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Mathematical Models and Methods of the Analysis and Optimal Synthesis of the Developing Pipeline and Hydraulic Systems 2022

Actual directions of digitalization of pipeline systems and methods of analysis of their properties as cyber-physical objects

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Abstract. The characteristics of the goals and objectives of digitalization of pipeline systems (PS) are given, during which they acquire new properties of cyber-physical objects. A structure of tasks of general importance for PS of various types and purposes (heat, water, oil, gas supply, etc.) is proposed. The characteristic of probabilistic state models is given. On this basis, a system of indicators for analyzing the controllability, identifiability and observability of PS is proposed, and their relationship is disclosed. For the first time, analytical dependences of the parametric identifiability of PS on the composition of measurements have been obtained.

1 Introduction

Currently, in most economically developed countries, the processes of digitalization of the economy and its industries, including energy, are developing [1, etc.]. The ultimate goal of such processes is the transition to fully automated production with real-time intelligent control in interaction with related systems and the external environment, with the prospect of merging into a global industrial network of things and services.

The goals and objectives of intellectualization were formulated in the early 2000s as part of the concept of "Smart grid" (smart networks), adopted as the main direction of energy development in the economically developed countries of the West [2, 3]. This direction is being actively developed in the Russian electric power industry [4]. The vast territory of Russia, most of which is in harsh climatic conditions, has led to the presence here of unique in scale and importance for the energy, industry and social sphere of the PS, and the problems of energy and resource conservation here act as the most important components of energy policy [5, 6, etc.]. Therefore, the creation of intelligent PSs based on digitalization should be considered as a strategic direction for their innovative transformation.

At the same time, the scientific and methodological basis for the intellectualization of PS, which can be the basis for practical application, is currently absent. In this article, based on the analysis of world experience and trends in the digitalization of energy systems, the main goals and scientific and methodological tasks that arise on the path of transforming

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traditional PS into intelligent cyber-physical systems are formulated. The term "cyber-physical systems" here is intended to reflect the following aspects: 1) such systems acquire cybernetic properties as a result of the synthesis of basic equipment, information and computing resources; 2) they are supposed to be considered in the space of real state parameters (pressures, flow rates, temperatures, etc.); 3) instead of formal models linking "input" and "output", models based on the physical laws of the flow of the working medium (water, oil, gas, etc.) for individual elements and for the system as a whole are used.

2 Goals and objectives of PS digitalization

An analysis of the literature, program documents from different countries, as well as practical experience in the intellectualization of energy systems, allows us to formulate the following, fairly general, interpretation and systematization of the key goals, means of digitalization, and the differences between intelligent PSs and traditional ones [7–10].

The goals of PS intellectualization coincide with the traditional ones - increasing the efficiency, reliability, quality and environmental friendliness of their operation. The main prospects are associated with new opportunities to meet these requirements based on the joint application of: 1) modern information, telecommunication and computer technologies, methods of mathematical modeling and optimization in real circuits of analysis and decision-making; 2) energy efficient main and maneuverable control equipment; 3) market mechanisms for coordinating interests, supply and demand.

The synthesis of these components opens up new possibilities for effectively harmonizing the interests, requirements and capabilities of all parties involved in the processes of production, transport and consumption of the target product. The main properties acquired by PS as a result of such a transformation are: 1) the presence of a common information (digital) space [11], as the main system-forming factor responsible for the observability of the processes of production, distribution and consumption for all participants in these processes; 2) a high level of controllability as the main way to harmonize the requirements of consumers and the capabilities of manufacturers (suppliers); 3) dynamic pricing that encourages consumers to save energy.

Achieving the above goals based on the use of new technologies will require a revision of the existing practice of design, operation and dispatch control of PS to solve the following urgent problems: 1) ensuring the operational efficiency of designed and reconstructed PS with a focus on new concepts and control technologies; 2) development and application of regulations and standards for controllability and observability; 3) synthesis of control and information-measuring systems; 4) full-scale transition to the practice of planning repairs based on the actual state of the equipment; 5) wide application of methods of active identification, technical diagnostics, analysis and damage prediction; 6) optimization of plans for repair and restoration work, taking into account the actual state of the equipment, statistics of damage and damage to consumers; 7) transition to a new concept of control of PS modes as dynamic and stochastic objects operating under conditions of uncertainty of the internal state and external influences [8, 9]; 8) optimal planning and management in the main (operational), post-accident and "repair" modes; 9) continuous monitoring of the actual state of the PS; 10) overcoming departmental or corporate disunity of technologically related parts of the PS and related subsystems.

The solution of these applied problems raises the following new scientific and methodological tasks from life: 1) analysis - a quantitative assessment of the controllability, identifiability, efficiency and other properties of the PS, taking into account the dynamics and stochastics of external influences, the uncertainty factors of the internal state of the PS, the characteristics of control and monitoring systems; 2) synthesis - joint optimization of the structure and parameters of both the PS themselves and control and measurement systems; 3) management - development of concepts and methods of management, decision-making systems, tuning and adaptation of control loops; 4) identification - tracking modes, parameters, their prediction, diagnostics of the state of both the PS and measurement and control systems.

3 Models of hydraulic circuits as stochastic controlled objects

In real conditions, the change in the operating modes of the PS is a random process under the influence of three main factors: 1) regular environmental influences (vector G); 2) purposeful control (vector u); 3) sudden changes in the internal state of the PS of a random nature. Analysis of the latter type of (relatively rare) disturbances (equipment failures, accidents, etc.) is the subject of reliability theory and is not considered here.

The parameters that simultaneously belong to the hydraulic circuits (HC) and the environment will be called boundary conditions. In this case, it is possible to carry out the decomposition of the mode parameters $R=\{Y,G\}$. Uncertainty factors are also related to the approximation of information about equipment parameters (vector α). Therefore, the equations of controlled flow distribution in can be written as $Y=\varphi(G,u,\alpha)$, where φ is an implicit vector function that uniquely determines the values of the dependent mode parameters Y for given values of the vectors G, u and α.

Assume that the control vector is deterministic, and random vectors G,α obey the normal distribution law with parameters obtained as a result of solving prediction or identification problems. That is $G \sim N_{mg}(\hat{G},C_G)$ and $\alpha \sim N_{na}(\hat{\alpha},C_\alpha)$, where $\hat{G}=E(G)$ is the mathematical expectation of G; $\hat{\alpha} \approx E(\alpha)$ is the vector of estimates; $C_G=E(\xi_G\xi_G^T)$, $C_\alpha=E(\xi_\alpha\xi_\alpha^T)$ are covariance matrices; $\xi_G=(G-\hat{G})$ is the vector of random deviations; $\xi_\alpha=(\alpha-\hat{\alpha})$ is the vector of estimation errors; $mg=\dim(G)$; $na=\dim(\alpha)$. Assuming the identification results α to be uncorrelated with the boundary conditions, the probabilistic flow distribution model can be represented as:

$$U(Y,G,u,\alpha)=0 \quad \Leftrightarrow \; Y=\varphi(G,u,\alpha), \tag{1}$$

$$G \sim N_{mg}(\hat{G},C_G), \quad \alpha \sim N_{na}(\hat{\alpha},C_\alpha), \quad E[(G-\hat{G})(\alpha-\hat{\alpha})^T]=0\,. \tag{2}$$

In most practical cases, the nonlinear distortion of the distribution $\varphi(G,u,\alpha)$ can be neglected, which makes it possible to approximate it as $Y \sim N_{mY}(\hat{Y},C_Y)$. In this case, the problem of probabilistic modeling is reduced to determining the parameters of this distribution. Applying the linearization method, we obtain:

$$\hat{Y} \approx \varphi(\hat{G},\hat{a})\,. \tag{3}$$

$$C_Y=E(\xi_Y\xi_Y^T) \approx \frac{\partial\varphi}{\partial G}C_G\left(\frac{\partial\varphi}{\partial G}\right)^T+\frac{\partial\varphi}{\partial\alpha}C_\alpha\left(\frac{\partial\varphi}{\partial\alpha}\right)^T. \tag{4}$$

The matrix C_Y reflects the degree of uncertainty Y, and the last term is the contribution of identifiability.

The probabilistic analysis of the consequences of control u is reduced to solving the traditional problem of flow distribution (3) according to the initial data $\{\hat{G},\hat{a}\}$ in combination with an additional calculation procedure C_Y for given matrices C_G, C_α and matrices of derivatives $\partial\varphi/\partial G$, $\partial\varphi/\partial\alpha$, calculated at a point $\{\hat{G},\hat{a}\}$ [12]. The probabilistic model (1), (2), in turn, forms the basis for the analysis of the cyber-physical properties of the PS, the most important of which are controllability and identifiability.

These properties are complex, which follows from the multipurpose nature of real control (ensuring the feasibility, reliability, efficiency of modes, etc.) and identification (structural identification, identification of equipment parameters, identification of modes, technical diagnostics, etc.).

4 Indicators and the task of analyzing the controllability of hydraulic circuits

In [13], a system of primary (integral over the system, but local in time) probabilistic indicators is proposed to quantify the degree of achievement of the main control goals: 1) membership of the feasible region 2) mode reliability 3) efficiency. The analysis of controllability according to the proposed indicators is reduced to the search for controls u that give them extreme values under conditions(1), (2) and

$$\underline{u} \le u \le \overline{u} \ . \tag{5}$$

Accordingly, we have a set of integral criteria for analyzing the controllability of the PS:
1) maximum probability of belonging to the feasible region

$$p_F^* = \max_u p(\underline{Y} \le Y(u) \le \overline{Y}) \,, \tag{6}$$

where p is the probability; $\underline{Y}$, $\overline{Y}$ are the boundaries of the feasible region of the random vector Y;
2) minimum probability of violation of the feasible region

$$p_V^* = \min_u \left(\max_i \left(\max\left(p_i^+(u), p_i^-(u) \right) \right), \, i = \overline{1, ny} \right), \tag{7}$$

where p_i^+, p_i^- are the probabilities of violation of the upper and lower boundaries of a single mode parameter Y_i, $ny = \dim(Y)$;
3) the maximum mathematical expectation of the useful use of the input power flow

$$\hat{\eta}^* = \max_u \ \eta(Y(u), G, \alpha), \ \ \underline{Y} \le Y \le \overline{Y}, \tag{8}$$

where η is the efficiency factor of the supplied flow power.

The obtained criteria (6) – (8) can be used as the basis for the analysis of potential effects from the introduction of various components of the PS intellectualization, as well as the development of controllability standards. In the presence of such standards, the considered multi-criteria tasks of controllability analysis can be reduced to single-criterion tasks by converting some of the indicators into restrictions. For example, if we take efficiency as the main criterion, then we get problem (8) taking into account conditions (1), (2), (5) and the requirements for mode reliability:

$$p_i^+(u) \ge \delta_i^+, \quad p_i^-(u) \ge \delta_i^-, \quad i = \overline{1, ny} \,, \tag{9}$$

where δ_i^+, δ_i^- are the given cutoff for the probability of violating the boundaries of the feasible interval for each mode parameter.

On this basis, various estimates of controllability for the period T can be built, for example: average probability for the period $\frac{1}{T}\int_0^T \overline{p}(t)dt$; the time during which the condition $\overline{p} > \delta$ is satisfied, where δ is the given cutoff of feasible probability; average time of continuous fulfillment of this condition, etc.

Note that the introduced probabilistic models and indicators make it possible to directly assess the contribution of identifiability to controllability. The uncertainty of the indicator η is similar to the uncertainty factors of the mode parameters (4). Having the parameters of a one-dimensional distribution $N(\hat{\eta},\sigma_{\eta}^{2})$, it is enough to simply obtain the probabilities of deviation $\hat{\eta}^{*}$ from a certain value (required, planned, desired), belonging to a certain range, and others. Also, at the extremum point (8), it is possible to check the probabilities of deviation of the optimal power of the PS and its active elements from the declared, installed, etc.

5 Dependence of identifiability on the composition of measurements

In [14], finite expressions for the covariance matrices of the experiment $C_X = F_X^{-1}$, independent mode parameters $C_{XR} = F_{XR}^{-1}$ and element parameters $C_{\alpha} = F_{\alpha}^{-1}$ were presented, as well as concepts were introduced and expressions for the matrices were obtained:

1) experiment

$$F_X = \left[\begin{array}{c|c} J_R^T C_{R1}^{-1} J_R & J_R^T C_{R1}^{-1} J_{\alpha} \\ \hline J_{\alpha}^T C_{R1}^{-1} J_R & J_{\alpha}^T C_{R1}^{-1} J_{\alpha} + I_{\alpha 1}^T C_{\alpha 1}^{-1} I_{\alpha 1} \end{array}\right]; \tag{10}$$

2) observability

$$F_{XR} = J_R^T H_{\alpha} J_R, \tag{11}$$

3) identifiability

$$F_{\alpha} = J_{\alpha}^T H_{R1} J_{\alpha} + I_{\alpha 1}^T C_{\alpha 1}^{-1} I_{\alpha 1}, \tag{12}$$

where

$$H_{\alpha} = C_{R1}^{-1} - C_{R1}^{-1} J_{\alpha} (J_{\alpha}^T C_{R1}^{-1} J_{\alpha} + I_{\alpha}^T C_{\alpha 1}^{-1} I_{\alpha})^{-1} J_{\alpha}^T C_{R1}^{-1}, \tag{13}$$

$$H_{R1} = C_{R1}^{-1} - C_{R1}^{-1} J_R (J_R^T C_{R1}^{-1} J_R)^{-1} J_R^T C_{R1}^{-1}, \tag{14}$$

$X = \{X_R, \alpha\}$ – vector of independent parameters of the model; X_R is the vector of independent mode parameters, which can (as can be seen from (1)) be the vector G; $J_R = \partial R_1 / \partial X_R$; $J_{\alpha} = \partial R_1 / \partial \alpha$; C_{R1} – covariance matrix of measurement errors of some subset of mode parameters (R_1); $C_{\alpha 1}$ is the covariance matrix of "pseudo-measurement" errors of a priori specified equipment parameters (α_1), $I_{\alpha 1}$ is the correspondence matrix of the components of the vectors α_1 and α.

An important consequence of the introduced concepts and relations is that: 1) if the system is identifiable, then it is observable, but not vice versa, therefore observability is a particular property of identifiability; 2) the obtained expressions for covariance matrix allow to perform a differentiated analysis of the accuracy of any parameter of the model or their combination, including by constructing confidence intervals or regions that cover the true values of the estimated parameters with a given probability; 3) such an analysis can be performed depending on the goals of the experiment (observation or parametric identification); 4) this technique can be used for both a posteriori and a priori differentiated quantitative analysis of identifiability. The only difference is that the derivatives are taken either at the point of solving the estimation problem or at the point of some expected values of the vectors X_R, α.

For an integral assessment of the observability and parametric identifiability of the PS, it is proposed to use the following indicators:

$$D_{XR} = \det(C_{XR}), \quad D_{\alpha} = \det(C_{\alpha}). \tag{15}$$

The lower the values of these indicators, the lower the uncertainty in the corresponding parameters and, accordingly, the higher the degree of observability and parametric identifiability of PS.

Consider the relationship of determinants (15). It is known from the theory of matrices [15] that for an arbitrary block matrix $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, which has an inverse $B = A^{-1} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$, the following relations hold true:

$$\det(A) = \frac{\det(A_{11})}{\det(B_{22})} = \frac{\det(A_{22})}{\det(B_{11})},\ \det(B) = \frac{\det(B_{11})}{\det(A_{22})} = \frac{\det(B_{22})}{\det(A_{11})}\ .$$

Applying these relations to matrix (10) and taking into account that $C_X = \left[\begin{array}{c|c} C_{XR} & C_{XR\alpha} \\ \hline C_{XR\alpha}^T & C_\alpha \end{array}\right]$ we get $\det(F_X) = \det(\bar{F}_{XR}) / \det(C_\alpha) = \det(J_\alpha^T C_{R1}^{-1} J_\alpha + I_{\alpha 1}^T C_{\alpha 1}^{-1} I_{\alpha 1}) / \det(C_{XR})$. Hence it follows that

$$\det(F_\alpha) = \det(F_X) / \det(\bar{F}_{XR})\,. \tag{16}$$

Consider the question of the dependence of the indicator $\det(F_\alpha)$ on the composition of the measurements. Let the measurement covariance matrix have the form $C_{R1} = diag\{\sigma_1^2, \sigma_2^2, ..., \sigma_l^2\}$, where , $l = |I_1|$ is the set of indices of the measured mode parameters.

As shown in [14], in relation to the simplest case, when the vector α is known and deterministically specified, and the observability matrix takes the traditional form $\bar{F}_{XR} = J_R^T C_{R1}^{-1} J_R$, when adding some dimension with index $l+1$, we obtain the matrix $\bar{F}_{XR}(l+1) = \bar{F}_{XR}(l) + (J_R)_{l+1}^T \sigma_{l+1}^{-2} (J_R)_{l+1}$. Its determinant changes to

$$\det[\bar{F}_{XR}(l+1)] = \det[\bar{F}_{XR}(l)] \left(1 + \frac{\bar{\sigma}_{l+1}^2}{\sigma_{l+1}^2} \right), \tag{17}$$

where $\bar{\sigma}_{l+1}^2$ is the estimate of the variance of the *l*+1 non-measurable parameter, calculated on the basis of l measurements; σ_i^2 - dispersion, determined by the metrological characteristics of the measuring device. A similar relationship also takes place for the experimental matrix

$$\det[F_X(l+1)] = \det[F_X(l)] \left(1 + \frac{\hat{\sigma}_{l+1}^2}{\sigma_{l+1}^2} \right), \tag{18}$$

except that here $\hat{\sigma}_{l+1}^2$ is the estimate of the variance of the *l*+1 parameter, calculated on the basis of l measurements under the uncertainty of the true value α (taking into account the correlation X_R and α). Combining relations (17) – (18) we get

$$\det[F_\alpha(l+1)] = \det[F_\alpha(l+1)] / \det[\bar{F}_{XR}(l+1)] = \det[F_\alpha(l)] / \det[\bar{F}_{XR}(l)] \times K = \det[F_\alpha(l)] \times K_{l+1},$$

где $K_{l+1} = (\sigma_{l+1}^2 + \hat{\sigma}_{l+1}^2) / (\sigma_{l+1}^2 + \bar{\sigma}_{l+1}^2)$.

It can also be shown that when the *l*-th measurement is excluded from the current composition, the relation takes place $\det[F_\alpha(l-1)] = \det[F_\alpha(l)] \times K_{l-1}$, where $K_{l-1} = (\sigma_l^2 - \hat{\sigma}_l^2) / (\sigma_l^2 - \bar{\sigma}_l^2)$ and $\hat{\sigma}_l^2$ is the estimate of the variance of the *l*-th (excluded) measurement, calculated from the results of processing *l* measurements.

The above relations, considered as decision rules, can be used as the basis for algorithms for finding the optimal arrangement of measurements to ensure the required level of parametric identifiability of PS.

6 Conclusion

An analysis of the ongoing processes of digitalization and intellectualization shows the presence of all the necessary prerequisites for the PS transformation into cyber-physical systems. The article attempts to structure the applied and formalize new scientific and methodological problems that arise along the way of such a transformation and are of general importance for PS of various types and purposes (heat, water, oil, gas supply, etc.). The problem of probabilistic analysis of PS controllability as a complex property is formulated, which can be used to assess the potential effects from the introduction of various components of PS digitalization and the development of controllability standards. Integral indicators of PS identifiability and observability are proposed, their relationship between each other, as well as with the composition of involved measurements, is disclosed. For the first time, analytical dependences have been obtained for changing the indicator of parametric PS identifiability when adding (removing) a measurement point, which can be taken as the basis for algorithms for optimal synthesis of information-measuring systems.

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