Dynamic calculation of a plane "Earth Dam-Base" system under seismic impact

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Abstract. Design, construction, as well as reliable and safe operation of earthen hydraulic structures (earth dams) in the Republic of Uzbekistan, located in a seismically hazardous zone, require constant improvement of load calculation methods, as required by building codes, including constant loads (static - gravity, hydrostatic) and temporary dynamic loads (seismic). The current normative method does not take into account the non-one-dimensional behavior and piecewise heterogeneity of the soil characteristics of the structure and foundation. It does not allow determining the stress-strain state (SSS) of an earthen dam, which is especially important for reliable and safe operation in seismic areas. A mathematical formulation of the dynamic problem of an earthen dam in a flat elastic formulation is given. The problem is solved numerically by the finite element method. The eigenfrequencies and modes of vibrations of the flat system "structure - foundation" are determined taking into account the piecewise inhomogeneous characteristics of the foundation soil. Based on the results of these parameters, an appropriate behavior analysis is performed. The SSS of the "dam - foundation" system was studied at the calculated natural frequencies. The result of the calculation was the isocline of equal displacements, normal and shear stresses in the "dam - foundation" system.

1 Introduction

Due to the fact that the republic is located in a seismically active zone, much attention is paid to the issues of seismically safe operation of hydraulic structures (earth dams). Damage or an accident in a dam can lead to colossal consequences of a material nature on the downstream territories [1-4]. In the republic, at all critical hydraulic structures, such as high (more than 100 m) earth dams, automated monitoring of the state is performed based on the results of the operation of the engineering seismometric service (in particular, the vibration spectrum of the structure recorded during an earthquake) [2, 3] and data from...
The authors of articles [1-4] analyze the field data (settlement, horizontal displacements, stresses, pore pressure) for 47 years of observations for the earth dam of the Charvak HPP, the highest earth dam in the republic. A number of publications [5, 6] are devoted to the results of field observations of dams built in the former USSR and abroad. Statement and solution of dynamic problems for a number of earth dams in a plane statement, taking into account the adjacent earth base, was conducted in [7, 8]. The study of the stress-strain state of a plane elastic earth dam under static loads for the selection of building materials for a reinforced concrete screen in earth dams was done by the authors of [9,10].

Strains in the structure base, its defects and damage affect the technical condition of all building elements, the main reasons are dynamic loads (seismic and explosive ones, etc.); aging of waterproofing materials; change in the groundwater level, and underestimation of the physical and mechanical soil characteristics of the base during construction and operation. The following features are the most characteristic ones: no uniform and local subsidence; actual precipitation exceeding the allowable values; bulging of the base rock from under the base foot, washing out of the base; soil heaving.

At present, when calculating earth dams for various types of loads, the soil of the structure body is considered as piecewise nonhomogeneous (continuous, isotropic) soil. According to the current design standards, the base of earth dams should be of stable soils (rocky, semi-rocky, etc.) otherwise, it is necessary to develop strengthening measures. Sometimes, during the operation of an earth dam, unfavorable areas of inclusion type (weakened (anhydride) zone) with a low modulus of elasticity are found in the base. The purpose of this article is to develop a methodology and algorithm for solving a dynamic problem for an earth dam considering this kind of inclusions. The problem was solved for a concrete earth dam operated in Uzbekistan in a plane elastic statement (the operational period is more than 30 years; the seismicity of the territory is 9 points). The JSC Hydroproject design institute provided the initial data on the geometric parameters, as well as the physical and mechanical soil characteristics of the structure for the calculation; the data was obtained as a result of pilot drilling of wells at a depth of more than 100 m from the dam foot.

At present, it is not possible to analytically solve the problem to assess the stress-strain state of an earth dam under loads, taking into account design features, real physical and mechanical characteristics of soil that make up the body and the base. Numerical research methods make it possible to investigate the stress-strain state of an earth dam, as well as to plot oscillations for structures of complex geometry. Currently, the most widely used method for solving problems of structure dynamics is the numerical finite element method with the discretization of the domain by plane triangular elements. The relatively simple logic makes it almost ideal for solving many problems in the mechanics of rigid bodies.

2 Methods and Materials

- The earth structure under plane strains is considered (Fig. 1).
The mathematical formulation of the problem of dynamic behavior of a plane model (Figure 1) is based on the variational Lagrange principle, according to which the sum of the work of all forces (elastic, inertia, weight and hydrostatic pressure) on virtual displacements is zero [11]:

$$\delta A = -\sum_{n} \left( \int_{V_n} \sigma_{ij} \delta \varepsilon_{ij} dV - \int_{V_n} \rho_n \ddot{u}_i \delta u_i dV + \int_{V_n} f_{n,i} \delta u_i dV \right) + \int_{S} \bar{p} \delta \tilde{u} dS = 0$$

where $u_i$, $\varepsilon_{ij}$, and $\sigma_{ij}$ are the displacement vector, strain and stress tensors, respectively; $\rho_n$ is the density of materials of the structure; $f_{n,i}$ is the vector of body forces; index $n$ takes the following values: for the structure $n = 1$, for the inclusion ($n = 2$) and the base ($n = 3$); $\bar{p}$ is the hydrostatic pressure on surface $S$ of the system in contact with the water medium (at the base, on the upper and lower slopes).

Boundary conditions accepted are:
1. The lower boundary of the base is rigid, which is expressed in the absence of horizontal and vertical virtual displacements:

$$y = \delta u|_{y=0} = \delta v|_{y=0} = 0$$

2. There are no horizontal displacements of contour points on the hinged (vertical) side boundaries of the base:

$$\delta u|_{x=x_0} = \delta v|_{x=x_0} \neq 0$$

3. The crest and side faces of the structure, that are not in contact with water, are stress-free:

$$\delta u|_{x=l} = \delta v|_{x=l} \neq 0$$

Fig. 1. Plane model of a longitudinal section of an earth dam (1) in piecewise nonhomogeneous soil (3) with an inclusion (2).
\[ \sigma_{ij} n_j = 0 \]
\[ p = \rho g z \]

where \( n \) is the normal vector to the surface.

\[ g_z p = \rho f \]

where \( z \) is the depth measured from the free water surface; \( g \) is the free fall acceleration.

The use of numerical FEM involves the partitioning of a plane model into finite elements with approximating functions [11].

To obtain a resolving system of equations using the finite element discretization method, the elements are combined at the nodal points, the displacements of which are the solution of the variational equation (1), realizing the extremum (minimum) of the work functional.

The technique developed for solving the problem by the finite element method is reduced to determining the eigen frequencies and modes of oscillations, i.e. to solving the problem of eigenvalues, where \( \omega \) are eigen frequencies; \( \{u\} \) is the vector of the corresponding eigenmode.

\[ [K] - \omega^2 [M] \{\ddot{u}\} = 0 \]

\( [K] \) and \( [M] \) are the stiffness and mass matrices of the entire model, formed from the corresponding matrices of individual elements; \( \{\ddot{u}\} \) is the sought-for vector of nodal displacements; \( \{0u\} \) is the base acceleration.

The modes calculated in the course of solving system (6) represent the displacements \( \{u\} \) of the nodal points of the model (determined using approximating functions and the displacements within each element), then the strains in the elements (the Cauchy relations) are determined, the resulting strains are used to determine the stresses by the Hooke law [11].

The forms calculated when solving system (6) are the displacements \( \{u\} \) of the nodal points of the model (determined using approximating functions and displacements within each element), then the deformations in the elements (Cauchy relations) are determined, the resulting deformations are used to determine the stresses according to Hooke's law [11].

When conducting dynamic calculations of hydro-technical structures for kinematic (seismic) impact, it is necessary to determine the seismic load, which, in accordance with the spectral method of calculating the seismic resistance of structures, is determined by the formula specified in the acting regulatory documents:

\[ Q_k A \beta_i \eta_{ik} \]

where \( Q_k \) is the weight falling to the \( k \)-th node; \( A \) is the estimated seismicity (0.1-7 points, 0.2-8 points, 0.4-9 points); \( \beta_i \) is the coefficient of impact (inversely proportional to the period of natural oscillations); \( \eta_{ik} \) are the coefficients of eigen modes normalized by mass, obtained when solving the problem for eigenvalues.

Thus, the first step in dynamic calculations is to find the fundamental frequencies and modes of natural vibrations of the object under study, which enter the formula for the load.
To test the developed methodology and program algorithm, eigen frequencies of the Nurek earth dam were compared with the results previously obtained in [12, 13].

### Table 1. Comparison of fundamental eigen frequencies of the Nurek Dam

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<th>№</th>
<th>Values of eigen frequencies of the Nurek Dam</th>
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<td>Ω3</td>
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<td>1.4749</td>
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### 3 Results

In the course of solving the problem, the dynamic characteristics (frequencies and modes) of rock fill Tupolang dam (height \( H = 165 \) m) were obtained for the options: homogeneous rock base (\( E = 13000 \) MPa, \( ρ = 2.56 \) t/m\(^3\), Poisson's ratio \( ν = 0.3 \)) [13] and rock base in the presence of an inclusion, called an anhydride zone, with physical and mechanical characteristics of soil \( E = 16 \) MPa, \( ρ = 1.58 \) t/m\(^3\), Poisson's ratio \( ν = 0.3 \). Soil parameters of the dam are slope ratio 2 and 1.9 for the upstream and downstream slopes, respectively, \( E = 3500 \) MPa, \( ρ = 2.04 \) t/m\(^3\), Poisson's ratio \( ν = 0.3 \). There is an anhydrite zone, 50 m high stretching along the entire diagonal at the base (two finite elements in height). In the first option for a homogeneous base the physical and mechanical parameters of soil are the same as for the surroundings [13]. In the second option, the parameters of this strip correspond to the parameters of the inclusion of the anhydride zone.

Results of the calculations eigen modes are - shear in the transverse direction (the 1st mode), vertical form (subsidence) (the 2nd mode) and a complex form of shear (the 3rd mode). The rigid rocky base is not deformed. The dynamic characteristics of the dam on a nonhomogeneous base (with a weakened zone) are shown in Fig. 2.
4 Discussions
When comparing the results obtained for a dam on a homogeneous base [13], where vertical displacements (±0.02 m) are the result of shear in the dam section on the dam slopes, in the option with a weakened zone (Fig. 3b), the vertical subsidence increases uniformly over the entire base section, reaching 0.07 m on the lower retaining prism (Fig. 3b). Vertical subsidence in the base is observed in the area above the weakened zone. The maximum stress values in a nonhomogeneous base fall on the contact zone with a weakened zone (0.1 MPa) – Fig. 3c and are significantly inferior to the stresses that occur at the dam contact with a homogeneous base (2.1 MPa). Thus, the weakened zone, located along the entire central section of the dam, is a kind of shock absorber, which takes on the subsidence of the dam along with the upper part of the rock base.

Fig. 3. Distribution of displacements: horizontal (a) and vertical (b), equivalent stresses (c) in the section of the dam on a nonhomogeneous base under fundamental mode of vibration.
Vertical strain under other fundamental modes (the second and third modes) of the dam model on a nonhomogeneous base is shown in Fig. 4.

Thus, for all fundamental modes of oscillation of the dam model on a nonhomogeneous base (Fig. 4), it is mainly the weakened layer of the nonhomogeneous base that is subject to vertical strains. At the first mode of vibration (Fig. 4), the base under the lower prism is subjected to strains. At the second mode, the tension-compression deformation covers the entire area under the structure (in Fig. 4a, sign "+" means tension under the upper prism, and sign "-" means compression under the lower prism).

At the third mode of vibration, the area located at the foot of the upstream slope is subjected to compression (Fig. 4b).

5 Conclusions

When comparing the results of calculations of a plane “structure – base” system on a homogeneous and nonhomogeneous base, it should be noted that in the calculation model on a homogeneous rock base, vertical displacements of the structure predominate and there are practically no displacements at the base. At that, for all modes of oscillation, the displacements are symmetrical, relative to the central axis of the dam. The presence of a weakened layer at the base violates the noted symmetry at all fundamental vibration modes. The weakened layer initiates vertical displacements of the part of the rocky base located above it, which leads to uneven subsidence of the structure (dam). Below the weakened (anhydrite) zone, the vertical displacements of soil are insignificant.

Thus, the presence of a weakened zone in the base of the dam causes nonuniform subsidence of the structure, which can lead to shear and fracturing in the rock base and structure.
References


