Mathematical models of magnetic circuits of high currents induction sensors for electric power supply systems devices of electric transport

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Abstract. The magnetic circuits of the developed high currents induction sensors for the electric power supply systems devices of electric transport are researched, analytical expressions of the magnetic flux in ferromagnetic connecting elements and magnetic voltage, the replacement circuit of the magnetic circuit elementary section, as well as mathematical models of the magnetic circuit of the developed sensors, are obtained.

1 Introduction

Currently, in the leading countries of the world, research work is being intensively carried out on the creation and development of monitoring sensors in electric transport power supply systems that provide high sensitivity, accuracy, and reliability of characteristics with great functionality. But along with this, not enough attention has been paid to creating high current sensors that can be used to convert both direct and alternating with several output windings inductively disconnected between the sub or simultaneous connection to circuits for automatic metering of electrical energy, relay protection, and automation.

Nowadays, research work is being intensively carried out on the creation and development of monitoring sensors in electric transport power supply systems in the world's leading countries. Therefore, great importance is given to the research of induction sensors of high currents (ISHC), characterized by high sensitivity, accuracy, and reliability.

To determine the characteristics of these sensors, it is necessary to research and develop their mathematical models. The electromagnetic circuit of the developed induction sensors of high currents (ISHC) is a circuit with distributed electrical and magnetic parameters [1-8]. These parameters include the linear values of the magnetic resistance of C-shaped ferromagnetic concentric rings, the magnetic capacitances of the gap between these rings and the longitudinal ampere-turns per unit of the angle of the magnetic circuit, the linear values of the magnetic resistance of the connecting ferromagnetic elements, the magnetic capacitances of the gap between these connecting elements and the longitudinal ampere-turns of the modulation winding, per unit the lengths of these connecting elements.

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The main factors are the change in magnetic voltage and magnetic flux along the length of the magnetic circuit in induction sensors with distributed parameters, particularly in the developed ISHC [9-12].

It is necessary to develop their mathematical models considering the distribution of the electromagnetic parameters of the circuits and the nonlinearity of the main magnetization curve of the ferromagnetic material for the theoretical research of the new ISHC electromagnetic circuits and their main characteristics.

These models are based on analytical expressions of magnetic flux and magnetic voltage as a function of the location coordinates of the windings.

2 Methods

The theory of electric circuits with distributed parameters, the theory of the electromagnetic field, and experimental research methods, are used.

The modulation magnetic circuits of the developed ISHC [2, 13, 14, 15, 16] consist of two C-shaped parallel sections with cutouts (a rod pair form) along the profile, interconnected by two ferromagnetic connecting elements made with rectangular cutouts (a rod pair form) and modulating windings arranged evenly on each rod pair of C-shaped sections and the connecting element (fig. 1).

Each section of C-shaped magnetic circuits with corresponding modulating windings and a connecting element with modulating windings represent a separate magnetic circuit, practically magnetically unrelated to each other.

The conditions for creating a magnetic field are almost the same in all three magneto modulation circuits: the modulating magnetic field is created by magnetizing windings evenly distributed along parallel rods.

Fig. 1. Design diagram of the developed device for converting current into voltage with wide functionality: 1 and 2 are C-shaped parallel sections of the magnetic circuit with cutouts, 3 and 4 are ferromagnetic connecting elements, 5 and 6 are wedges, 7 is conductive bus, 8 is modulating windings, 9, 10, 11 are three measuring (output) windings.

The magnetic flux of the converted current in C-shaped sections of a magnetic circuit is determined based on Ohm's law as:

\[ E_{out,l} = \frac{U_{mod}}{n_1}, \quad E_{out,p} = \frac{U_{mod}}{n_2} \]
$$Q^c_{\mu x} = \frac{F_x}{\Sigma \mu} = \frac{I_x w_x}{\mu_0 + \mu_0 \delta}$$

$$Z_{\mu 0} = \frac{2 \pi R_T \sigma_m}{\mu_0 (r_n - r_u) b_1 360^0}, \quad Z_{\mu 0} = \frac{\delta_p}{\mu_0 b_p x_m}$$

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$$Q^c_{\mu x} = \frac{F_x}{\mu_0 \delta}$$

$$B^c_c = \frac{Q^c_{\mu x}}{2 b_2 b_3}$$

$$H^c_x = \frac{b_2}{\mu_0}$$

$$Q^l_{\mu x} - U^l_{\mu x} C_{\mu p} dx - Q^l_{\mu x} - dQ^l_{\mu x} = 0$$

$$-U^l_{\mu x} + Q^l_{\mu x} dx + U^l_{\mu x} + dU^l_{\mu x} + Z_{\mu p} Q^l_{\mu x} dx = 0$$

$$Q^l_{\mu x}, U^l_{\mu x}$$

$$F_x = I_x w_x \quad Z_{\mu p 3} = \frac{1}{2 \mu_0 b_2 b_3}, \quad C_{\mu p 3} = \mu_0 \frac{2 b_2}{\delta_p}$$
Differentiating (6) and substituting (5) into it, we obtain the following second-order differential equation:

$$\frac{d^2 U}{d x^2} = 2 Z \mu_p^3 C \mu_p^3 U_{\mu x}.$$  \(7\)

The general solution of this differential equation, according to [4], has the following form:

$$U_{\mu x} = A_1 e^{\gamma_3 x} + A_2 e^{-\gamma_3 x}. \quad \text{(8)}$$

where \(\gamma_3 = \sqrt{2 Z \mu_p^3 C \mu_p^3}\) is the propagation coefficient of the magnetic flux in the magnetic circuit; \(A_1, A_2\) are constant integrations.

Differentiating (8) and substituting it into (6), we find the expressions of the magnetic flux:

$$U_{\mu x} = -\gamma_3 A_1 e^{\gamma_3 x} + \gamma_3 A_2 e^{-\gamma_3 x}. \quad \text{(9)}$$

The constant integrations \(A_1, A_2\) are determined from the following boundary conditions:

$$U_{\mu x} = \left| \begin{array}{c} x = 0 = F_x - Q_{\mu x}^{n} \\ Z_{\mu 0} \end{array} \right., \quad \text{(10)}$$

Fig. 2. Magnetic circuit of a ferromagnetic connecting element with a converted current

Fig. 3. Replacement scheme of the magnetic circuit elementary section of a ferromagnetic connecting element with a converted current
\[ U_{\mu x}^L = |x = X_m = Q_{\mu x}^L |_{x = X_m} \cdot Z_{\mu T} \]

\[ Z_{\mu p} = \begin{cases} \left(1 - \frac{y_3 Z_{\mu 0} Z_{\mu T}}{2Z_{\mu p3}}\right)A_1 + \left(1 - \frac{y_3 Z_{\mu 0}}{2Z_{\mu p3}}\right)A_2 = F_x \\ \left(1 + \frac{y_3 Z_{\mu 0}}{2Z_{\mu p3}}\right)e^{y_3 M A_3} \left(1 + \frac{y_3 Z_{\mu 0}}{2Z_{\mu p3}}\right)e^{-y_3 M A_2} = 0 \end{cases} \]

\[ A_1 = -\frac{F_x}{2a} e^{-y_3 M} + \frac{F_x y_3 Z_{\mu T}}{4A_4 Z_{\mu p3}} e^{-y_3 M} \]

\[ A_2 = -\frac{F_x}{2a} e^{-y_3 M} + \frac{F_x y_3 Z_{\mu T}}{4A_4 Z_{\mu p3}} e^{-y_3 M} \]

\[ \Delta_4 = \left(1 + \frac{y_3 Z_{\mu 0} Z_{\mu T}}{4Z_{\mu p3}}\right)sh \beta_3 + \frac{y_3 Z_{\mu 0} Z_{\mu T}}{4Z_{\mu p3}} \cdot ch \beta_3 \]

\[ U_{\mu x}^L = \frac{F_x}{4A_4} \left(\text{sh}_3 \{y_3 (X_M - x)\} + \frac{y_3 Z_{\mu T}}{2Z_{\mu p3}} \cdot \text{ch}_3 \{y_3 (X_M - x)\}\right) \]

\[ Q_{\mu x}^L = \frac{y_3 F_x}{2a Z_{\mu p3}} \cdot \text{ch}_3 \{y_3 (X_M - x)\} + \frac{y_3 Z_{\mu T}}{2Z_{\mu p3}} \cdot \text{sh}_3 \{y_3 (X_M - x)\} \]

\[ U_{\mu x} = U_{\mu x}^L + U_{\mu x}^p = \frac{F_x}{4a_4} \left(\text{sh}_3 \{y_3 (X_M - x)\} + \frac{y_3 Z_{\mu T}}{2Z_{\mu p3}} \cdot \text{ch}_3 \{y_3 (X_M - x)\}\right) \]

\[ Q_{\mu x} = Q_{\mu x}^L + Q_{\mu x}^p = \frac{y_3 F_x}{2Z_{\mu p3}} \cdot \text{ch}_3 \{y_3 (X_M - x)\} - \text{ch}_3 \{y_3 (X_M - x)\} + \frac{y_3 Z_{\mu T}}{2Z_{\mu p3}} \cdot \text{sh}_3 \{y_3 (X_M - x)\} - \text{sh}_3 \{y_3 (X_M - x)\} \]
Expressions (20) and (21) will be simplified if, in the first approximation, we assume 
\[ Z \rightarrow \infty, Q^p_{\mu} \Big|_{x=X_M} = 0 \text{ and } Q^p_{\mu} \Big|_{x=0} = 0 \]

\[ U_{\mu x} = \frac{F_x}{d_2} \{ ch[\gamma_3 (X_m - x)] + ch(\gamma_3 x) \}, \]

\[ U_{\mu x} = \frac{\gamma_3 F_x}{Z_{\mu 0} a_5} \{ sh[\gamma_3 (X_m - x)] - sh(\gamma_3 x) \}, \]

\[ \Delta_5 = ch(\gamma_3 X_m) + \frac{\gamma_3 Z_{\mu 0}}{Z_{\mu 0} a_5} sh(\gamma_3 X_m). \]

Then, taking into account these assumptions, expressions (20) and (21) will have the following form:

\[ U_{\mu x} = \frac{F_x}{\gamma_3 a_5} \{ ch(\beta_3 (0.5 - x^*)) \}, \]

\[ Q_{\mu x} = \frac{F_x}{\gamma_3 a_5} \{ sh(\beta_3 (0.5 - x^*)) \}, \]

\[ \beta_3 = \gamma_3 X_m. \]

The obtained expressions (24) and (25) on the analysis of the magnetic circuit of ferromagnetic connecting elements with the converted current of the developed ISHC and their curves in relative units shows that the magnetic voltage along the magnetic circuit is unstable and has a minimum value at the magnetic neutral point, and the magnetic flux is distributed non-linearly and changes its sign at the magnetic neutral point, moreover, with an increase in the attenuation coefficient of the magnetic flux \( \beta_3 \), the variability of the magnetic voltage and the degree of nonlinearity of the distribution of the magnetic flux along the length of the magnetic circuit, it increases.

Fig. 4 shows the curves of dependence \( U_{\mu x}^* = f(x^*) \) and \( Q_{\mu x}^* = f(x^*) \) at different values of \( \beta_3 \). Fig. 5 shows the curves of dependence \( U_{\mu x}^* = f(x^*) \) at different values of \( \beta_3 \).
The modulating magnetic field strengths in the corresponding sections of the ferromagnetic connecting element are as follows:

\[ H_m(x_1) = \frac{1}{0.5X_m} Z_{\mu p2} \int_0^{0.5X_m} Q_{\mu}(x_1) dx_1 = f_{m2} \left[ 1 - \frac{k_2}{d_3} sh(0.5X_m) \right] \]

\[ H_m(x_2) = \frac{1}{0.5X_m} Z_{\mu p2} \int_0^{0.5X_m} Q_{\mu}(x_2) dx_2 = f_{m2} \left[ 1 - \frac{k_2}{d_3} sh(0.5X_m) \right] \]

3 Results and discussion

4 Conclusions

References

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