the cutting edge, since in chopping cutting, the entire blade profile is filled with material and the contact area is close to saturation. Under these conditions, R_0 can be represented as the sum of elemental forces acting behind infinitely small areas dS of the blade:

$$R_0 = \int_S \sigma_n dS \tag{2}$$

where: σ_n is normal stress in the half-finished product; S is the contact area of the cutting edge with the half-finished product.

Applying solution of the problem of the penetration of a chop into a half-infinite medium [15] it is possible to obtain the law of distribution of normal stresses in a semi-finished product depending on the angle of immersion of the blade into it:

$$\sigma_n = \sigma_S(1 - \frac{\pi}{2} - \varepsilon_0); \tag{3}$$

where: ε_0 is the angle of immersion of the blade.

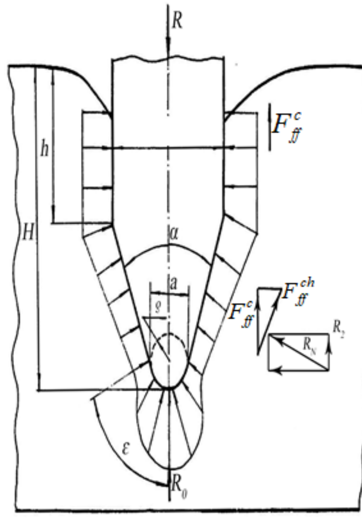

Fig.1. Power scheme for calculating the total cutting force.

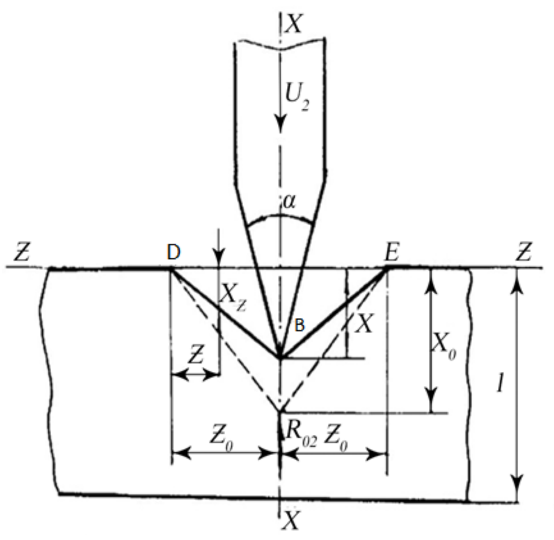

Fig. 2. Deformation of the material by a cutting wedge.

Considering the accepted shape of the cross section of the blade, the value of the elementary area can be represented as follows:

$$dS = \rho d\varepsilon_0 \tag{4}$$

Based on geometric considerations, it is possible to determine the radius of rounding of the cutting edge:

$$\rho = \frac{a}{2\cos\alpha/2} \tag{5}$$

The angle of immersion as the blade is penetrated will vary from 0 to ε_k determined from the expression:

$$\varepsilon_k = \frac{\pi}{2} - \frac{\alpha}{2} \tag{6}$$

Substituting the value σ_n from (3) into expression (2) and considering expressions (4) and (5), for the initial period of penetration of the cutting edge into the material we obtain:

$$R_0 = 2\int_0^{\varepsilon_\kappa} \sigma_S \left(1 + \frac{\pi}{2} - \varepsilon_0\right) \frac{a}{2\cos\alpha/2} d\varepsilon_0 \tag{7}$$

For the case when blade is completely immersed into half-finished product, the value R_0 is determined by the formula:

$$R_0 = \sigma_S\left(2 + \frac{\alpha}{2}\right)\left(\pi - \frac{\alpha}{2}\right)\frac{\alpha}{2\cos\alpha/2}, \tag{8}$$

In order to determine the resistance R_N at the deformation by a lateral facet, we consider the stress diagram taking into account the modulus of elasticity of the material. Evidently, at the point of intersection of the blade surface and the facet, the stress will have the value $\frac{a}{2b_0}E$ (where E is the modulus of elasticity). With an increase in the immersion depth of the knife (δ is the knife thickness), the stresses will increase proportionally until they reach the value $\frac{\delta}{2b_0}E$. The value b_0 characterizes the design of the cutting tool.

In view of the above-mentioned points, R_N is defined as the resultant of compressive stresses along the length $\frac{\delta}{2\sin\alpha/2}$ in the following form:

$$R_N = \frac{\delta(\delta - a)}{8b_0 \sin\alpha/2} E \tag{9}$$

Then for the vertical component of this force, we can write:

$$R_B = \frac{\delta(\delta + \alpha)}{8b_0} E \tag{10}$$

The presence of the force R_N causes a friction force acting along the facet in the opposite direction to the cutting direction. For a correct assessment of its influence on the cutting force, it is necessary to obtain a projection onto the vertical axis:

$$F_{ff}^{ch} = \frac{\delta(\delta + \alpha)}{8b_0} \cdot E \cdot f \cdot ctg\frac{\alpha}{2} \tag{11}$$

The friction force of the material on the side surface of the knife is conditioned with the action of compressive stresses equal in magnitude to the stress at the point of transition of the side facet to the side face, but on an area that differs in magnitude. The area of action of these stresses will depend only on the depth of immersion of the side edge of the knife h, since we consider a knife of unit length along the blade. The immersion depth can be calculated from geometric considerations as follows:

$$h = H - \frac{B}{2\cos\alpha/2}\left(1 - \sin\frac{\alpha}{2}\right) - \frac{\delta - a}{2tg\alpha/2}; \tag{12}$$

where: H is total depth of immersion of the knife; B is the width of the knife blade.

In this case, the friction force can be determined from the expression:

$$F_{ff}^{ch} = \frac{\delta}{2b_0} Ef\left[H - \frac{a}{2\cos\alpha/2}(1 - \sin\alpha/2) - \frac{\delta - a}{2tg\alpha/2}\right] \tag{13}$$

where f is the friction coefficient.

Therefore, we have obtained expressions for all terms included in equation (1). After substitution and transformations, we obtain the total cutting force under the conditions of the introduction of the cutting wedge:

$$R = \sigma_S\left(2 + \frac{\alpha}{2}\right)\left(\frac{\pi}{2} - \frac{\alpha}{2}\right)\frac{a}{2\cos\alpha/2} + \frac{\delta E}{8b_0}\{(\delta + a)(1 + fctg\alpha/2) + 4f \cdot$$

$$\cdot \left[B - \frac{a}{2\cos\alpha/2}\left(1 - \sin\frac{\alpha}{2}\right) - \frac{\delta - a}{2tg\alpha/2}\right]\} \tag{14}$$

Here it is considered that the knife in the steady state is completely immersed in the material and therefore $H = B$.

The analysis of relation (14) using a computer allowed establishing the following:

– the angle of sharpening the knife α slightly affects the change in the cutting force R. An increase in the angle α two times with other parameters constant leads to an increase in R by 5-10%.
– the thickness of the knife significantly affects the magnitude of the cutting force. When the thickness δ increases in two times, R increases by 90-100%.
– the modulus of elasticity of the material significantly affects the force R: an increase in the modulus E in two times causes an increase in the cutting force by 50% with other parameters constant:
– the friction coefficient f also significantly affects the value of the total cutting force. Therefore, for example, an increase in the friction coefficient in two times causes increase in cutting by 40-50% with the remaining parameters being equal.

During sliding cutting, the vector $\bar{R}$ does not coincide with the direction of movement of the cutting tool due to the tangential movement of the knife along the blade at a speed u_1. This circumstance makes it necessary to consider two components and the total cutting force, the direction of action of which coincides with the vectors $\bar{u}_1$ and $\bar{u}_2$, respectively. Thus, in the case of chopping cutting R_1=0; $\bar{R}_2 = \bar{R}$and for sliding cutting $\bar{R} = \bar{R}_1 + \bar{R}_2$.$R_1$=0.

From the physical model, it follows that the presence of force $\bar{R}_2$in the case of a sliding movement of the knife is not a sufficient condition for the formation of a new surface. The feed force in this case provides only the necessary preliminary deformation of the material, and its destruction occurs with the simultaneous action of macro- and microcutting mechanisms, the latter of which determines the presence of force R_1.

Therefore, when transiting from a mathematical model of chopping cutting in the form of formula (14) to a sliding cutting model, it is advisable to consider the features of the blade and their influence on R_0, leaving other components of R (compression and friction forces on the facets and side surfaces of the knife) unchanged. In accordance with the physical model, the field of tensile stresses σ_s in the volume of the sample is provided by the movement of the cutting wedge with the taper angle α in the direction u_2 (Figure 2). During the sliding cutting, the value of the speed u_2 is much less than with chopping cutting. Therefore, the three-component Maxwell-Thomson model can be used in this case to describe the deformation behavior of a material in a microvolume. An analytical description of the model leads to a differential equation:

$$\frac{\eta}{E_1}\frac{d\sigma}{dt} + \left(1 + \frac{E_2}{E_1}\right)\sigma = E_2\varepsilon + \eta\frac{d\varepsilon}{dt}, \tag{15}$$

Considering that

$$\frac{d\varepsilon}{dt} = \frac{d(\Delta l)}{ldt} = \frac{u}{l}$$

where u is material deformation rate, η is viscosity, l is initial length of the deformable body.

Then at a constant deformation rate, equation (15) is reduced to the form:

$$\frac{\eta \cdot u}{E_1 l}\cdot\frac{d\sigma}{d\varepsilon} + \left(1 + \frac{E_2}{E_1}\right)\sigma = \frac{\eta \cdot u}{l} + E_2\varepsilon$$

Its solution relative to the initial conditions $\varepsilon = 0$, $\sigma = 0$ and has the form:

$$\sigma = \frac{E_1}{E_1 + E_2}\left[E_2\varepsilon + \eta\frac{u}{l}\left(\frac{E_2}{E_1 + E_2}\right)\left(\exp\left(-\frac{E_1 + E_2}{\eta \cdot u}l\varepsilon\right) - 1\right)\right] \tag{16}$$

If we assume that the amount of tension $z_0$is constant in the steady cutting process and that the material is deformed in a triangular shape (*DBE*), then we can determine the amount of stress σ_z, acting on the section *DB*, as a function of the distance z along the length z_0.

We rewrite equation (16) for the voltage σ_z at unconditioned distance z:

$$\sigma_z = \frac{E_1}{E_1+E_2}\left[\varepsilon_z E_2 + \frac{\eta \cdot u_z}{l}\left(\frac{E_2}{E_1+E_2}\right)\left(\exp\left(-\frac{E_1+E_2}{\eta \cdot u_z} l\varepsilon_z\right)-1\right)\right] \quad (17)$$

where $\varepsilon_z = \frac{X_z}{l}$; u_z is deformation rate on axis z.

From Fig. 2 we can see that

$$\varepsilon_z = \frac{X}{l \cdot z_0} z \quad (18)$$

$$\text{and } u_z = u_2 \frac{z}{z_0} \quad (19)$$

For ultimate deformation of the material $\varepsilon_z = \varepsilon_p, z = z_0$. This gives grounds to rewrite equation (17) in the following form:

$$\sigma_z = \frac{E_1}{E_1+E_2}\left[\varepsilon_p E_2 + \frac{\eta \cdot u_2}{l}\left(\frac{E_2}{E_1+E_2}-1\right)\left(\exp\left(-\frac{E_1+E_2}{\eta \cdot u_2} X_0\right)-1\right)\right] \quad (20)$$

In this case, the value of σ_z is the limiting tensile stress, the excess of which leads to the rupture of material volumes, as a rule, out of cutting plane.

Comparison of the numerical values of σ_z, obtained by calculation by formula (20) using data on E_1, E_2, η_*, ε_p and experimental data of σ_z under tension showed their close correspondence for the studied range of change of these parameters. The value of the relative error (ε = 12-16%) is greater for more rigid half-finished products.

The force R_{02} acting per unit length of the blade and providing movement in the feed direction is defined as the sum of the elementary stresses acting on the section *DBE*:

$$R_{02} = 2\int_0^{z_0} \sigma_z dz \quad (21)$$

After integration, considering (18) and (19), we obtain:

$$R_{02} = \frac{E_1 z_0}{E_1+E_2}\left[\frac{X_0}{l}E_2 + \frac{\eta \cdot u_2}{l}\left(\frac{E_2}{E_1+E_2}-1\right)\left(exp\left(-\frac{E_1+E_2}{\eta \cdot u_2}X_0\right)-1\right)\right]; \quad (22)$$

where: X_0 is the ultimate strain of the material in the direction of the vector u_2; Z_0 is ultimate stretching of the material in the perpendicular direction.

In order to determine the relationship between z_0 and X_0, we imagine that the cross section of the surface of the deformed material is described by the equation of the parabola $X=a_0z^2$. Differentiating this equation, we obtain the expression for the angle coefficient

$$K = \frac{dX}{dz} = 2a_0 z \quad (23)$$

The facet of the knife is tangent to the parabola and its angle coefficient is equal to $tg(90^0 - \frac{\alpha}{2}) = ctg\frac{\alpha}{2}$.

$$\text{Then } K = 2a_0 z = ctg\frac{\alpha}{2} \text{ or } a_0 = ctg\frac{\alpha/2}{2z} \quad (24)$$

Based on the final deformation conditions ($X=X_0$; $z=z_0$) and substituting (24) into the original parabola equation, after numerical transformations, we obtain

$$z_0 = 2X_0 \cdot tg\frac{\alpha}{2} \quad (25)$$

Therefore, the feed force can be represented in its final form:

$$R_{02} = \frac{2E_1}{E_1+E_2} \cdot \frac{X_0}{l} tg\frac{\alpha}{2}\left[X_0 E_2 - \frac{E_1}{E_1+E_2}\eta \cdot u_2\left(e^{-\frac{E_1+E_2}{\eta \ u_2}}-1\right)\right] \quad (26)$$

The problem of determining the stress state in the zone of action of the blade microteeth differs from the usual problems of determining the stress concentration in that the geometrically linearized formulation of the boundary conditions and the physically linear theory of elasticity lead to infinite stress gradients at the end of a thin cut (at X=0, $\sigma_y \to \infty$) [16-19]. In this case, to determine the stresses and deformations, the methods of the theory of plasticity can be used, according to which, with an increase in the external load, the area where nonlinear effects begin to appear also increases.

As the calculations show, the interaction of the blade with the material corresponds to the plastic type of contact. Therefore, the deformation in the blade zone can be described using the critical crack opening (CCO) parameter, proposed by G. Wells, and used in nonlinear destruction mechanics [7]:

$$CCO = \frac{4}{\pi}\frac{K_{1c}^2}{E_{din}\sigma_S}; \tag{27}$$

where K_{1c} is stress intensity factor. Under plane stress condition:

$$K_{1c} = 1{,}4\sigma_z\sqrt{\pi l_0}, \tag{28}$$

where l_0 is crack length.

Mentioned data show that the contact in the blade zone must correspond to microcutting conditions, characterized by high values of the h/R_{max} ratio. In our case, the formation of the initial micronotch will be localized in the zone of action of the microteeth. At the same time, its depth does not exceed the working height $l_0 \leq h_p$, and the critical opening does not exceed the thickness of the blade: CCO$\leq a$. Compliance with these conditions depends on the depth of penetration of microroughnesses into the cut material.

From equations (27) and (28), considering the above limitations, it is possible to obtain an expression for the depth of the initial micronotch, which ensures the beginning of the formation of a new surface:

$$l_0 = \frac{aE_{din}\sigma_S}{8\sigma_z^2}$$

Where E_{din} is dynamic modulus of elasticity.

If we substitute the limiting value $\sigma_z = \sigma_S$ into this formula, then we can obtain the minimum value l_0 required to start the growth of a destruction crack:

$$l_0 = \frac{aE_{din}}{8\sigma_S} \tag{29}$$

Calculations by formula (29) show that when using sharp knives (*a*=4-10 mcm) for all cutting objects, the value of l_0 is approximately in the range of 5-30 mcm. With an increase of $a \geq 40$ mcm, the calculated value of l_0 begins to exceed the values of R_{max} for real blade microprofiles, which indirectly indicates the inefficiency of sliding cutting under these conditions. Efficient operation of the blade microteeth is possible under the following condition: $l_0 = h_p$.

The mutual influence of kinematic and geometrical parameters that determine the value of h_p can be considered when analyzing the contact interaction of microteeth with the material being cut.

Based on works on sclerometry [20, 21], the depth of penetration of a microtooth, modeled by a cone with an apex angle γ, can be represented as:

$$h_p = \sqrt{\frac{2P}{\sigma_S \pi tg\alpha/2\sqrt{(tg\gamma/2)^2+(sin\,\gamma/2 - f\,cos\,\gamma/2)^2}}} \tag{30}$$

The resistance to tangential movement of the blade microteeth depends on the value of the friction coefficients during cutting and the number of active microteeth. Therefore, considering (30), we can calculate the cutting force:

$$R_{01} = 0{,}5\sigma_S n_p \mu_1 \pi h_p^2 tg\frac{\gamma}{2}\sqrt{tg\frac{\gamma}{2} - f\,cos\frac{\gamma}{2}} \tag{31}$$

In the case of γ= 20° and f= 1, the force R_{01} can be expressed by a simplified formula:

$$R_{01} = 0{,}23\sigma_S n_p \mu_1 h_p^2, \tag{32}$$

where n_p is the number of active microteeth per unit length of the cutting edge.

Determining the “clean” cutting force $\bar{R}_0 = \bar{R}_{01} + \bar{R}_{02}$ after considering the value of other components of the force *R* (formula 1) allows evaluating the value of the efficiency of the knife during sliding cutting of food materials.

4 Conclusion

Mathematical model for the generalized criterion of the process, i.e. the total cutting force R and its components in the directions of movement of the knife R_1 and the supply of material R_2 has been developed. The "clean" cutting component is associated with the formation of a new surface, represents a significant part of the total force R and is determined by the cutting ability of the blade, which in turn depends on the parameters of the microgeometry. In order to improve the efficiency of knives, it is necessary to strive to reduce the coefficient of friction (tribopassivation) on the sides and facets and increase this parameter (triboactivation) on the blade.

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