A study of problems of the multilevel probabilistic modeling for hydraulic conditions in pipeline systems

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Abstract. The article is devoted to of the multilevel probabilistic modeling problem for hydraulic conditions of pipeline systems (PSL) having a hybrid topology. The essence of the multilevel approach is to decompose of the design topology into a multi-loop part and tree-shaped branches by applying calculation techniques appropriate for each of the parts. A numerical study of the proposed approach was carried out on a example, which showed that to calculate all parameters except for the nodal pressure dispersions, it is sufficient to use the methods previously developed by the authors. To calculate the nodal pressure dispersions, it is necessary to take into account the correlation of the pressure field of the multi-loop and tree-like levels of the design scheme.

Keywords: probabilistic modeling; pipeline systems; multilevel approach; hydraulic conditions.

1. Introduction

The basic problems of analysis and validation of operating conditions of PLS of various types and purposes are load flow problems Traditionally, to solve them, deterministic mathematical models and methods are employed, which, however, do not allow one to assess the degree of uncertainty of PLS operating conditions as formed as a result of many random factors (consumer loads, pressures at flow sources, etc.). The above makes it relevant to have probabilistic statements of the problems of load flow analysis. The latter consist in obtaining the results of calculations in a form that lends itself to a probabilistic interpretation based on utilizing the information about the so-called boundary conditions (BC) in their probabilistic form.

2. Literature review and problem statement

The research published abroad deals with the stochasticity when either accounting for damage from water not served [1] or factoring in the uncertainty of input data used to solve reliability-related problems [2-5]. Stochastic statements of the problem of analysis of operating conditions of pipeline systems were considered in Ref. [6-8], mainly to estimate the uncertainty range of operation of reservoirs or pumping stations, with such estimates obtained for the most part based on the application of the Monte Carlo method. Ref. [9] investigated the problem of applying probability metrics (expected values and variances) to assess the performance of water supply systems but without specifying the finite methods to be used for its solution.

The Melentiev Energy Systems Institute SB RAS proposed an approach [10-12] to obtain analytical probabilistic models of steady-state hydraulic conditions, which can be applied, for example, to water supply and heating systems operating under stochastic consumption. The general (GM) and matrix-based methods presented in the above studies are known to be versatile and have acceptable accuracy and low computational cost if compared to the classical Monte Carlo method. For the analysis of tree-shaped configurations, the authors have developed the topological method (TM) [13], which allows us to obtain the probabilistic parameters of hydraulic conditions relying only on finite analytical formulas, without time-consuming matrix operations involved. The above methods allow calculating all probabilistic characteristics of hydraulic conditions established in PLSs of any configuration and any composition of BC specified in a probabilistic form (nodal pressures, nodal flow rates, friction loss at branches).

Under real-world settings, there is only a limited number of PLSs (e.g., systems of water supply, gas supply, etc.), consisting of multi-loop or tree-shaped configurations only, most often it is a combination thereof, and such where tree-shaped segments outnumber multi-loop ones. Therefore, we propose to apply a multilevel (hierarchical) approach to the probabilistic modeling of such systems. The essence of the approach is to decompose an equivalent network layout into its multi-loop part and tree-shaped branches. The connection node of such branches becomes an aggregated consumer for the multi-loop part (top layer) and an aggregated source for the tree-shaped segments (bottom layer). The GM is used for the top layer, whereas the less computationally-demanding TM is used for the bottom one. In addition to reducing the computational cost of PLS analysis as a whole, it enables parallel calculations of multiple tree-shaped segments.

3. Object, aim, and purpose of the study

The object of the study is PLSs of mixed configuration, consisting of the structures of the top (multi-loop part) and bottom (tree-shaped part) layers.

The aim is to investigate the possibility of probabilistic modeling of hydraulic conditions of PLSs based on the multilevel (hierarchical) approach, consisting in a preliminary decomposition of equivalent network layouts and subsequent alignment of calculation results for individual segments.

The paper considers a particular but most common problem statement, in which the probabilistic nature of hydraulic conditions is determined by the stochasticity of loads (consumption of the operating fluid at the nodes). Givens: 1) topology of the equivalent network layout in the form of the $(m\text{-}1)\times n$-matrix of incidence relations of nodes and branches A (m, n – the number of nodes and branches); 2) hydraulic characteristics of all its branches in the form of the n-dimensional vector-valued function $f(x)$ for dependencies of pressure drops (vector y) on flow rates (vector x); 3) pressure at one of the nodes of the top layer $P_m = \text{const}$; 4) BC in the form of the $(m\text{-}1)$- dimensional vector of mathematical expectation (ME) $\bar{Q}$ and diagonal covariance matrix (CM) C_Q .

The goal is to determine the MEs and CM of parameters of hydraulic conditions: i.e., flow rates (x) and pressure drops (y) at branches; nodal pressures at m-1 nodes (vector P) and nodal flow rate at the balancing node m (Q_m).

4. Models and methods

The general method presupposes two stages: 1) load flow analysis by conventional methods to obtain the EVs of parameters of hydraulic conditions [14] subject to the given MEs of BC; 2) calculation of CM to calculate the variances of parameters of hydraulic conditions, the derivation of which was presented in [13]. Here we give only the finite CM formulas for the unknown parameters of hydraulic conditions P, x, y:

$$C_P \approx M^{-1}C_Q M^{-1}, \quad C_x \approx (f_x')^{-1} A^{\mathrm{T}} C_P A (f_x')^{-1}, \quad C_y \approx f_x' C_x f_x'. \tag{1}$$

where $M = A(f_x')^{-1}A^{\mathrm{T}}$ is the symmetric non-degenerate Maxwell matrix, T – is the transposition sign, f_x' is the diagonal matrix of partial derivatives at the ME point $\bar{x}$ with elements $f_{x,i}' = \partial f_i(x_i)/\partial x_i$, $i = \overline{1,n}$ on the main diagonal.

The topological method was detailed in [13]. It boils down to sequentially visiting the nodes of the equivalent network layout by tiers (starting from the nodes for which BCs are specified), applying at each step relatively simple algebraic relations to determine the unknown MEs, variances, and covariances of parameters of hydraulic conditions. For the use of TM, the schemes are numbered according to the following principle: 1) the numbering goes sequentially from the last tier to the source; 2) the node with the initial distribution of links in the path from the source is assigned the first number; 3) a sensitive index with the index of its end node.

Next, we state the finite formulas for the MEs and variances of the unknown parameters of hydraulic conditions:

1) flow rates at branches

$$\bar{x}_i = \sum_{t \in I_i^-} \bar{x}_t + \bar{Q}_i, \quad \sigma_{x,i}^2 = \sigma_{Q,i}^2 + \sum_{t \in I_i^-} \sigma_{x,t}^2, \quad i = \overline{1,n}\,, \tag{2}$$

where I_i^- is the set of branches originating from node i, and this takes into account the fact that by the time of the calculation of $\bar{x}_i$ all $\bar{x}_t,\, t \in I_i^-$ are already known. For "pendant" branches $I_i^- = \varnothing$, therefore $\sigma_{x,i}^2 = \sigma_{Q,i}^2$;

2) branch pressure drops

$$\bar{y}_i \approx f_i(\bar{x}_i), \quad \sigma_{y,i}^2 = \left(f_{x,i}'\right)^2 \sigma_{x,i}^2, \; i = \overline{1,n}\,; \tag{3}$$

3) nodal flow rate at the root of the tree

$$\bar{Q}_m = \sum_{i \in I_m} \bar{x}_i, \; \sigma_{Q,m}^2 = \sum_{j=1}^{m-1} \sigma_{Q,j}^2\,; \tag{4}$$

4) nodal pressures

$$\bar{P}_j = \bar{P}_{\mathrm{H}(j)} - \bar{y}_j, \; \sigma_{P,j}^2 = \sigma_{P,\mathrm{H}(j)}^2 + \sigma_{y,j}^2 + 2 f_{x,j}' \sigma_{x,j}^2 f_{\Sigma,\mathrm{H}(j)}', \; j = \overline{m-1,1}\,, \tag{5}$$

where $H(j)$ is the initial node of branch i=j; $f_{\Sigma,j}' = \sum_{l \in R_j} f_{x,l}'$; R_j is a set of branch indices on the path from the tree root (m) to node j (for nodes of the first tier $H(j)$=m, $R_m = \varnothing$, and $f_{\Sigma,\mathrm{H}(j)} = 0$).

Thus, the TM is reduced to two passes over the tree:

1) the backward pass ($j = \overline{1,n}$) to determine $(\bar{x}_i,\ \sigma_{x,i}^2)$, $(\bar{y}_i, \sigma_{y,i}^2)$, and $(\bar{Q}_m,\ \sigma_{Q,m}^2)$ using relations (2) - (4);

2) the forward pass ($j = \overline{n,1}$) to calculate $(\bar{P}_j,\ \sigma_{P,j}^2)$ by recurrence relations (5).

The multilevel approach to probabilistic modeling of hydraulic conditions (MA) in the considered problem statement is based on the fact that random flow rate disturbances propagate from bottom to top (from consumption nodes), whereas pressure disturbances proceed from top to bottom (from pressure sources). Accordingly, the proposed approach consists of the following basic steps.

1. The backward pass: calculation of statistical characteristics of loads of aggregated consumers of the top layer as per relations (4) (as well as flow rates and pressure drops at branches as based on (2), (3)) for each tree-shaped segment of the bottom layer.
2. Probabilistic load flow analysis of the top layer: calculation of MEs and CMs of all parameters of hydraulic conditions of the top-layer layout as based on the use of conventional methods of load flow analysis at the ME point of BC and relations (1).
3. The forward pass: calculation of statistical characteristics of node pressures for each tree-shaped segment of the bottom layer as per relations (5) and statistical characteristics of pressures at decomposition points calculated at the previous stage.

5. Numerical examples

Below we present several numerical examples that illustrate the application of the MA to the probabilistic load flow analysis juxtaposed with the results obtained with the GM as a reference. All the examples consider passive networks. Element characteristics and derivatives have the form $f_i(x_i) = s_i\,|\,x_i\,|\,x_i$, $f'_{x,i} = 2s_i\,|\,x_i\,|$, $i = \overline{1,n}$ where s_i is the friction loss of the i-th branch. Figure 1 shows a network, which include 8 nodes and 8 branches, nodes 3 and 7 are decomposition nodes. Table 1 shows the input data and the calculation results for the nodes. Table 2 shows the input data on branch friction losses and the results of calculations of flow rates and losses pressures at the branches.

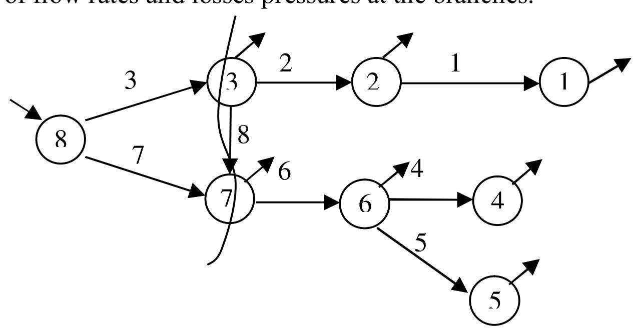

Fig.1. An 8-node nodal layout of a PSL. The wavy line indicates the network decomposition nodes.

Table 1. Input data (MEs and variances) and calculation results (ME and variances) of nodal pressures and nodal flow, for design network, obtained by GM and MA.

Node, j	Input data				Calculation results					
					GM			MA		
	$\bar{P}_j$	$\sigma^2_{P,j}$	$\bar{Q}_j$	$\sigma^2_{Q,j}$	$\bar{P}_j$	$\sigma^2_{P,j}$	$\bar{Q}_j$	$\bar{P}_j$	$\sigma^2_{P,j}$	$\bar{Q}_j$
1			5.7	0.86	23.99	0.43		23.99	**0.56**	
2			13.6	0.87	24.03	0.42		24.03	**0.25**	
3			3.6	0.53	27.42	0.07		27.42	0.07	
4			7.6	0.38	22.60	0.35		22.60	**0.24**	
5			6.7	0.68	22.94	0.34		22.94	**0.24**	
6			7.7	0.56	23.01	0.33		23.01	**0.24**	
7			3.7	0.36	27.40	0.07		27.40	0.07	
8	31	0.1					48.6			48.6

Table 2. Friction losses and calculation results for flow rates at the branches (ME and variances) and pressure drop variances at the branches of design network, obtained by GM and MA.

Branch, i	Friction loss	$\bar{x}_i$ as per GM	$\bar{x}_i$ as per MA	$\sigma^2_{x,i}$ as per GM	$\sigma^2_{x,i}$ as per MA	$\sigma^2_{y,i}$ as per GM	$\sigma^2_{y,i}$ as per MA
1	0,00150	5,7	5,7	0,74	0,74	0,0002	0,0002
2	0,00908	19,3	19,3	1,50	1,50	0,1840	0,1840
3	0,00608	24,3	24,3	0,72	0,72	0,0630	0,0630
4	0,00708	7,6	7,6	0,14	0,14	0,0016	0,0016
5	0,00150	6,7	6,7	0,46	0,46	0,0002	0,0002
6	0,00908	22,0	22,0	0,92	0,92	0,1470	0,1470
7	0,00608	24,3	24,3	0,69	0,69	0,0610	0,0610
8	0,00708	1,37	1,37	0,66	0,66	0,0002	0,0002

The calculation results indicate that the MEs of all parameters of hydraulic conditions as obtained by different methods coincide. The variance values for branch flow rates and pressure drops also coincide. Differences arise in the values of the variances of nodal pressures of the tree-shaped network. In this particular case, it is nodes 1, 2, 4 to 6 (the values that do not match are highlighted in bold). Next, we give the index expressions for the variance of pressures at nodes 1. $\sigma^2_{P,1}(MA) = \sigma^2_{P2} + \sigma^2_{y,1} + 2f'_{x,1}\sigma^2_{x,1}\left(f'_{x,2}\right)$

$$\sigma^2_{P,1}(GM) = \sigma^2_{P2} + \sigma^2_{y,1} + 2f'_{x,1}\sigma^2_{x,1}\left(f'_{x,3} + f'_{x,2}\right)\left(\frac{f'_{x,7} + f'_{x,8}}{f'_{x,3} + f'_{x,7} + f'_{x,8}}\right).$$

The analysis of these expressions allows us to distinguish the term responsible for taking into account the correlations of the pressure fields of different layers, which we denote as ϑ_m. The formula for calculating the variances of nodal pressures of the tree-shaped segment will be

$$\sigma^2_{P,j} = \sigma^2_{P,\mathrm{H}j} + \sigma^2_{y,j} + 2f'_{x,j}\sigma^2_{x,j}f'_{\Sigma,\mathrm{H}(j)}\vartheta_m, \quad j = \overline{m-1,1}, \tag{6}$$

where $f'_{\Sigma,j} = \sum_{l \in R_j} f'_{x,l}$; R_j is the set of branch indices on the path from the initial node of the top-layer branch ($H(m)$) that reaches the root of the tree-shaped segment (m) to the node j of this segment; in this particular case $\vartheta_m = \left(\sum_{t=1}^{nk} f'_{x,t} - f'_{\mathrm{H}(m)}\right) \Big/ \sum_{t=1}^{nk} f_{x,t}$, nk is the number of branches of the top-layer layout.

6. Discussion of results

The following conclusions can be drawn from the above examples:

1) the calculation accuracy of MEs of all parameters of hydraulic conditions, as well as the variances of flow rates and losses on the branches by the MA is acceptable;

2) it is necessary to take into account the correlation of the top-layer load flow and the pressure field for the bottom-layer layouts;

3) for such consideration, instead of the TM formula (5), ratio (6) was proposed for decomposition points with an indeterminate coefficient ϑ_m that depends in a complex way on the values of derivatives for the branches of the top-layer layout.

Algebraic computations of the CM of nodal pressures have shown that the inverse Maxwell matrix $\left(M^{-1}\right)$, which is obtained incidentally when calculating the CM of the top-layer ME, contains information ϑ_m about each node of the top layer. Table 4 shows the results of calculations of the variances of nodal pressures of bottom-layer nodes obtained by the GM and MA as per the formula (6).

Table 3. The results of calculations of bottom-layer nodal pressures obtained by the GM and MA, given $\vartheta_{K,j}$, for the network layouts shown in Figs. 1.

Node, *j*		1	2	4	5	6
Methods	GM	0,43	0,42	0,35	0,34	0,33
	MA	0,43	0,42	0,35	0,34	0,33

7. Conclusion

We have attempted - for the first time in the literature - a formulation and formalization of a solution to the problem of multilevel probabilistic modeling of hydraulic conditions of pipeline systems. We have proposed techniques to decompose an equivalent layout and arrange the sequence of multilevel calculations. Numerical and analytical studies of the proposed approach that was tested on simplified cases have shown that to calculate the MEs of all parameters of hydraulic conditions, as well as the variances of nodal flow rates, flow rates, and pressure drops at branches, it is sufficient to use the methods previously developed by the authors (GM and TM). To calculate the variances of nodal pressures, it is necessary to take into account the correlation of the pressure field of the top and bottom layers of the equivalent layout. We have identified what causes the correlation and outlined possible directions on how to address it. Providing an analytical foundation for finite rules and dependencies to be used for factoring it in (as applied to the general case of an arbitrary topology of the top-layer layout) should be the subject of further research.

The research was carried out under State Assignment Project (no. FWEU-2021-0002) of the Fundamental Research Program of Russian Federation 2021-2030

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