Methods of increasing the resolution of signals in optical systems

V.E. Makhov¹, I.I. Sytko²*, and V.V. Shirobokov¹

¹ Mozhaisky Military Space Academy, St. Petersburg, Russia
² Saint-Petersburg Mining University, 199106 St. Petersburg, Russia

Abstract. Optical observation is the most effective way to control the state of various objects. The dynamism of the objects of the observed scene makes it an urgent task to improve the algorithms for obtaining coordinate and non-coordinate information about the observed objects. The issues of increasing the accuracy of measuring the distance and increasing the resolution between the light objects observed by the optical system by the method of double continuous wavelet transformation are considered. It is shown that the use of the second continuous wavelet transform to the curves of the coefficients of the first transform leads to an increase in the maxima of the scalegram and the curves of the coefficients, providing the coordinate sensitivity of the position of the signals by more than two times. The use of different types of wavelets in each continuous wavelet transform of signals gives many options for constructing a processing algorithm and can be used for additional filtering of noise taking into account the characteristics of signals. In this regard, it is proposed to use parallel mathematical models and real signals in a neural network for determining the coordinates of signals and their characteristics, which leads to an increase in accuracy for each type of signal. The indicated approach can be used in systems for multiple signaling from different sources or for combining images in multi-position systems.

1 Introduction

When registering an ensemble of small-sized remote objects with a photodetector, in particular, a matrix photodetector (MPD) [1], they have dimensions comparable to the dimensions of the MPD photodiode, which does not provide sufficient resolution accuracy of their brightness structure and the accuracy of determining their coordinates. Small-sized objects include sufficiently remote objects, for example, objects of near and far space [2]. Monitoring of a set of objects is carried out to eliminate dangerous situations or as a means of navigation. Determining the parameters of small-sized light objects is also relevant in photogrammetry [3] when controlling the shape of the surface of distant objects. Measurement of true or relative coordinates, size and other characteristics of small or distant objects in practice is always associated with difficulties. The small area of object images on the MPD [4] and the blurring of their brightness structure highly affect the accuracy of

* Corresponding author: ivan-sytko@yandex.ru

© The Authors, published by EDP Sciences. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (https://creativecommons.org/licenses/by/4.0/).
determining their coordinates and geometric parameters [5]. A large optical zoom does not increase the accuracy of measuring the coordinates and parameters of objects. Using most known algorithms, the highly localized brightness structure of objects on a photodiode array does not lead to correct and reliable results. Therefore, the improvement of high-precision methods for processing optical signals is relevant primarily for the evaluation of transport systems, space monitoring, astronomical observation systems in military engineering.

2 Level of previous research

Algorithms for determining the coordinates of small-sized light signals of images are known, which are based on the methods of analyzing the brightness structure [5], analyzing clusters of a binarized image structure, as well as the correlation analysis of two-dimensional signals with a known pattern [6, 20], digital correlators of the light field (by analogy with holographic correlators [7, 24]). However, in all cases, the accuracy of resolution and determination of the coordinates of objects depends on the quality of the image [8, 25], determined by known methods [9]. One of the ways to improve the image quality and the accuracy of measuring the parameters of objects is to obtain an image using a hybrid optical system, for example, with an array of microlenses [10]. Such a system forms an array of subaperture small-sized images (light field file), from which a two-dimensional image can then be formed based on the four-dimensional Fourier transform [11, 29]. The light field file allows us to algorithmically form the viewing angle, scale and brightness structure of objects (focusing distance). This, in turn, makes it possible, for example, to improve the accuracy of measuring the distance between light objects [12, 23, 28]. However, the use of all the above algorithms is not able to provide guaranteed measurement accuracy.

3 Research

Further, it is assumed that the second scalogram of the continuous wavelet transform (CWT) [13] from the curves of coefficients (CC) $C_{\Psi_2}$ of the first CWT signal $g(x)$ with a new wavelet $\Psi_2$ can provide greater information content in determining the coordinates and shape of the signals [14, 28]:

$$CC'_{\Psi_2}(x,a) = CWT_{\Psi_2}(CWT_{\Psi_1}(g(x))).$$  \hfill (1)

Since from the CC of the first CWT we again obtain a new scalogram (1) with the possibility of choosing the wavelet $\Psi_2$ and then the CC of the second CWT, it will display the features of the shape of the CC of the first CWT [15]:

$$CC^2_{\Psi_1\Psi_2}(x) = \int CC^2_{\Psi_1}(a,x) \cdot da$$  \hfill (2)

To study this approach, we used a mathematical model of an ensemble of small-sized light spots (see Fig. 1), described by two-dimensional functions [15, 21], including Gaussian functions [16, 22]. Obviously, in this case, the integral projection of the intensities of light spots on the coordinate axes $OX$ and $OY$ will also be a Gaussian function.

The coordinates, mutual position $(x_i, y_i)$ and amplitudes $(A_i)$ of the signals varied over a wide range. The main parameters also include the width of the generated Gaussians and the level of various noise factors [17, 26].
Figure 1. Three-dimensional intensity model of an ensemble of light spots.

Figure 2 shows the scalogram $CWT (\psi=\text{bior 3_1})$ of the projection of the integral intensity of two noisy light spots. On the graph along the $X$-axis – shift, along the $Y$-axis – scale, along the $Z$-axis (color scale) – amplitude.

Thus, the method of analyzing the $CWT$ scalograms (1) and (2) makes it possible to determine a number of signal features using the coordinates of the sample curves of the $CWT$ coefficients, such as: the relative position and shape of a single signal, the distance between two or more spaced signals, the nature of superimposed other signals. The basis for obtaining the maximums and minimums of the scalogram and $CC CWT$ are the slew rate (first derivative), inflections (second derivative). Thus, it is possible to determine the coordinates of the maxima of the signal sample:

$$\langle u_j \rangle_x = \frac{1}{N} \sum_{i=1}^{N} u_{ij}, \quad X_k = \max_{k=1}^{K} \langle u_j \rangle_x, \quad X_k = \max_{k=1}^{K} \langle u_j \rangle_x$$

Signal sample maxima (3) can be determined by various methods: weighted average, derivative maximum, approximation by analytical functions, determination of $CC CWT$ extrema [5]. This approach makes it possible to more accurately determine the coordinates $x_i, y_i$ of individual light objects, as well as the ability to analyze all objects in a random ensemble.

The presence of several maxima and minima in the $CC CWT$ of a given wavelet $\psi$, makes it possible to further improve the accuracy of determining the coordinates [14], for example, by their simple or weighted averaging. Since $CC CWT$ extrema are generated by their behavior along the entire axis, in particular, this is the basis for increasing accuracy by
determining the extremum coordinate, it can be considered possible to apply CWT to CC and also repeated CWT, which initializes the process of obtaining new CC (2). Obviously, the issues of the degree of information content of such CCs to the signal parameters of interest and resistance to factors of various distortions (nonlinearity of the transmission path, sampling, limitation) and noise require a detailed study.

Table 1 shows the CC peaks of the single CWT (ψ₁=biør_3_1) and second CWT (ψ₂=db02), CC of the first CWT for the sum of two offset (Δx) Gaussian waveforms

\[ G_{12}(x) = e^{\frac{x-x_0+\Delta x/2}{b_1^2}} + A_{12} e^{\frac{x-x_0-\Delta x/2}{b_2^2}} \]  \hspace{1cm} (4)

with the ratio of their amplitudes \( A_{12}=A_2/A_1=0.5 \), and width \( b_1^2=b_2^2=1 \).

<table>
<thead>
<tr>
<th>No.</th>
<th>Δx=1.0</th>
<th>Δx=2.0</th>
<th>Δx=3.0</th>
<th>Δx=4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>129</td>
<td>35</td>
<td>109</td>
<td>92</td>
</tr>
<tr>
<td>2</td>
<td>190</td>
<td>96</td>
<td>154</td>
<td>77</td>
</tr>
<tr>
<td>3</td>
<td>136</td>
<td>188</td>
<td>114</td>
<td>194</td>
</tr>
<tr>
<td>4</td>
<td>163</td>
<td>224</td>
<td>138</td>
<td>241</td>
</tr>
<tr>
<td>5</td>
<td>203</td>
<td></td>
<td>163</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>255</td>
<td></td>
<td>197</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td>234</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td>280</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x_{av}</td>
<td>159.5</td>
<td>148.0</td>
<td>168.8</td>
<td>152.6</td>
</tr>
</tbody>
</table>

Graphs of two shifted (Δx=4.0 Pix) noisy (signal-to-noise ratio = 5) signals are shown in Fig. 3.

Graphs of CC CWT (ψ₁) and CC CWT ψ₂ are shown in Fig. 4. It can be seen from the graphs that the CC of the second CWT are smoother than the CC of the first CWT. However, the scalograms themselves also have surface unevenness in the region of small wavelet scales (see Fig. 5). This non-uniformity depends on the noise parameters, which may be the basis for constructing new CC CWT:

\[ CC_{\psi_1\psi_2}(x) = \int_0^\infty w(a) \cdot CC_{\psi_1\psi_2}(a,x) \cdot da \]  \hspace{1cm} (5)

Fig. 3. Two shifted noisy signals
From the graphs (Fig. 4) it can be seen that the CCs of the second CWT have more extremes, a multiple of the extremes of the first and second CWTs. In addition, the total coordinates and amplitudes of the CC peaks change depending on the mutual displacement $\Delta x$ of the signals. These patterns are complex, depending on the shape of the signals and other factors. In all specific cases, it is possible to build calibration dependences of measurements by comparing them with experimental data:

$$x_{12}^{w}(x_2 - x_1) = f\left(A_2 / A_1 \bigcup_{i=1,N}^{\varphi} \max_{\psi} \min_{\psi}\right), \quad (6)$$

$$x_{12}^{\psi_1\psi_2}(x_2 - x_1) = f\left(G_{12}^{\psi}, \{\max_{\psi} \{\min_{\psi}\}_{i=1,N}, \{\min_{\psi}\}_{i=1,N-1}\right) \quad (7)$$

Results (6) and (7) can be analyzed jointly by determining, in one way or another, the probabilistic share of their reliability $w_1$ and $w_2$. Similarly, it is necessary to take into account the types of the selected wavelet types:

$$\bar{x}^{(1)}(x_2 - x_1) = \left\langle \frac{1}{N_{\varphi}} \sum_{\psi} w_\psi \cdot y_{12}^{\psi}(x_2 - x_1) \right\rangle, \quad (8)$$

$$\bar{x}^{(2)}(x_2 - x_1) = \left\langle \frac{1}{N_{\psi_1\psi_2}} \sum_{\psi} w_{\psi_1\psi_2} \cdot y_{12}^{\psi_1\psi_2}(x_2 - x_1) \right\rangle, \quad (9)$$

Noise factor parameters can be determined by determining the correlation of CC CWT extrema for different types of wavelets $\psi_1, \psi_2$. Obviously, having determined in advance the
dominant features of the signal components, for more accurate identification of the type of signals and determination of their coordinates, it is advisable to use the technology of building artificial neural networks (ANN) of some type [18, 21] or a Bayesian classifier [19, 27]. Thus, the proposed approach, due to obtaining a larger number of extrema, gives a better alignment of images [14] in multichannel optoelectronic systems. For example, ANN provides the correct selection of the weight coefficients ANN ($w_i$) in the algorithm for summing the coordinates of the peaks of $CC\ CWT$, depending on the types of wavelets and signal parameters. The next step is to study the ANN in the impact of various noise factors.

4 Conclusion

The proposed method of signal analysis based on double continuous wavelet transform gives a larger number of peaks of the coefficient curves distributed over a wider area on the abscissa axis, which provides a higher accuracy in determining the relative position of optical signals obtained by an optoelectronic system for observing remote objects.

The use of the proposed method, due to the possibility of choosing two different types of wavelets, provides a variety of algorithms and expands the possibilities for choosing the type of filtering for various noise factors of the studied signals.

The use of different types of wavelets for each continuous wavelet transform of signals gives many options for constructing a processing algorithm, so it can be used for optimal filtering and identifying signal features. Parallel use of mathematical models and real signals in a neural network gives a more accurate determination of the signal coordinates and their specific features, which leads to an increase in accuracy for the selected signal type. This approach can be used in systems for multiplexing signals from different sources or combining images in multi-position systems, which expands the capabilities of optoelectronic systems.

References


8. V. E. Makhov, S. E. Shaldaev, A. I. Potapov, Ya. G. Smorodinsky, Influence of image quality in optical-electronic systems on the accuracy of determining the parameters of
16. V. E. Makhov, V. V. Shirobokov, A. I. Potapov, Identification and determination of the position of light spots when they are applied in optical control systems, Control Diagnostics, 7, 4-13 (2019) DOI: 10.14489/td.2019.07.pp.004-013


