Features of using the smoothed instantaneous wave energy history (SIWEH) to analyze the group structure of sea waves

Alexander Zapevalov[1*]

[1]Marine Hydrophysical Institute RAS, 299011Sevastopol, Russian Federation

Abstract. The methodological features of determining the characteristics of wave groups of sea surface waves based on the function smoothed instantaneous waves through energy (SIWEH) are analyzed. The analysis is carried out by numerical simulation based on an analytical model of the group structure of waves. It is shown that when the grouping of sea waves is investigated using the SIWEH function, it is advisable to calculate the Groupiness Factors of Height (GFH) in two stages. At the first stage, transform the SIWEH function into a wave envelope, and at the second stage, calculate the GFH using a known envelope. The previously proposed direct calculation of GFH by the SIWEH function leads to significant errors. The effect of the change in the cutoff frequency of the Bartlett filter at a fixed frequency of dominant waves on the calculated values of GFH is investigated. It is shown that when the cutoff frequency changes $\pm 30\%$, the GFH changes do not exceed 5%.

1 Introduction

The group structure of surface waves plays an important role in various technical applications. The effect of wave groups on ships and coastal structures is stronger than the effect of ungrouped waves [1-3]. The group structure has a significant impact on the generation of infrasound by the sea surface [4], high power of infrasound in stormy conditions can lead to the appearance of microseisms in the earth crust [5] The difficulty of including the effects created by wave groups in an engineering project is due to their strong variability, and also partly due to the variety of theories and parameters used to describe the group structure [6]. Several methods are used to isolate a group of waves: Hilbert transform [7], Markov chain model [8], wavelet transform [9].

The function smoothed instantaneous waves through energy (SIWEH) has become widespread in the study of the group properties of marine surface waves [10-12]. The SIWEH function proposed in [13] is based on the instantaneous wave energy process. The advantage of SIWEH functions is their simple analytical representation and clarity.

* Corresponding author: sevzepter@mail.ru

When using the SIWEH function, it is necessary to take into account a number of methodological features. The dimensions of the envelope function and function SIWEH are different. As a result, we cannot directly map these functions. The approximate relation between them can be given by [6]

$$E(t) \approx A(t)^2/2, \tag{1}$$

where $E(t)$ is function SIWEH, $A(t)$ is wave envelope, t is time.

A feature of the construction of the SIWEH function is the use of a low pass filter. The Bartlett window is used [13]. Different types of filters and the difference in their parameters can affect the characteristics of wave groups obtained using the SIWEH function [10]. Changing the shape and parameters of the filter does not have a noticeable effect on the estimates of group characteristics if one system of waves is observed on the surface and the wave spectrum is unimodal [11]. Errors can occur if there are two or more wave systems with comparable energy (multimodal wave spectrum).

The purpose of the work is to analyze the features of using the SIWEH function to isolate groups of waves in a complex wave field, when two wave systems are present on the sea surface.

2 Materials and methods

2.1 Analytical model of the wave group

The profile of a wave having a group structure is usually described as the product of an wave envelope and a carrier wave

$$\eta(x,t) = aA(x,t)\xi(x,t), \tag{2}$$

where a – amplitude multiplier; $A(x,t)$ – wave envelope, $\xi(x,t)$ – carrier wave.

A simple analytical model describing the wave envelope and the carrier wave was proposed in [14]

$$A(x,t) = \exp\left[-\rho_1 \cos^2\left(\frac{k_0 x - (\omega_0/2)t}{2\rho_2}\right)\right], \tag{3}$$

$$\xi(x,t) = \exp\left[-\rho_0 \cos^2\left(\frac{k_0 x - \omega_0 t}{2}\right)\right], \tag{4}$$

where k_0 and ω_0 are the wave number and frequency (circular) of the carrier wave, the parameters ρ_1 and ρ_2 determine the shape of the envelope and the number of waves in the group, the parameter ρ_0 determines the asymmetry of the carrier wave. The properties of the model in the form (3) and (4) were analyzed in [11].

Groups create gravitational waves. If surface waves propagate in an area where the influence of the bottom can be neglected, then the parameters k_0 and ω_0 are related by the dispersion relation

$$\omega^2 = g\,k\,, \tag{5}$$

where g is the gravitational acceleration.

Usually, the geometric structure of the sea surface is studied from measurements at one or more points, using string gauges or wave buoys [15, 16]. If the parameter x is fixed, then equation (2) describing the change in the sea surface level at a given point can be used for numerical simulation and in situ analysis of wave measurement data.

The advantage of the model (2)-(4) is that it allows taking into account the observed deviations of the distribution of sea surface elevations from the Gaussian distribution [17]. Note that the average value $\overline{\xi(x,t)} \neq 0$, therefore, when modeling a group of waves in equation (2), the replacement is performed

$$\xi(x,t) \to \xi(x,t) - \overline{\xi(x,t)}\,. \tag{6}$$

The third statistical moment calculated according to (4) for the carrier wave $\xi(x,t)$ always has a positive sign, which corresponds to a pointed crest and a flat trough. It should be taken into account that numerous measurements carried out in different areas of the World Ocean show that the skewness of sea surface elevations can take negative values [16, 18]. To simulate a similar situation after replacing (6) in equation (2), it is necessary to change the sign $\xi(x,t)$.

The situation when two wave systems are present on the sea surface, we will consider in the approximation that these systems do not interact with each other. This makes it possible to describe the change in surface elevations by the superposition of two wave systems

$$\eta_\Sigma(x,t) = \eta_1(x,t) + \eta_2(x,t)\,. \tag{7}$$

For the sake of certainty, we will assume that $\omega_0^{(1)} < \omega_0^{(2)}$. Here and further, the superscript indicates the number of the wave system to which this parameter corresponds.

An example of how the elevation of the sea surface changes at a fixed point is shown in Fig. 1. When constructing $\eta_\Sigma(x,t)$, it was assumed that $\omega_0^{(2)} = 1.21\omega_0^{(1)}$ and variants with different ratios $a^{(2)}/a^{(1)}$ were considered. On the top fragment of Fig. 1 shows the profiles of two wave systems, normalized so that their maximum value is equal to one. On the abscissa, time is given in a dimensionless form, normalized to the period of the carrier wave $T_0^{(1)}$.

2.2 Characteristics of wave groups

The main parameter that describes the group structure of surface waves is the Groupiness Factors of Height (GFH). This parameter characterizes the depth of wave height modulation in the group. The GFH for a known wave envelope is given as [10]

$$\mathrm{GFH}_A = \sqrt{2}\sigma_A / \overline{A}\,, \tag{8}$$

where σ_A is standard deviation of the wave envelope, $\overline{A}$ is average of the wave envelope. The subscript GFH shows the calculation method

Another important parameter used in engineering calculations is the Group Length Factors (GLF) [19, 20]

$$\mathrm{GLF} = \omega_p / \omega_{pA} , \tag{9}$$

where ω_p and ω_{pA} are the peak frequencies of the wave spectrum and the wave envelope spectrum. If the model (2)-(4) is used, it can be shown that $\mathrm{GLF} = \rho_2$.

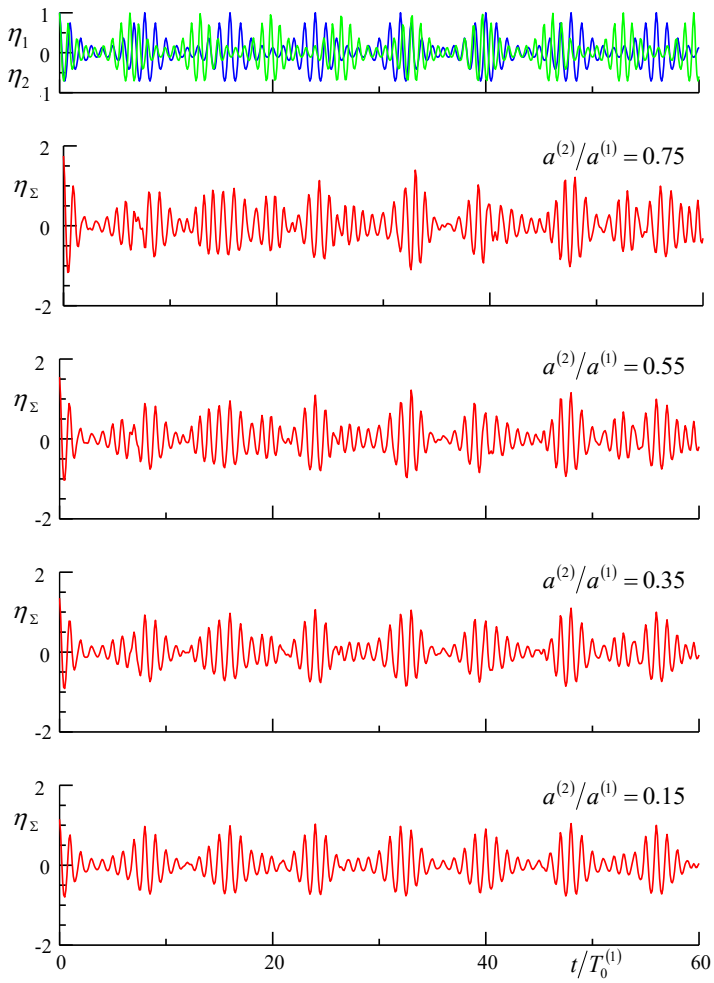

Fig. 1. Change of the wave profile in the presence of two wave systems: blue curve is $\eta_1(t)$; green curve is $\eta_2(t)$; red curve is $\eta_\Sigma(t)$

2.3 SIWEH function

The SIWEH function has the form [13]

$$E(t)=\frac{1}{T_p}\int_{-\infty}^{\infty}\xi^2(t+\tau)Q(\tau)d\tau \quad , \qquad (10)$$

where $Q(\tau)$ is the low pass filter; $T_p=1/f_0$ ($f_0=\omega_0/(2\pi)$ is the period of carrier waves. For sea waves, the parameter T_p is equal to the period of waves corresponding to the maximum of the wave spectrum. The $Q(\tau)$ has the form

$$Q(\tau)=\begin{cases}1-|\tau|/T_p & if\ \ |\tau|\le T_p\\ 0 & if\ \ |\tau|>T_p\end{cases} \quad . \qquad (11)$$

$Q(\tau)$ is Bartlett window.

The GFH for the SIWEH function is defined as [19]

$$\mathrm{GFH}_S=\frac{1}{\overline{E}}\sqrt{\frac{1}{T_w}\int_0^{T_w}\left[E(t)-\overline{E}\right]^2 dt} \quad , \qquad (12)$$

where T_w is the wave record length and $\overline{E}$ is the mean value of $E(t)$ over the recording time,

$$\overline{E}=\frac{1}{T_w}\int_0^{T_w}E(t)\,dt \quad . \qquad (13)$$

3 Numerical simulation and discussion

3.1 Wave envelope and SIWEH function

Using numerical simulation, let's compare the results of calculating the GFH by the wave envelope with the calculation using the SIWEH function. To do this, consider the situation when the model (2)-(4) describes one wave system. The wave envelope $A(t)$ is known, which allows us to obtain reliable estimates GFH_A, with which we compare GFH_S obtained according to (12). We also consider the estimates GFH_{AE} obtained by converting (1) the SIWEH function into a wave envelope $A_E(t)$, and calculating according to (10).

Based on Fig. 2, two conclusions can be drawn. First, in the area of maximum values of the modulated carrier wave, the inequality $A_E(t)<A(t)$ takes place. Second, estimates GFH_S are significantly higher, than GFH_A. Previously, similar discrepancies in calculations GFH by different methods were noted in [10]. The estimates GFH_{AE} are much closer to the estimates of GFH_A, than GFH_S.

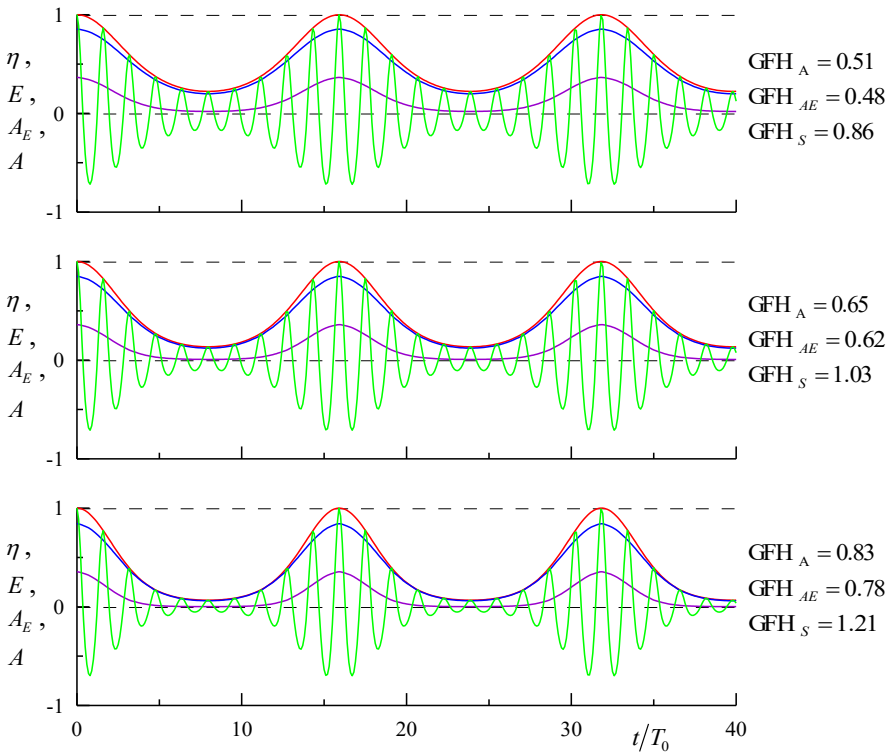

Fig. 2. The profile of the wave group and the functions characterizing it: the green curve is η, the purple curve is SIWEH functions *E*, the blue curve is A_E, the red curve is A

3.2 The effect of changing the frequency of the carrier wave

Both individual waves and groups of waves are characterized by strong variability in space and time. Not only the energy of the waves changes, but also their frequency. The Bartlett window $Q(\tau)$ is set for the period of the dominant wave averaged over the measurement session, which does not allow taking into account local period changes. To appreciate this effect, consider how changing the filter parameter affects GFH_{AE} and GFH_S. To do this, we will carry out calculations by replacing $T_p \to nT_p$. Changing the parameter *n* leads to a change in the Bartlett window width in the time domain, smaller values of *n* lead to a narrower window $Q(\tau)$. As a result of changing the parameter *n*, the filter cutoff frequency is shifted. Cutoff frequency is related to the filter width by the ratio $T_p = 1/f_0$.

The calculation results are shown in Fig. 3. It can be seen that changing the width of the Bartlett window has little effect on the calculated values GFH_{AE} and GFH_S. A relative change in the width of the window $Q(\tau)$ results in $\pm 30\%$ a change of GFH_{AE} and GFH_S less than 5%.

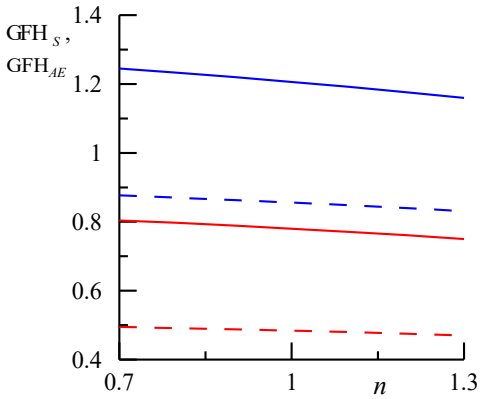

Fig. 3. Dependences of GFH_{AE} and GFH_S on the width of the Bartlett window (11). Blue lines are GFH_S, red lines are GFH_{AE}, solid lines correspond $\mathrm{GFH}_A = 0.83$, dashed lines correspond $\mathrm{GFH}_A = 0.51$.

3.3 Two wave systems

The ability to distinguish groups of waves using the SIWEH function when two wave systems are present is shown in Fig. 4.

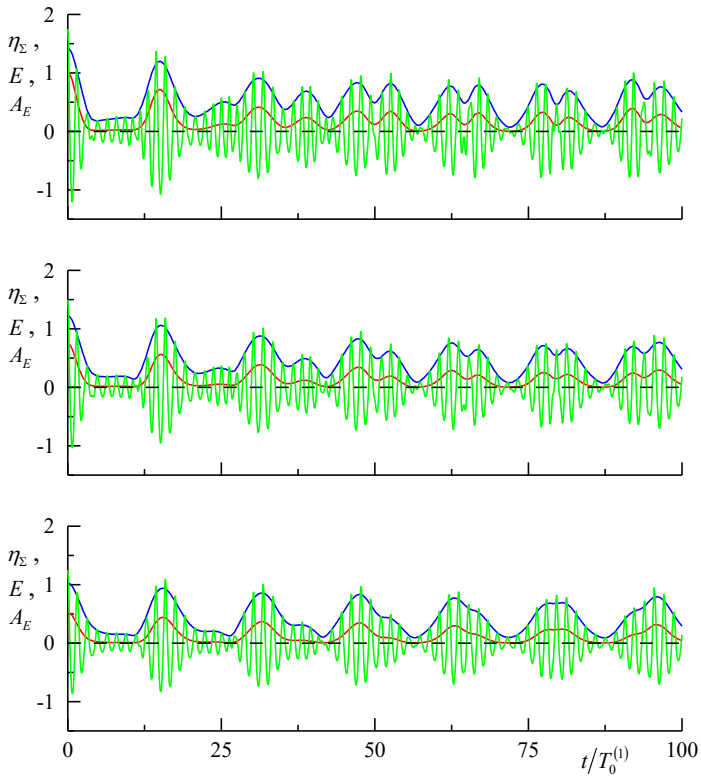

Fig. 4. The wave profile and its characterizing functions in the presence of two wave systems: the green curve is η_Σ, the red curve is the SIWEH function, the blue curve is A_E

The envelope describes a complex wave profile well. As in the case when the wave field is determined by a single wave system, small excesses $\eta_\Sigma > A_E$ occur only for the crests of high waves

4 Conclusion

The analysis of the possibilities and limitations of calculating the characteristics of wave groups of sea surface waves based on the SIWEH function is carried out. The analysis was carried out by numerical simulation based on an analytical model describing the group structure of waves.

It is shown that when the grouping of sea waves is investigated using the SIWEH function, it is advisable to calculate the GFH in two stages. At the first stage, transform the SIWEH function into a wave envelope, and at the second stage, calculate the groupness factor using a known envelope. The previously proposed direct calculation of GFH by the SIWEH(12) function leads to significant errors. It is also shown that the change in the cutoff frequency of the Bartlett window has little effect on the calculated values of GFH, when the filter cutoff frequency changes $\pm 30\%$, the pair changes do not exceed 5%.

5 Acknowledgments

The work was carried out within the framework of the state task on the Theme: FNNN-2021-0004 “Fundamental studies of oceanological processes which determine the state and evolution of the marine environment influenced by natural and anthropogenic factors, based on observation and modeling methods”

References

1. X.J. Ma, Z.C. Sun, Z.M. Zhang, G.P. Yang, F. Zhou, J. Hydrodyn **23(2)**, 145–153 (2011)
2. L. Wang, Y. Tang, X. Zhang, J. Zhang, Ocean. Eng. **163**, 391–399 (2018)
3. B.Ö. Özbahçec, A thesis submitted to the graduate school of natural and applied sciences of Middle East Technical University. 157 p. (2004)
4. A.S. Zapevalov, Physical Oceanography **30(2)**, 160-170 (2023)
5. G. Dolgikh, V. Chupin, E. Gusev, IEEE Geoscience and Remote Sensing Letters, **17(5)**, 750–754 (2020)
6. J.R. Medina, R.T. Hudspeth, Coast. Eng. **14(6)**, 515–542 (1990)
7. R.T. Hudspeth, J.R. Medina, Proceedings of 21st International Conference on Coastal Engineering 884–898. Costa del Sol–Malaga, Spain, June 20–25, (1988)
8. M.S. Longuet–Higgins, Philosophical Transactions of the Royal Society of London. Series A. Mathematical and Physical Sciences **312(1521)**, 219–250 (1984)
9. G.Dong, Y. Ma, X. Ma, Cross–shore variations of wave groupiness by wavelet transform. Ocean. Eng. **35(7)**, 676–684 (2008)
10. W. Huang, S. Dong, Ocean Eng. **225**, 108745 (2021)
11. I.P. Shumeyko, M.I. Ozhiganova, N.N. Voronina, M.V. Kril, S.V. Narivonchik, Processes in GeoMedia **3(16)**, 1077-1081 (2018)
12. Y. Liu, S. Li, J. Waterw. Port, Coast. Ocean Eng. **144(6)**, 04018022 (2018)

13. E.R. Funke, E.P.D. Mansard, Hydraulics Laboratory Rept. LTR–HY–66, National Res. Council of Canada **54**, (1979)
14. A.S. Zapevalov, Processes in GeoMedia, Springer, Cham **3**, 139-145 (2021)
15. A.S. Zapevalov, A.N. Bol'shakov, V.E. Smolov, Oceanology 49(1), 31-38 (2009)
16. C.Guedes Soares, C. Cherneva Z., E.M. Antão Ocean Eng. **31**, 1147–1167 (2004)
17. A.V. Babanin, V.G. Polnikov, Physical Oceanography **3**, 241–245 (1994)
18. A.S. Zapevalov, A.V. Garmashov, Izvestiya, Atmospheric and Oceanic Physics **58(3)**, 263-269 (2022)
19. H. Mase, J. Waterway, Port. Coastal, and Ocean Engineering **115(1)**, 105–121 (1989)
20. A.V. Presnukhin Proceedings of the international conference dedicated to the 100th anniversary of the birth of Professor V.V. Longinov (September 14–17, 2009, Moscow). GEOS, Moscow, pp 31–33 (2009)