Non-stationary problem of high earth dam under seismic impact

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Abstract. The design, construction, and operation of high earth dams on the territory of seismic zones of Uzbekistan set the task for researchers to improve the calculation methods and algorithms for various kinds of loads to which the structure is subjected. At present, the application of the dynamic method (wave theory) to the calculation of an earth structure, i.e., the dynamic stress-strain state of high earth dams under seismic impact within the framework of wave theory, is the most difficult problem in the mechanics of a deformable rigid body. In the example of the operating Charvak high earth dam, the numerical method of finite differences solved the problem of studying the stress-strain state with shear dynamic impact on the base (such as the records of a real seismogram of an earthquake). At the boundaries of the considered finite region of the base, the so-called radiation conditions are set, i.e., diffraction is not considered. The result of solving the non-stationary problem is presented as isolines of displacements and stresses along the dam's body, depending on time. At that, the most vulnerable zones of the considered earth dam were identified.

1 Introduction

At present, one of the most important priority tasks for developing the national economy is to ensure the safe and reliable operation of structures in seismic regions, such as the territory of the Republic of Uzbekistan. In particular, the category of especially important structures includes earth hydraulic structures (dams of hydroelectric power stations, dam reservoirs) since their possible destruction during an earthquake can lead to catastrophic consequences and death of people.

At present, various methods of calculation are used to assess the strength under loads to which the body of the water-retaining structure itself is subjected. The current normative methods for calculating earth dams for seismic resistance are based on the linear-spectral theory. This method considers a structure as a cantilever rod (fixed to the base). A one-dimensional problem is solved to determine the frequencies and modes of vibrations. One of the disadvantages of this method is that it does not consider the non-one-dimensional nature of the oscillation and the real physical and mechanical characteristics of the soil.

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2 Objects and methods of research

Using Hooke's elastic law, the relationship between the stress and strain tensors is represented as:

\[
\begin{align*}
\sigma_{xx}^i &= \frac{\partial u^i}{\partial x} \\
\sigma_{yy}^i &= \frac{\partial v^i}{\partial y} \\
\tau_{xy}^i &= \frac{\partial u^i}{\partial y} + \frac{\partial v^i}{\partial x}
\end{align*}
\]
\[ \sigma_{xx}^i = \frac{E_i}{1 - \nu_i^2} \left[ \varepsilon_x^i + \nu \varepsilon_y^i \right] \]
\[ \sigma_{yy}^i = \frac{E_i}{1 - \nu_i^2} \left[ \nu \varepsilon_x^i + \varepsilon_y^i \right] \quad (i=1,2) \]
\[ \tau_{xy}^i = \frac{E_i}{1 + \nu_i} \varepsilon_{xy}^i \]

\[
p_x = \sigma_{xx}^i l_x + \tau_{xy}^i m_y
\]
\[
p_y = \tau_{xy}^i l_x + \sigma_{yy}^i m_y
\]

\[
\tau_{xy}^2 = 0; \quad \sigma_{yy}^2 = 0
\]

\[
\sigma_{xx}^i l_x + \tau_{xy}^i m_y = 0
\]
\[
\tau_{xy}^i l_x + \sigma_{yy}^i m_y = 0
\]

\[
u^x = \frac{\partial u}{\partial t}
\]
\[
u^y = \frac{\partial v}{\partial t}
\]

\[
\sigma_{yy} = 0
\]
\[
\tau_{yx} = 0
\]
Displacements and velocities in the entire area of the earth dam at the initial time \( t=0 \) are zero. The time when the wavefront approaches the lower part of the dam base was taken as the initial condition \( t=0 \), i.e.

\[
\begin{align*}
\nu' &= \nu' \\
\tau' &= \tau'
\end{align*}
\]

The problem is solved numerically by the method of finite differences using an explicit scheme, and therefore we select the calculated finite domain for the base of the dam, the so-called fictitious boundary, and set the boundary conditions along the contour of the calculated domain. The displacement for the dam foundation soil is represented as the sum of the incident \((u_0, \nu_0, \sigma_{xx0}, \tau_{xy0})\) and reflected \((u_1, \nu_1, \sigma_{xx1}, \tau_{xy1})\) waves, i.e.

\[
\begin{align*}
\sigma_{xx} &= \rho a \frac{\partial u}{\partial t} \\
\tau_{xy} &= \rho b \frac{\partial \nu}{\partial t}
\end{align*}
\]

\[
\begin{align*}
\sigma_{xx} &= -\rho a \frac{\partial u}{\partial t} \\
\tau_{xy} &= -\rho b \frac{\partial \nu}{\partial t}
\end{align*}
\]

\[
\begin{align*}
\sigma_{yy} &= \rho a \frac{\partial \nu}{\partial t} \\
\tau_{yx} &= \rho b \frac{\partial u}{\partial t}
\end{align*}
\]

On the fictitious boundary of the base from the side of the upstream slope, the following conditions are set:

\[
\begin{align*}
\partial_x (u + u) &= \partial_x (\nu + \nu) \\
\partial_y (u + u) &= \partial_y (\nu + \nu)
\end{align*}
\]

On the bottom of the fictitious boundary of the base, the following conditions are set:

\[
\begin{align*}
\partial_y (\nu + \nu) &= \partial_y (\sigma_{yy} + \tau_{yx}) \\
\partial_x (\nu + \nu) &= \partial_x (\sigma_{xx} + \tau_{xy})
\end{align*}
\]

The calculated area of the earth dam, taking into account the deformable subsoil, is conditionally divided into three areas with a step: \( h_x = h_{x1} \) (upper retaining prism); \( h_x = h_{x2} \) (dam crest), \( h_x = h_{x3} \) (lower retaining prism); \( h_y \) vertically (Fig. 1).
For internal nodes of the computational domain at \((i+1/2, j)\) at time \(k\tau\), the components of the strain tensor in the difference form have the following form:

\[
\varepsilon_{xx}^{k} = \frac{u_{i+1,j}^{k} - u_{i,j}^{k}}{h_x} \quad \varepsilon_{yy}^{k} = \frac{u_{i,j+1}^{k} - u_{i,j}^{k}}{h_y} \\
\varepsilon_{xy}^{k} = \frac{u_{i+1,j}^{k} - u_{i,j}^{k} - u_{i,j+1}^{k} + u_{i-1,j}^{k}}{h_x} + \frac{u_{i,j}^{k} - u_{i+1,j}^{k} - u_{i,j}^{k}}{h_y}
\]

where on the upper retaining prism, it is \(h_x = h_x1\), on the lower retaining prism - \(h_x = h_x3\); on the crest of the dam - \(h_x = h_x2\).

Since the problem under consideration does not take into account the water pressure on the upstream slope, i.e. \(p_x = p_y = 0\), then conditions (4) have the form:

\[
\sigma_{yy} = \sigma_{xx} d \\
\tau_{xy} = -\sigma_{xx} d
\]
\[ \frac{\partial v}{\partial y} = D \frac{\partial u}{\partial x} \]
\[ \frac{\partial u}{\partial y} = D \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} \]

\[ D = \frac{d}{\lambda + G - \frac{\lambda}{\lambda + G - \lambda d}} D = - \frac{d}{\lambda + G + \lambda D} \frac{\lambda}{\lambda + G - \lambda d} \]

\[ \nu_{i,j+1}^k = \sqrt{h} y \left( D \frac{u_{i+1,j}^k - u_{i-1,j}^k}{hx} \right) + \nu_{i,j-1}^k \]
\[ u_{i,j+1}^k = \sqrt{h} y \left( D \frac{u_{i+1,j}^k - u_{i-1,j}^k}{hx} - \frac{v_{i+1,j}^k - v_{i-1,j}^k}{hx} \right) + u_{i,j-1}^k \]

\[ i=j, \; j=1, \ldots, J_1 - 1, \; k=1, \ldots, K. \]

\[ \nu_{i,j+1}^k = \sqrt{h} y \left( D \frac{u_{i+1,j}^k - u_{i-1,j}^k}{hx} \right) + \nu_{i,j-1}^k \]
\[ u_{i,j+1}^k = \sqrt{h} y \left( D \frac{u_{i+1,j}^k - u_{i-1,j}^k}{hx} - \frac{v_{i+1,j}^k - v_{i-1,j}^k}{hx} \right) + u_{i,j-1}^k \]

\[ i=J_2 + j, \; j=1, \ldots, J_1 - 1, \; k=1, \ldots, K \]

\[ d_i = l, \; m, \; D = \frac{d}{\lambda + G - \lambda d} D = - \frac{d}{\lambda + G + \lambda D} \frac{\lambda}{\lambda + G - \lambda d} \]

\[ \sigma_{xx} = \frac{G \lambda + G \frac{\partial u}{\partial x}}{\lambda + G} \]

\[ \sigma_{xx} = \frac{G \lambda + G \frac{\partial u}{\partial x}}{\lambda + G} \]

\[ \sigma_{xx} = \frac{G \lambda + G \frac{u_{i+1,j}^k - u_{i,j}^k}{hx}}{\lambda + G} \]

\[ i=J_2, \ldots, J_2 - 1, \; j=J_1, \; k=1, \ldots, K. \]
Results and discussion

\[ \frac{\partial^2 u}{\partial t^2} = Ae^{-\omega t} \]

\[ u = \frac{A}{\omega^2 + s^2} \left( e^{-s t} (s \omega t + \omega^2) + \cos \omega t \right) \]
which represents the model (23) for $A=0.2g$ and coefficient $s=0.3$, which is equivalent to an 8-point earthquake.

For example, the Charvak earth dam was considered with the following parameters:

- Height $H=168m$
- Crest width $Lo=12m$
- $m_1=2.2$
- $m_2=2.2$
- $h=2m$
- $V=0.3$
- $\rho=1950 \text{ kg/m}^3$
- Longitudinal wave propagation velocity in the dam's body $V_{pr}=1000m/s$
- Poisson's ratio $\nu=0.3$
- Soil density $\rho=1950 \text{ kg/m}^3$
- Time step for the calculation $t=0.001s$

As the impact on the dam foundation, the law of change of displacements is taken in the horizontal direction in the form of (24) and in the vertical direction $\nu_0=\nu$, i.e., there is no vertical component.

The calculations were performed at frequencies $f=1Hz$ and $f=5Hz$, which enter the frequency range of strong earthquakes and are the key parameters of many building codes when assessing seismic risk.

Isolines of velocities of (horizontal) tangential stresses $(t=0.3 \text{ sec})$ at frequency $f=1Hz$ are shown in Figures 2-3, and the dam's foundation is assumed to be non-deformable.

It can be seen that at the initial time point, a shear wave propagates along the dam at a large amplitude. Vertical oscillations of the dam occur due to wave reflection from the upstream and downstream slopes.

After the time point $t=0.3s$, the shear wave front travels a distance of approximately 160m (80x2m) at a dam height of 168m. In the lower part of the retaining prisms, the particle velocities in the horizontal direction have a negative value ($-0.1m/s$), which shows that the particles in this part of the dam have changed their direction.

Figure 3 at the time point $t=0.3\text{sec}$ shows the isolines of the maximum values of shear stresses $\tau_{max}$, and its maximum value of 0.41 MPa occurs in the area of the dam crest.

Fig. 2. Isolines of horizontal velocity $\frac{\partial u}{\partial t}$ m/s in the body of the dam at $t=0.3s$ and frequency $f = 1 \text{ Hz}$.  

Fig. 3.
Fig. 3. Isolines of the maximum value of shear stress (max MPa) in the body of the dam at t=0.3s and frequency f=1 Hz.

Fig. 4. Distribution of shear stress (max MPa) in the body of the dam at the bases at t=10 sec and frequency f=1 Hz.

Acceleration amplitude according to formula (23) is A=0.2g, coefficient s=0.3.

Longitudinal wave propagation velocity at the dam base is $a_1=1000\,\text{m/s}$, density $\rho_1=1950\,\text{kg/m}^3$, Poisson's ratio $\nu_1=0.3$.

For the body of the dam, $a_2=600\,\text{m/s}$, density $\rho_2=1750\,\text{kg/m}^3$, Poisson's ratio $\nu_2=0.3$. 

Fig. 4. Distribution of shear stress (max MPa) in the body of the dam at the bases at t=10s.
4 Conclusion

In contrast to the studies conducted in [1–8], in this article, following the current regulatory methods for calculating structures for seismic effects, the stress-strain state was calculated using the example of the high earth dam of the Charvak HPP, which has been in operation for about 50 years. A real record of earthquakes was taken as a seismic impact. The problem was solved numerically using the finite difference method. Thus:

- a mathematical formulation was developed, and a solution to a non-stationary problem was derived by the numerical method of finite differences. In the example of a specific operated high earth dam, the calculation of the stress state together with the base and the physical and mechanical parameters of the soil of the structure and the base was conducted under shear seismic impact;
- the results of solving the problem are of horizontal velocities, normal, tangential, and maximum tangential stresses in the body of the dam at the frequency of seismic action $f=1\text{Hz}$ and $f=5\text{Hz}$, which enter the frequency range of strong earthquakes.
- for the dam under consideration, it was shown that low-frequency seismic impacts are more unfavorable than high-frequency ones. At that, the crest and slopes are the most vulnerable areas of high earth dam in terms of the possibility of soil sliding.

References


