Geodetic monitoring of high-rise structures according to satellite determinations

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Abstract. The results of satellite determinations, the recalculation of geocentric coordinates into topocentric ones, and the technology for creating a reference network and its use for transmitting coordinates to mounting horizons are shown. The article compares the coordinates obtained from satellite determinations without taking into account the deviation of the plumb line with the data on the proposed technology. The results obtained indicate the effectiveness of using the topocentric coordinate system in assessing and monitoring horizontal displacements of high-rise structures. GNSS technology eliminates the disadvantages of traditional methods, especially with the use of plumb lines and sequential sectional measurements, in which errors accumulate. The accuracy of the proposed technology practically does not depend on the height of structures.

1 Introduction

The development and widespread use of satellite measurement methods and the accumulated experience allow them to be used in rather specific, responsible work, such as the assessment of vertical deviations of high-rise buildings and structures. Regulatory document: SP 126.13330.2017 Geodetic work in construction method (CODE OF RULES), which regulates these works, allows the use of satellite definitions to assess horizontal displacements of buildings and structures. At the same time, the problem arises of how to orient the marks along the plumb lines.

Articles [1-3] describe the advantages of a topocentric coordinate system in construction. Comparisons with a flat rectangular projection of the Gauss-Kruger coordinates are given. It satisfies the requirement of choosing an independent coordinate system for structures and has a simple but close mathematical relationship with a rectangular geocentric coordinate system. This is very convenient when carrying out engineering work using GNSS technology. The main advantage of the topocentric coordinate system is that the z-axis can be directed along a plumb line, which is important when monitoring the verticality of buildings, especially high-rise ones.

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2 Materials and methods

Currently, GNSS technology is a common tool for creating geodetic terrestrial networks. After the transformation of spatial geocentric coordinates into a local topocentric coordinate system, the coordinates of the points are recalculated into the local coordinate system of the construction (geodetic grid base) used to monitor the verticality of the building. The main requirement for tall buildings is to ensure their verticality. According to [4] GOST 24846-2019 Soils. Methods for measuring deformations of the foundations of buildings and structures (INTERSTATE STANDARD) The roll of the foundation and buildings should be measured by leveling. The roll of the foundation is determined by the relative difference in the settlement of deformation marks located on its opposite edges. The roll of buildings, structures and individual structural elements is determined by the results of measuring the horizontal movements of the top and bottom of buildings. Limit errors in determining the horizontal displacements of the top of the building, depending on the height H of the observed building (structure) should not exceed the following values, mm: - 0.0001 N - for civil buildings and structures; - 0.0005N - for industrial buildings and structures, chimneys, blast furnaces, masts, towers, etc.; - 0.00001 N - for foundations for machines and units. Thus, when checking the verticality of high-rise buildings using GNSS technology, the use of a topocentric coordinate system seems to be the most appropriate, due to its orientation along a plumb line.

The transformation of the coordinates of GNSS measurement points from the geocentric coordinate system to the local topocentric coordinate system is performed according to the formula [2, 3, 5, 6, 11]:

\[
\begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix}
= 
\begin{bmatrix}
    -\sin B_0 \cos L_0 & -\sin B_0 \sin L_0 & \cos B_0 \\
    -\sin L_0 & \cos L_0 & 0 \\
    \cos B_0 \cos L_0 & \cos B_0 \sin L_0 & \sin B_0
\end{bmatrix}
\begin{bmatrix}
    X - (N_0 + H_0) \cos B_0 \cos L_0 \\
    Y - (N_0 + H_0) \cos B_0 \sin L_0 \\
    Z - [N_0(1 - e^2) + H_0] \sin B_0
\end{bmatrix}
\]

Where: \((x, y, z)\) are the coordinates of the point in the topocentric coordinate system; \((X, Y, Z)\) – spatial rectangular coordinates of the point in the geocentric coordinate system; \((B_0, L_0, H_0)\) – geodetic coordinates of the central point of the grid (or the origin of the topocentric coordinate system); \(N_0\) is the radius of curvature of the first vertical passing through the origin of the topocentric coordinate system.

The accuracy of determining topocentric coordinates depends only on the SCP measurement of geocentric coordinates [7]. Modern satellite receivers can provide position determination with SCP ~ 3 mm [8]. The results of the studies also show that in most of the territory of the Russian Federation the deviation of the plumb line is no more than 4" [7]. According to the results of studies [9], the deviation of the plumb lines in the territory of Vietnam can be 12.58" more). At the same time, the deviation of the sheer lines in the Hanoi region is about 10" and can be 18" in the mountains in the north of Vietnam relative to the WGS-84 ellipsoid.

According to [4], the marginal error in determining the slope (roll) of buildings and structures is determined by their type and the height of the object (Table 1).

<table>
<thead>
<tr>
<th>Types</th>
<th>Limit error</th>
<th>SCP linear</th>
<th>SCP corner</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Multistory building</td>
<td>0.0001H</td>
<td>0.0005H</td>
<td>10.3&quot;</td>
</tr>
<tr>
<td>Industrial chimneys</td>
<td>0.0005H</td>
<td>0.00025H</td>
<td>51.5&quot;</td>
</tr>
<tr>
<td>TV tower, radio antenna</td>
<td>0.0001H</td>
<td>0.00005H</td>
<td>10.3&quot;</td>
</tr>
</tbody>
</table>

\(H\) - building height
The results in column 4 (Table 1) show that: in most cases, the errors in determining the plumb line do not exceed the regulatory requirements for determining the slope. Therefore, it is possible to use the topocentric coordinate system to control verticality in the construction of high-rise structures.

### 3 The principle of determining the verticality of the structure according to GNSS technology

Depending on the mode of relative GNSS measurements, the receiver is installed at different levels in accordance with the construction schedule.

![Fig. 1. Principal diagram for determining the verticality of a structure according to GNSS technology.](image)

On Figure 1 point 1 - the beginning of the topocentric coordinate system. According to (1), we calculate the topocentric coordinates of the points as follows (in full form). The results of data processing make it possible to determine the increment of the coordinate vector, thereby determining the coordinates of the i-th point in the geocentric coordinate system from the expression (Figure 1):

\[
\begin{align*}
X_i &= X_0 + \Delta X_{oi}, \\
Y_i &= Y_0 + \Delta Y_{oi}, \\
Z_i &= Z_0 + \Delta Z_{oi}
\end{align*}
\]

(2)

Where: \(X_0, Y_0, Z_0\) are the coordinates of the point O in the geocentric coordinate system; \(X_i, Y_i, Z_i\) - coordinates of the i-th point in the geocentric system; \(\Delta X_{oi}, \Delta Y_{oi}, \Delta Z_{oi}\) is the difference in coordinates between points O and i-th in the geocentric coordinate system.

Topocentric coordinates are determined from the expression:
\[
x_i = -(X_i - X_1) \sin B_1 \cos L_1 - (Y_i - Y_1) \sin B_1 \sin L_1 + (Z_i - Z_1) \cos B_1
\]
\[
y_i = -(X_i - X_1) \sin L_1 + (Y_i - Y_1) \cos L_1
\]
\[
z_i = (X_i - X_1) \cos B_1 \cos L_1 + (Y_i - Y_1) \cos B_1 \sin L_1 + (Z_i - Z_1) \sin B_1
\]

(3)

Where: \(i = 2, 3, \ldots, n\).

According to (3), we have:

\[
x_i = -\Delta X_{0i} - \Delta X_{o1} \sin B_1 \cos L_1 - (\Delta Y_{0i} - \Delta Y_{o1}) \sin B_1 \sin L_1 + (\Delta Z_{0i} - \Delta Z_{o1}) \cos B_1
\]
\[
y_i = -\Delta X_{0i} \sin L_1 + (\Delta Y_{0i} - \Delta Y_{o1}) \cos L_1
\]
\[
z_i = \Delta X_{0i} \cos B_1 \cos L_1 + (\Delta Y_{0i} - \Delta Y_{o1}) \cos B_1 \sin L_1 + (\Delta Z_{0i} - \Delta Z_{o1}) \sin B_1
\]

(4)

It can be seen from formulas (2) - (4) that topocentric coordinates are calculated through the increments of the spatial coordinates of the baseline vectors \(\overrightarrow{O_i}\), regardless of the coordinates of the reference geodetic point - \(O\). However, if the position of the \(O\) point is unstable during coordinate determinations, this will lead to a change in the length of the line vectors. This point must be taken into account when choosing the location of point \(O\), as well as when placing control points so that they remain stable throughout the construction process. If points \(2, 3, \ldots, n\) are on the same line, they must satisfy the condition:

\[
x_i = y_i = 0.
\]

(5)

When point \(O\) is not too far from the job (<200 m), the linear method with the ellipsoid passing through point \(O\) and point 1 can be considered parallel to each other. Then you can choose the point \(O\) as the origin of the geographic coordinate system, and the formula (4) from the expression:

\[
x_i = -\Delta X_{0i} \cdot \sin B_0 \cos L_0 - \Delta Y_{0i} \cdot \sin B_0 \sin L_0 + \Delta Z_{0i} \cdot \cos B_0
\]
\[
y_i = -\Delta X_{0i} \cdot \sin L_0 + \Delta Y_{0i} \cdot \cos L_0
\]
\[
z_i = \Delta X_{0i} \cdot \cos B_0 \cos L_0 - \Delta Y_{0i} \cdot \cos B_0 \sin L_0 + \Delta Z_{0i} \cdot \sin B_0
\]

(6)

Where: \(i = 2, 3, \ldots, n\); \(B_0, L_0\) are the coordinates of the point \(O\) (known geodetic coordinates).

In this case, if points \(2, 3, \ldots, n\) are on the same route passing through point \(l\), then the following conditions must be met:

\[
x_2 = x_3 = \cdots = x_1; y_2 = y_3 = \cdots = y_1
\]

(7)

Formulas (5) or (7) allow you to check the verticality of the building in the topocentric coordinate system when used using satellite positioning technology.

4 Experimental studies

The technique for determining the degree of verticality of structures was tested on the project Tower A in the complex of high-rise buildings Keang Nam (Hanoi, Vietnam) during their construction on floors 40, 43 and 46 (measurement cycles: 13, 14 and 15). To carry out measurements using GNSS technology, a reference geodetic ground network was created. The network consisted of four starting points M1-M4 (Figure 2).
Where:

\[ i = 2, 3, \ldots, n. \]

According to (3), we have:

\[ \text{(4)} \]

It can be seen from formulas (2) - (4) that topocentric coordinates are calculated through the increments of the spatial coordinates of the baseline vectors, regardless of the coordinates of the reference geodetic point \(-O\). However, if the position of the point \(-O\) is unstable during coordinate determinations, this will lead to a change in the length of the line vectors. This point must be taken into account when choosing the location of point \(-O\), as well as when placing control points so that they are stable throughout the construction process. If points \(-2, 3, \ldots, n\) are on the same line, they must satisfy the condition:

\[ x_i = y_i = 0. \]

\[ \text{(5)} \]

When point \(-0\) is not too far from the job (<200 m), the linear method with the ellipsoid passing through point \(-O\) and point \(-1\) can be considered parallel to each other. Then you can choose the point \(-O\) as the origin of the geographic coordinate system, and the formula (4) from the expression:

\[ \text{(6)} \]

Where:

\[ i = 2, 3, \ldots, n; \quad B_0, L_0 \text{ are the coordinates of the point } O \text{ (known geodetic coordinates)}. \]

In this case, if points \(-2, 3, \ldots, n\) are on the same route passing through point \(-1\), then the following conditions must be met:

\[ \text{(7)} \]

Formulas (5) or (7) allow you to check the verticality of the building in the topocentric coordinate system when used using satellite positioning technology.

### 4 Experimental studies

The technique for determining the degree of verticality of structures was tested on the project Tower A in the complex of high-rise buildings Keang Nam (Hanoi, Vietnam) during their construction on floors 40, 43 and 46 (measurement cycles: 13, 14 and 15). To carry out measurements using GNSS technology, a reference geodetic ground network was created. The network consisted of four starting points M1-M4 (Figure 2).

![Fig. 2. Schematization of measurements: M1-M4 - starting points of the geodetic network, X3Y18, X3Y21, X5Y21 points at the intersection of the main axes of the building.](image)

Points X3Y18, X3Y21 and X5Y21 (controlled) are formed by the intersection of the main axes of the structure, the same name with the geodetic points. These points are projected onto the upper floors from the 1st floor by a vertical design laser device and are used as control points. In each measurement cycle, 4 Trimble R3 GNSS receivers were installed at the initial points of the reference network and at the control points (Figure 2, 3+1 - 3 on the control and one on the reference). After observations of each cycle, the geodetic network was processed in the VN-2000 coordinate system, the central meridian \(L_0 = 105^\circ0'00''\) (UTM-zone 3\(^{o}\), \(m_e = 0.9999\)). The result of spatial rectangular coordinates of control points for each cycle is presented in tab. 2.

#### Table 2. Spatial coordinates.

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>X (m)</th>
<th>Y (m)</th>
<th>Z (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - cycle</td>
<td>X3Y18</td>
<td>-1620192.8789</td>
<td>5731855.5127</td>
<td>2273345.8485</td>
</tr>
<tr>
<td>1 - cycle</td>
<td>X3Y21</td>
<td>-1620210.7024</td>
<td>5731844.8009</td>
<td>2273360.0831</td>
</tr>
<tr>
<td>1 - cycle</td>
<td>X5Y21</td>
<td>-1620224.5473</td>
<td>5731847.0963</td>
<td>2273344.4577</td>
</tr>
<tr>
<td>2 - cycle</td>
<td>X3Y18</td>
<td>-1620227.1281</td>
<td>5731976.7115</td>
<td>2273394.2322</td>
</tr>
<tr>
<td>2 - cycle</td>
<td>X3Y21</td>
<td>-1620244.9573</td>
<td>5731965.9369</td>
<td>2273408.4499</td>
</tr>
<tr>
<td>2 - cycle</td>
<td>X5Y21</td>
<td>-1620258.8129</td>
<td>5731968.2988</td>
<td>2273392.8402</td>
</tr>
<tr>
<td>3 - cycle</td>
<td>X3Y18</td>
<td>-1620229.8591</td>
<td>5731986.3842</td>
<td>2273398.0823</td>
</tr>
<tr>
<td>3 - cycle</td>
<td>X3Y21</td>
<td>-1620247.7014</td>
<td>5731975.6742</td>
<td>2273412.3157</td>
</tr>
<tr>
<td>3 - cycle</td>
<td>X5Y21</td>
<td>-1620261.5477</td>
<td>5731977.9884</td>
<td>2273396.6981</td>
</tr>
<tr>
<td>4 - cycle</td>
<td>X3Y18</td>
<td>-1620233.7196</td>
<td>5732000.0011</td>
<td>2273403.5334</td>
</tr>
<tr>
<td>4 - cycle</td>
<td>X3Y21</td>
<td>-1620251.5474</td>
<td>5731989.2839</td>
<td>2273417.7477</td>
</tr>
<tr>
<td>4 - cycle</td>
<td>X5Y21</td>
<td>-1620265.4083</td>
<td>5731991.5982</td>
<td>2273402.1391</td>
</tr>
</tbody>
</table>

To determine the slope of the building in each cycle, a topocentric coordinate system was created with the origin at the point X3Y18.
Table 3. Topocentric coordinates.

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>X (m)</th>
<th>Y (m)</th>
<th>Z (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - cycle</td>
<td>X3Y18</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>X3Y21</td>
<td>15.2458</td>
<td>20.0652</td>
<td>0.0090</td>
</tr>
<tr>
<td></td>
<td>X5Y21</td>
<td>-1.4830</td>
<td>32.7637</td>
<td>-0.0181</td>
</tr>
<tr>
<td>2 - cycle</td>
<td>X3Y18</td>
<td>-0.0080</td>
<td>-0.0091</td>
<td>134.9190</td>
</tr>
<tr>
<td></td>
<td>X3Y21</td>
<td>15.2431</td>
<td>20.0787</td>
<td>134.8670</td>
</tr>
<tr>
<td></td>
<td>X5Y21</td>
<td>-1.4950</td>
<td>32.7694</td>
<td>134.9079</td>
</tr>
<tr>
<td>3 - cycle</td>
<td>X3Y18</td>
<td>-0.0190</td>
<td>-0.0121</td>
<td>145.6820</td>
</tr>
<tr>
<td></td>
<td>X3Y21</td>
<td>15.2231</td>
<td>20.0707</td>
<td>145.6970</td>
</tr>
<tr>
<td></td>
<td>X5Y21</td>
<td>-1.5049</td>
<td>32.7694</td>
<td>145.6899</td>
</tr>
<tr>
<td>4 - cycle</td>
<td>X3Y18</td>
<td>-0.0071</td>
<td>-0.0010</td>
<td>160.8490</td>
</tr>
<tr>
<td></td>
<td>X3Y21</td>
<td>15.2212</td>
<td>20.0697</td>
<td>160.8470</td>
</tr>
<tr>
<td></td>
<td>X5Y21</td>
<td>-1.4999</td>
<td>32.7785</td>
<td>160.8469</td>
</tr>
</tbody>
</table>

Condition (7) was used to determine the verticality of the structure (Table 4).

Table 4. Deviations of the structure from the vertical at control points.

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Coordinate deviation</th>
<th>Total deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Δx (m)</td>
<td>Δy (m)</td>
</tr>
<tr>
<td>2 - cycle</td>
<td>X3Y18</td>
<td>0.008</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>X3Y21</td>
<td>0.003</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>X5Y21</td>
<td>0.012</td>
<td>0.006</td>
</tr>
<tr>
<td>3 - cycle</td>
<td>X3Y18</td>
<td>0.019</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>X3Y21</td>
<td>0.023</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>X5Y21</td>
<td>0.022</td>
<td>0.002</td>
</tr>
<tr>
<td>4 - cycle</td>
<td>X3Y18</td>
<td>0.007</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>X3Y21</td>
<td>0.025</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>X5Y21</td>
<td>0.017</td>
<td>0.015</td>
</tr>
</tbody>
</table>

The above results are included in the report on the monitoring of horizontal displacements of the high-rise tower in Keang Nam [10].

5 Conclusion

Based on the results of theoretical studies and field measurements presented in the article, the following conclusions can be drawn: the topocentric coordinate system is very effective in controlling the verticality of high-rise buildings and structures. It is convenient to use it in conjunction with GNSS technology, since the data obtained online through recalculation make it possible to obtain topocentric coordinates. This technology eliminates the disadvantages of traditional methods, especially with the use of plumb lines and sequential sectional measurements, in which errors accumulate. The accuracy of the proposed technology practically does not depend on the height of structures.

In addition, the application of GNSS positioning technology is fully associated with computer information processing technologies. In this regard, the transfer of coordinates from the geocentric system to any other provides for their implementation in an automated mode. In this case, it is possible to optimize not only the algorithm for converting coordinates into a flat projection oriented along a plumb line, but also the choice of projection and area size, which, for solving a specific engineering problem, will distort the spatial coordinates to the least degree. In this case, it is highly advisable to have the recalculation of coordinates to the normative projection in order to maintain the correctness.
of the solution of the problem, which is important from both scientific and practical points of view.

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