Modern trends in the design of leaf spring suspensions of trucks

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Abstract. The article provides an analysis of the improvement of methods for calculating of leaf spring suspensions in retrospect; features of modern calculation methods and own experience in developing of a spring suspension design for a medium-tonnage truck with a gross weight of 6 tons.

1 The main aspects of designing of leaf springs

Nowadays, the use of spring suspensions on trucks do not lose their relevance, according to the totality of modern requirements imposed on them. This is primarily due to the fact that the spring combines the functions of an elastic element and a suspension guide; the spring suspension has a relatively simple design; there is an extensive experience in exploitation of such suspensions and predicting their reliability.

Known disadvantages of spring suspensions, such as the interdependence of kinematic and power factors on the left and right wheels, as well as layout limitations, are not so significant for trucks.

Let us present general analytical dependencies that reflect the characteristic properties of springs as elastic elements and underlie the calculation methods, as well as determine the directions for improving of spring designs.

It is known from the resistance courses of materials that the deflection of a solid section beam (Fig. 1) with width B, height H, length $L = 2l$, which is loaded in the middle by force F, is determined by the formula

$$y = \frac{2Fl^3}{EBH^3} \quad (1)$$

The deflection of a beam of the same cross-section ($H= mh$; $B=b$), which is a package of m sheets (Fig.2) with width b and thickness h, laying on each other and having the possibility of relative sliding without friction, is determined by the formula

$$y' = \frac{2Fl^3}{Ebmh^3}. \quad (2)$$

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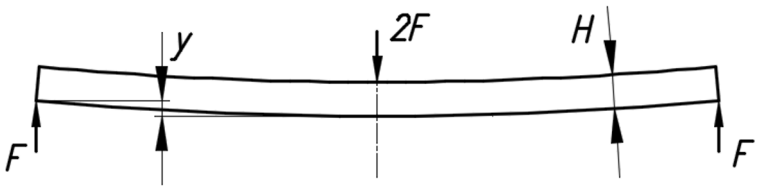

Fig. 1. Solid section beam.

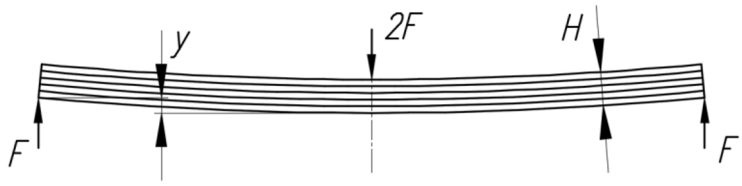

Fig.2. Beam – a package of *m* sheets

Then the ratio of deflections of these beams has the form

$$\frac{y'}{y} = m^2. \quad (3)$$

That is, the transition from a solid beam to a beam of m sheets (while maintaining overall dimensions) allows you to increase the deflection by m^2 times.

It is known from the Strength of Materials subject that bending stresses in a solid beam are determined by the expression

$$\sigma = \frac{3Fl}{BH^2} = \frac{3Fl}{bm^2h^2}. \quad (4)$$

And bending stresses in a package of *m* sheets are determined by the formula

$$\sigma' = \frac{3Fl}{bmh^2}. \quad (5)$$

Then, the ratio of bending stresses has the form

$$\frac{\sigma'}{\sigma} = m. \quad (6)$$

Thus, with an increase in deflection by m^2 times, the stresses at the same time increase by only m times. This fact determines the effectiveness of the spring design as a package of *m* sheets.

From formulas (2), (5) follows another expression of the deflection of a package of *m* sheets

$$y' = \frac{2}{3}\frac{\sigma' l^2}{Eh}. \quad (7)$$

That is, to increase the deflection (and reduce the frequency of natural vibrations), it is necessary to increase the spring length and operating stress, as well as reduce the thickness of the sheets.

However, an increase in the length of the spring and a decrease in the thickness of the sheets entails other disadvantages, such as complexity of the design, increased hysteresis, less lateral and torsional stiffness of the spring.

It is possible to achieve an increase in the deflection of the spring without these negative consequences by the approximation of the spring to the shape of the beam of equal bending resistance. The idea of approaching a multi-leaf spring to a beam of equal bending resistance is carried out by using sheets of different lengths that work with approximately equal stresses (Fig.3).

The use of a multi-leaf spring with sheets of constant thickness does not allow achieving a sufficiently close match with a beam of equal resistance due to some design limitations.

One of the significant disadvantages of a multi-leaf spring is the presence of dry friction between the sheets during bending, which increases the stiffness of the spring (leads to the presence of a "blocking zone"), and also causes wear of the sheets. Schematically, the dependence of the spring deflection on the load is shown in Fig.4.

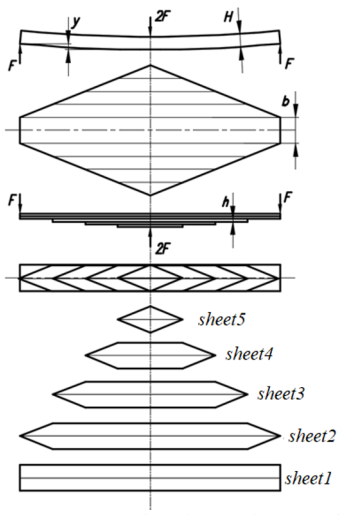

Fig.3. Approximation of a multi-leaf spring to a beam of equal resistance.

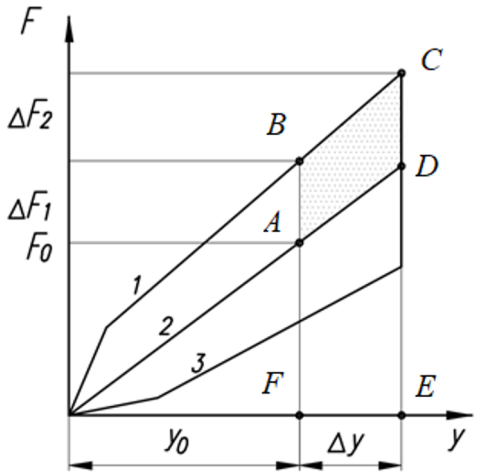

Fig.4. The "deflection-load" dependencies for spring, taking into account friction: 1 – loading line; 2 – calculation line; 3 – unloading line.

According to Fig.4, point A corresponds to static load F_0. With an increase in the load on ΔF_1 (point B), the deflection does not increase, since friction prevents the relative movement of the sheets. With a further increase in the load on ΔF_2, the deflection of the spring will increase by Δy. Then the stiffness of the spring, taking into account its inter-sheet friction, is determined by the formula

$$c' = \frac{\Delta F_1 + \Delta F_2}{\Delta y}. \quad (8)$$

The stiffness corresponding to a static load is equal to

$$c = \frac{F_0}{y_0} < c'. \quad (9)$$

Friction in a multi-leaf spring is estimated by the relative friction coefficient, which is equal to the relative work of the friction forces

$$\mu = \frac{S(ABCD)}{S(FBCD)} = 2\varphi(n-1)\frac{h}{l}, \quad (10)$$

where S(ABCD) is the area of the ABCD quadrilateral representing the work of friction forces; S(FBCD) is the area of the FBCD quadrilateral representing the work of elastic forces; $\varphi=0.2...0.3$ is the coefficient of friction between the sheets; n is the number of leaf springs; h is the height of the spring leaf; l is the length of the spring. To limit the harmful effect of dry friction, the value of μF_0 should be 5-6% F_0 (no more) at $\varphi = 0.3$.

A further step in improving the designs of springs are few-leaf springs, which largely eliminate the disadvantages inherent in multi-leaf springs.

Few-leaf springs, in comparison with multi-leaf springs, have a higher coefficient of use of spring steel (a lower coefficient of material consumption), since the spring sheet approaches to a beam of equal resistance. In addition, few-leaf spring suspensions have a lower coefficient of dry friction, which has a positive effect on the running smoothness.

Varieties of designs of few-leaf spring suspensions are simplified mainly to the peculiarities of the geometric shape of the spring leaf and the designs of its supports on the frame. The most common are spring sheets of constant width with a parabolic law of thickness change. The use of few-leaf springs with a variable cross-section profile of the sheet was held back for some time due to the complexity of manufacturing periodic rolled products. The beginning of their widespread use on mass-produced vehicles abroad can be counted from the middle of the last century. Thus, replacing multi-leaf springs with similar few-leaf springs is an effective solution.

2 Modern spring design methodology

When designing a vehicle suspension, it is necessary to have analytical dependencies that link its geometric parameters with operational indicators. The main domestic methods for calculating few-leaf spring suspensions, in which these analytical dependencies are established, are set out in the works of I.G. Parkhilovsky [2], A.M. Gorelik [1].

Any spring design technique proceeds from a finite number of parameters: part of the parameters is subject to initial acceptance, and the other part is determined (usually compromise) on the basis of rational considerations.

The first group of parameters includes the static deflection of the spring (the derived values from the static deflection are the stiffness of the suspension and the frequency of natural vibrations), the dynamic deflection of the spring and the allowable bending stresses for spring steel. These values determine the following: static deflection - the properties of the suspension to smooth (filter) the harmful effects transmitted from the uneven profile of the road; dynamic deflection of the spring – the probability of suspension breakdown; permissible bending stresses – the durability of the spring and its material consumption.

The second group of parameters includes, first of all, the length of the spring L, the number of sheets n and the thickness of the sheets h. The values of these parameters should preferably be chosen as minimal as possible in order to reduce the dimensions and material consumption of the spring.

In the theory of Strength of Materials, two fundamental dependencies are determined that link the parameters of the first and second groups:

- spring deflection:

$$y = \frac{4FL^3}{Ebh^3}\delta, \quad (11)$$

where F is the load acting on the spring; L is the length of the spring; $E = 2.1\times10^5$ MPa; b is the width of the spring; h_0 is the thickness of the spring; δ is the coefficient of spring deflection increase;

- stresses arising in the leaf springs:

$$[\sigma] = \frac{FL}{W} = \frac{6FL}{bh^2}, \tag{12}$$

where $W = \frac{bh^2}{6}$ is the moment of the spring resistance at the area of its attachment to the vehicle axle.

Combining these dependencies (11), (12) and expressing design parameters from them (from the second group) through initially accepted parameters (from the first group), we obtain the main dependencies used in the design of few-leaf springs:

$$L = \frac{3}{\Delta\sigma}\sqrt[3]{\left(\frac{E}{\delta}\right)^2\frac{c}{2nb}}; \tag{13}$$

$$n = \frac{3}{\Delta\sigma L}\sqrt[3]{\left(\frac{E}{\delta}\right)^2\frac{c}{2b}}; \tag{14}$$

$$h = \frac{3}{\Delta\sigma}\sqrt[3]{\frac{Ey}{\delta}\left(\frac{c}{2nb}\right)^2}, \tag{15}$$

where $\Delta\sigma = \frac{[\sigma]}{y}$ is the specific tension; $c = \frac{F}{y}$ is the spring stiffness.

As well as such design criteria as:

- when assumed as the previously known $\Delta\sigma$, b, c:

$$L^3 n = \text{const}, \tag{16}$$

that is, the more sheets, the smaller the spring length; moreover, the degree of influence of the spring length is much higher;

- when assumed as the previously known b, c, L:

$$m = \frac{\text{const}}{(\Delta\sigma)^2}, \tag{17}$$

that is, the mass of the spring is inversely proportional to the square of the spring tension.

For example, we deduce from the fundamental dependencies (11), (12) dependence (13). From (11) follows

$$y_{\text{ст}} = \frac{4F_{\text{ст}}L^3}{Ebh^3}\delta;\ c = \frac{nF_{\text{ст}}}{y_{\text{ст}}} = \frac{nEbh^3}{4L^3\delta}, \tag{18}$$

where n is the number of sheets of a small leaf spring; c is the stiffness of the spring.

From (12) follows

$$[\sigma] = \frac{FL}{W} = \frac{6F_{\text{ст}}L}{bh^2n};\ \Delta\sigma = \frac{[\sigma]}{y_{\text{ст}}} = \frac{6cL}{bh^2n}. \tag{19}$$

We express from (18) the value of the spring length L

$$L^3 = \frac{nEbh^3}{4\delta c};\ L = h\sqrt[3]{\frac{nEb}{4\delta c}}. \tag{20}$$

We express from (19) the thickness of the spring h

$$h = \sqrt{\frac{6cL}{\Delta\sigma bn}}. \tag{21}$$

Substitute (21) in (20)

$$L = \sqrt{\frac{6cL}{\Delta\sigma bn}}\sqrt[3]{\frac{nEb}{4\delta c}}; \frac{L}{\sqrt{L}} = \sqrt{L} = \sqrt{\frac{6c}{\Delta\sigma bn}}\sqrt[3]{\frac{nEb}{4\delta c}}; L = \frac{6c}{\Delta\sigma bn}\sqrt[3]{\left(\frac{nEb}{4\delta c}\right)^2},$$

and we finally get the formula (13).

From the theory of vehicle springing, as well as the fundamental dependencies of the theory of vibrations, it is known that, in the first approximation, the vibration-insulating properties of the suspension allow for a correct assessment by one global parameter – the natural oscillation frequency ω_0 (when modeling the suspension with a single-mass oscillatory system) or the corresponding partial frequency in the case of adopting a multi-mass model.

The frequency of natural oscillations is determined by the formula

$$\omega_0 = \sqrt{\frac{c}{m}}, \tag{22}$$

where c is the stiffness of the suspension; m is the part of the sprung mass that falls on the suspension.

The static deflection of the suspension is determined by the formula

$$y_{\text{ст}} = \frac{F_{\text{ст}}}{c}, \tag{23}$$

where $F_{\text{ст}} = mg$ is the static load on the suspension.

Taking into account the formulas (22), (23), we can write

$$\omega_0 = \sqrt{\frac{c}{m}} = \sqrt{\frac{cg}{mg}} = \sqrt{\frac{cg}{F_{\text{ст}}}} = \sqrt{\frac{g}{y_{\text{ст}}}} = \frac{3.13}{\sqrt{y_{\text{ст}}}}. \tag{24}$$

The frequency of natural oscillations (24) is expressed in rad/sec; often the natural oscillation frequency is used, expressed in vibrations per second

$$f = \frac{\omega_0}{2\pi} = \frac{0.5}{\sqrt{y_{\text{ст}}}}. \tag{25}$$

Lower values of the natural oscillation frequency (respectively, higher values of static deflection) provide higher indicators of smooth running. The values of the natural oscillation frequency are normalized depending on the type of vehicle; for trucks they are $f = (1{,}7 \ldots 4)$ Hz.

From formula (25) we determine the static deflection of the suspension

$$y_{\text{ст}} = \frac{\omega_0}{2\pi} = \frac{0.25}{f^2}. \tag{26}$$

From formula (23) we determine the stiffness of the suspension

$$c = \frac{F_{\text{ст}}}{y_{\text{ст}}} = 4F_{\text{ст}}f^2. \quad (27)$$

When choosing the value of the dynamic deflection y_d of the suspension, the consideration of the absence of "breakdowns" of the suspension is made. To assess the probability of suspension "breakdowns", the concept of suspension energy intensity is used. The energy intensity of the suspension is understood as the work of deformation of the elastic suspension element from the fully unloaded state to the maximum deformation:

$$E = \int_0^{y_{max}} F(y)dy. \quad (28)$$

At the preliminary stage of suspension design, the dynamic stroke is estimated by the specific dynamic energy intensity

$$y_d = E_{ud} = \frac{E_d}{F_{\text{ст}}}, \quad (29)$$

where $E_d = E - E_{\text{ст}}$ is the dynamic energy intensity of the suspension; $E_{\text{ст}}$ is the static energy intensity of the suspension.

Then for trucks $E_{ud} = 0{,}05 \ldots 0{,}07$m.

As is known, the efficiency of vibration damping is provided by a rational choice of elastic-inertial and damping parameters. The main share of vibration energy dissipation is provided by the suspension shock absorber. In the first approximation, when determining the damping coefficient of a shock absorber, the concept of the aperiodicity coefficient is used:

$$\Psi_0 = \frac{k}{2m\omega_0} = \frac{\delta}{\omega_0}. \quad (30)$$

The physical meaning of the aperiodicity coefficient is that it determines the attenuation of the amplitude in one oscillation cycle and is estimated by the ratio of the amplitudes of successive oscillations:

$$\Psi_0 = \frac{\ln p}{2\pi}, \quad (31)$$

where p is the decay decrement.

From the experience of designing suspensions, take $\Psi_0 = 0{,}2 \ldots 0{,}3$, then the formula for approximate determination of the damping coefficient of the shock absorber will take the form

$$k = \Psi_0 2m\omega_0. \quad (32)$$

The specific stress $\Delta\sigma$ is selected within the permissible values for bending stresses for reasons of the required durability of the spring.

Table 1 shows the main parameters of spring steels used in spring suspensions.

Table 1. Spring steel. Characteristics.

Steel	$\sigma_{\text{в}}$, MPa	$\sigma_{0.2}$, MPa	Ψ_0, %	δ, %
50HGFA	1300	1200	35	6
50HG	1270	1080	35	8
55S2	1270	1175	30	6
55HGR	1270	1175	35	7
69G	980	785	30	8

The basic requirements for the geometric parameters of few-leaf spring suspensions are given in State standard "GOST 3396-90 "Leaf springs of motor vehicles. General specifications" [3].

From the experience of designing few-leaf suspensions, it is known that, as a rule, a single-leaf spring is characterized by a longer length compared to a multi-leaf spring with equal energy intensity. In addition, single leaf springs are not preferable from the point of view of safety in case of breakage of the spring leaf. Therefore, few-leaf springs are used on trucks.

The peculiarity of truck suspensions is that they perceive a wide range of loads (from the curb weight to full weight). In this case, it is not possible to meet the requirements of smooth running by using one few-leaf spring. From the point of view of the vehicle springing theory it is necessary to have a suspension that, with variations in static load, would not change the frequency of its own oscillations significantly. The simplest and economically justified solution is the use of an additional spring, which is activated under heavy loads.

Some typical data on the geometric parameters of few-leaf springs used on vehicles are given in Table 2.

Table 2. Characteristics.

Spring weight, kg	From 31.3 to 78.8
Calculated length, mm	From 1280 to 1970
Width of sheets, mm	From 60 to 100
Thickness in the center of the spring sheet, mm	From 13 to 30
The thickness of the end sections, mm	From 7 to 15
Number of sheets included, pcs	Up to 4
Static stress, MPa	Up to 650

As it is known, the main purpose of the suspension is to smooth out the effects transmitted from the uneven profile of the road to the sprung mass, the driver, passengers and the transported cargo [4]. Therefore, meeting the standards of smooth running is one of the main criteria in the design of the suspension. The study of the running smoothness is carried out on the basis of the analysis of oscillatory models of springing and microprofile of the road. There are many methods for assessing the running smoothness, the most typical scheme for checking compliance with the standards of running smoothness is a technique based on the spectral theory of linear systems: according to a given spectrum of external influences (microprofile of the road), its transformation by a linear springing system into a spectrum of output variables (vibration displacement, vibration velocity and vibration acceleration of the sprung mass) is determined; further, according to the obtained values (the result of transformations of the microprofile spectrum), various integral estimates are formed, which are compared with the regulated criteria for running smoothness.

The main criteria for smooth running, regulated in regulatory documents, take into account the effect of vibration on a person in terms of intensity, frequency, direction and duration of the vibrations: International standard ISO 2631:78 "Guide for the evaluation of human exposure to whole-body vibration", as well as State standard "GOST 12.1.012-90 "Occupational safety standards system. Vibration safety. General requirements", All-Union standard "OST 37.001.291-84 "Motor vehicles. Technical standards of running smoothness".

The assessment of human impacts is carried out in the frequency range 1-90 Hz, which is divided into octaves and a third-octaves. Each octave, as a frequency band, is characterized by an upper and lower bound and an average frequency, moreover, the upper boundary frequency is twice as large as the lower frequency. For a more differentiated assessment of human impacts, octaves, in turn, are divided into a third-octaves, each of which is also characterized by a lower, upper and average geometric frequency (Tables 3,4).

Table 3. Characteristics of octave frequency bands

Average geometric frequencies of octave bands, Hz	Boundary frequencies of octave bands, Hz		Sensitivity weight coefficients		$L_{k\ddot{z}_i}$, Db	$\sigma_{\ddot{z}_i}$, m/sec^2	$L_{\sigma\ddot{z}_i}$, Db
	lower	upper	Z_0	X_0			
1,0	0,8	1,4	0,50	1,00	-6	1,10	121
2,0	1,4	2,8	0,71	1,00	-3	0,79	118
4,0	2,8	5,6	1,00	0,50	0	0,57	115
8,0	5,6	11,2	1,00	0,25	0	0,6	116
16,0	11,2	22,4	0,50	0,125	-6	1,13	121

Table 4. Characteristics of octave frequency bands

Average geometric frequencies of octave bands, Hz	Boundary frequencies of octave bands, Hz		Sensitivity weight coefficients		$L_{k\ddot{z}_i}$, Db	$\sigma_{\ddot{z}_i}$, m/sec^2	$L_{\sigma\ddot{z}_i}$, Db
	lower	upper	Z_0	X_0			
0,8	0,7	0,89	0,45	1,00	-7	0,71	117
1,0	0,89	1,12	0,50	1,00	-6	0,63	116
1,25	1,12	1,41	0,56	1,00	-5	0,56	115
1,6	1,41	1,80	0,63	1,00	-4	0,5	114
2,0	1,80	2,25	0,71	1,00	-3	0,45	113
2,5	2,25	2,81	0,8	0,8	-2	0,4	112
3,15	2,81	3,55	0,9	0,63	-1	0,355	111
4,0	3,55	4,5	1,00	0,50	0	0,315	110
5,0	4,5	5,6	1,00	0,4	0	0,315	110
6,3	5,6	7,07	1,00	0,315	0	0,315	110
8,0	7,07	9,00	1,00	0,25	0	0,315	110
10	9,00	11,2	0,8	0,2	-2	0,4	112
12,5	11,2	14,1	0,63	0,16	-4	0,5	114
16,0	14,1	18,0	0,50	0,125	-6	0,63	116
20,0	18,0	22,4	0,4	0,1	-8	0,8	118

With regard to vehicles, International standard ISO 2631-78 recommends that when designing a driver's workplace, the main criterion is to use the "limit of reduced productivity from fatigue", and for passenger seats to proceed from the "threshold of reduced comfort".

When determining a person's response to vibration, four physical factors are taken into account: intensity, frequency, direction of action and duration of vibration exposure.

The following indicators are used to normalize the vibration load:

root-mean-square (RMS) values of vibration acceleration and vibration velocity:

$$\sigma_{\ddot{z}} = \sqrt{\sum_i^n \ddot{z}_i^2}; \quad (33)$$

$$\sigma_{\dot{z}} = \sqrt{\sum_i^n \dot{z}_i^2}; \quad (34)$$

logarithmic levels of RMS values of vibration accelerations and vibration velocity:

$$L_V = 20\lg\left(\frac{\sigma_{\dot{z}}}{5\cdot 10^{-8}}\right); \quad (35)$$

$$L_A = 20\lg\left(\frac{\sigma_{\ddot{z}}}{10^{-6}}\right); \quad (36)$$

the frequency-adjusted value of the controlled parameter and its logarithmic level

$$\tilde{u} = \sqrt{\sum_i^n (k_i u_i)^2}; \quad (38)$$

$$L_{\tilde{u}} = 10\lg\sum_i^n 10^{0.1\left(L_{u_i}+L_{k_i}\right)}. \quad (39)$$

The maximum technical standards of running smoothness of all categories of cars are given in All-Union standard OST 37.001.291-84 Tables 5, 6.

Table 5. Nomenclature and characteristics of road sections of motor-vehicle providing ground of NICIAMT for testing the running smoothness of motor vehicles

Part number	Type of road	Part length, m	Wavelength range, m	RMS of the height of irregularities, m
1	Cement-concrete dynamometer road	1000	0,4-40	0,6 10^{-2}
2	Cobblestone paved road without potholes	1000	0,25-25	1,1 10^{-2}
3	Cobblestone with potholes	500	0,12-12	2,9 10^{-2}

Table 6. Maximum technical standards of running smoothness of trucks

Part number of motor-vehicle providing ground of NICIAMT	Adjusted values of vibration accelerations on the seat, m/sec^2, no more			RMS of vertical vibration acceleration at the characteristic points of the sprung mass, m/sec^2, no more
	vertical	horizontal		
		longitudinal	transverse	
1	1	0,65	0,65	1,3
2	1,5	1	0,8	1,8
3	2,3	1,6	1,6	2,7

When calculating the running smoothness, there is a need for an analytical representation of the microprofile function

$$q = q(x). \quad (40)$$

Moreover, in most statements of smoothness problems, it is important to know not the exact, instantaneous values of the ordinates of the microprofile of the road, but the averaged integral estimates. The microprofile of highways usually has a complex shape with a random nature of changes in the heights of irregularities and therefore can be characterized statistically.

For a comparative assessment of various microprofiles, the ordinates of the microprofile are "centered", i.e., the count is made from the average value:

$$q_{\text{ср}} = \frac{1}{2L}\int_{-L}^{L} q(x)dx. \quad (41)$$

After centering the microprofile function on a section of length 2L, its average value on the same section becomes zero, and the standard deviation will have a minimum value.

$$q_{\text{ц}}(x) = q(x) - q_{\text{ср}} \quad (42)$$

$$q_{\text{ср}} = \frac{1}{2L}\int_{-L}^{L} q_{\text{ц}}(x)dx = q_{\text{ср}} - \frac{1}{2L}\int_{-L}^{L} q(x)dx = 0; \quad (43)$$

$$\sigma_q = \sqrt{\frac{1}{2L}\int_{-L}^{L} q_{\text{ц}}(x)^2 dx} = min\left(\sqrt{\frac{1}{2L}\int_{-L}^{L} q(x)^2 dx}\right). \quad (44)$$

An exhaustive probabilistic characteristic of the micro profile of the road as a random function is the correlation function:

$$R_q(s) = \frac{1}{2L}\int_{-L}^{L} q_{\text{ц}}(x) q_{\text{ц}}(x+s)dx. \quad (45)$$

The correlation function displays the degree of linear relationship between two ordinates of the microprofile, separated from each other by a value of s. At a certain critical value of the shift $s = s_0$, the correlation function becomes zero (the ordinate values do not depend on each other). Therefore, the value s_0, which characterizes the length of the probabilistic relationship of the ordinates of the road profile, is one of the indicators of the flatness of the road.

It follows from the formula of the correlation function that the correlation function at zero shift (s=0) is equal to the square of the variance of the ordinates of the road profile

$$R_q(0) = \frac{1}{2L}\int_{-L}^{L} q_{ц}(x)q_{ц}(x)dx = {\sigma_q}^2. \quad (46)$$

Therefore, in order to give universality (comparability) to correlation functions, normalized correlation functions are used.

$$\rho_q(s) = \frac{R_q(s)}{R_q(0)}. \quad (47)$$

From the accumulated data on the statistics of the ordinates of the microprofile of the road [4, 10], it was found that the normalized correlation functions for certain groups of road bases have a stable qualitative form (Fig.5).

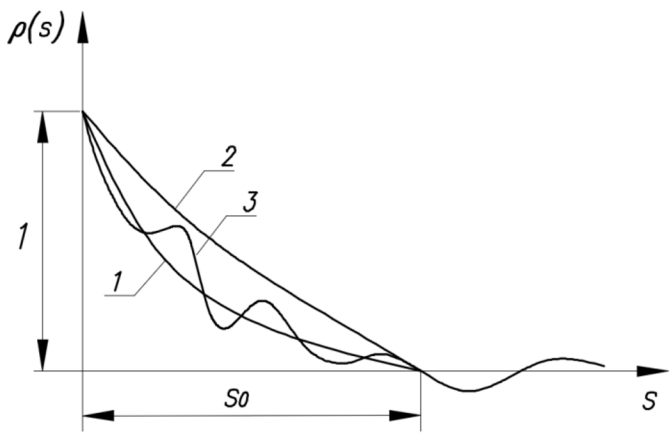

Fig.5 Typical diagrams of normalized correlation functions for certain types of roads.

The given typical normalized correlation functions (Fig. 5) are satisfactorily approximated by an expression of the form

$$\rho_q(s) = A_1 e^{-\alpha_1|s|} + A_2 e^{-\alpha_2|s|}\cos\beta s. \quad (48)$$

In some cases, the functions of the microprofile are approximated with a sufficient degree of accuracy by simplified dependencies (Table 7)

$$\rho_q(s) = A_1 e^{-\alpha|s|} \text{ или } \rho_q(s) = e^{-\alpha|s|}\cos\beta s. \quad (49)$$

Table 7. Coefficients of normalized correlation functions for certain types of roads

Type of road	Coefficients $\rho_q(s)$					RMS σ_q, sm
	A_1	A_2	α_1	α_2	β	
Broken soil road	0,55	0,45	0,085	0,080	0,235	3-10
Worn cobblestone highway with potholes	0,953	0,047	0,213	0,049	1,367	2,52
Large-boulder section of the highway	0,668	0,336	1,1	10,6	19,71	2,49
A little-worn cobblestone highway	1	0	0,45	0	0	1,35-2,29
Paved highway	0,85	0,15	0,2	0,05	0,6	0,8-1,26
Cement-concrete highway	1	0	0,15	0	0	0,5-1,24

In special courses in the theory of random processes, a connection is established between the energy spectrum of a random function and the correlation function, namely, the energy spectrum is the Fourier transform of the correlation function of the microprofile:

$$R_q(s) = \frac{1}{\pi}\int_0^\infty S_q \cos(s) p dp, \tag{50}$$

where

$$S_q(p) = 2\int_0^\infty R_q(s)\cos(s) p ds. \tag{51}$$

Considering that the area of the energy spectrum is equal to the dispersion of the microprofile function

$$(\sigma_q)^2 = \frac{1}{\pi}\int_0^\infty S_q dp, \tag{52}$$

consider the expressions for the normalized energy spectrum of the microprofile:

$$\overline{S_q}(p) = \sqrt{\frac{S_q(p)}{\pi(\sigma_q)^2}} = \sqrt{\frac{2\int_0^\infty R_q(s) cos(s) p ds}{\pi(\sigma_q)^2}}. \tag{53}$$

Let's substitute the approximating correlation functions into the last expression:

$$\overline{S_q}(p) = \frac{\sqrt{2}}{\pi\sigma_q}\sqrt{\frac{A_1\alpha_1}{p^2+\alpha_2} + \frac{A_2\alpha_2(p^2+(\alpha_2)^2+\beta^2)}{(p^2+(\alpha_2)^2+\beta^2)^2+4(\alpha_2)^2\beta^2}}. \tag{54}$$

In particular cases of approximating correlation functions, the normalized energy spectrum of the microprofile is determined by the expressions

$$\overline{S_q}(p) = \frac{\sqrt{2}}{\pi\sigma_q}\sqrt{\frac{\alpha_1}{p^2+\alpha_2}}; \tag{55}$$

$$\overline{S_q}(p) = \frac{\sqrt{2}}{\pi\sigma_q}\sqrt{\frac{p^2+(\alpha_1)^2+\beta^2}{(p^2+(\alpha_1)^2+\beta^2)^2+4(\alpha_1)^2\beta^2}}. \tag{56}$$

If we assume that the accumulation of physiological shifts in the body of a sitting person from the effects of vertical harmonic oscillations of various frequencies and amplitudes occur in a linear manner, then its overall functional state is adequately assessed by the RMS acceleration

$$\sigma_{\ddot{z}}^2 = \int_0^\infty |A(\omega)|^2 S_{\ddot{z}}(\omega) d\omega, \tag{57}$$

where $A(\omega)$ is the amplitude-frequency response, $S_{\ddot{z}}(\omega)$ is determined on the base of formula (56).

3 Conclusion

1. Based on the above formulas, which determine the deformations and stresses in leaf springs with different designs, rational ratios of spring parameters are shown.
2. Step-by-step improvement of spring designs reflects a compromise solution in an effort to meet the main criteria (running smoothness, increased suspension energy consumption, reduced hysteresis and material consumption, etc.), as well as in an effort to bring the spring closer to the "ideal" beam of equal bending resistance.
3. The main trends in improving of the methods of designing of leaf springs are based on analytical dependencies. And modern software methods for numerical calculation of stresses and deflections of spring sheets do not provide an understanding of the basic global optimal dependences of spring parameters but can be effectively used as verification calculations.

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References

1. Few-leaf springs / A.M. Gorelik. – M. : NIINavtoprom, 1981. – 52 p.
2. Parkhilovsky I.G. Automobile leaf springs. – 2nd ed., reprint. and additional – M.: Mechanical Engineering, 1978. – 232 p.
3. State standard “GOST 3396-90 “Leaf springs of motor vehicles. General specifications”
4. Rotenberg, R.V. Vehicle suspension / R.V. Rotenberg. - M. Mechanical engineering, 1972. - 392 p.
5. Kravets V.N. Legislative and consumer requirements for vehicles: study guide / NSTU, N. Novgorod, 2000 – 400 p.
6. State standard “GOST 12.1.012 90 “Occupational safety standards system. Vibration safety. General requirements”.
7. State standard “GOST 31191. 1-2004 (ISO 2631 – 1:1997) “Vibration and shock. The change in the general vibration and the assessment of its impact on humans”.
8. State standard “GOST 12. 1. 012-2004 “System of occupational safety standards. Vibration safety. General requirements”.
9. All-Union standard “OST 37.001.291 84 “Motor vehicles. Technical standards of running smoothness”.
10. Zezyulin, D., Makarov, V., Alexander, B., Ogorodnov, S., & Belyakov, V. (2014). Modeling of roads impacts for life prediction of light commercial vehicles parts. Paper presented at the FISITA 2014 World Automotive Congress - Proceedings