# Sea surface short-period roughness unsteadiness

*Irina* Shumeyko[1] and *Vyacheslav* Burdyugov[2]*

[1]Sevastopol State University, 299011Sevastopol, Russian Federation
[2]Marine Hydrophysical Institute RAS, 299011 Sevastopol, Russian Federation

**Abstract.** The effect of spatial inhomogeneity of sea waves on the accuracy of determining the level of the sea surface is studied. Based on the data of direct wave measurements, the variability of statistical characteristics of sea waves on scales smaller than the size of the area illuminated by the radar was analyzed. The case is considered when radio sounding is carried out at low angles of incidence by radar located on a spacecraft. It is shown that the displacement of the distribution median of surface elevations, which determines the skewness bias, is alternating. The absolute value of the median shift reaches two percent of the significant wave height. In most cases, the signs of skewness bias and electromagnetic bias coincide.

## 1 Introduction

Improving the accuracy of altimetric measurements of sea level remains the main task of modern satellite oceanography [1]. The main factor affecting the accuracy of sea level measurement is the change in the state of the sea surface. Due to this factor, the error in determining the sea level was called sea state bias (SMB). SSB is on the order of a few tens of centimeters, it must be properly corrected [2].

SSB is a combination of three components, they are skewness bias, electromagnetic bias, an instrument tracker bias [3]. In this paper, we will limit ourselves to analyzing the first component, which arises as a result of the deviation of the distribution of sea surface elevations from the Gauss distribution [4]. The median of the distribution of sea surface elevations does not coincide with the level of the undisturbed surface, as a result, an error occurs in determining the sea level [5].

The shape of the reflected pulse radio altimeter is approximated by convolution of functions describing the technical characteristics of the altimeter, measurement conditions and the space-time structure of the sea surface [6]. Currently, this approximation (Brown's model) is written in the form of [7, 8]

$$V(t) = F(t) * S(t) * q(t),$$

(1)

* Corresponding author: burdyugov@mail.ru

where $t$ is the time; $F(t)$ is the shape of the pulse reflected from a flat surface; $S(t)$ is the shape of the probing pulse; $q(t)$ is the function associated with the density of the specular reflection points; the symbol $*$ means convolution. The function is obtained by converting the probability density function $P(\xi)$ of sea surface elevations $\xi$ [9]. The transformation is carried out on the basis of an equation linking spatial and temporal $t$ coordinates

$$\xi = (c/2)t,$$

(2)

where $c$ is the speed of light. Equation (2) is linear, so the even statistical moments of the functions $q(t)$ and $P(\xi)$ coincide. Odd statistical moments have opposite signs, because lowering the sea surface level leads to an increase in the return time of the altimeter radio pulse.

The calculation of the sea surface level based on the model (1) assumes that the field of surface waves is spatially homogeneous. The spatial heterogeneity of the surface wave field leads to errors in determining sea level. The appearance of spatial heterogeneity is caused by the group structure of gravitational waves [10, 11], as well as the variability of the characteristics of individual groups of waves. In the presence of abnormally high waves (freak waves), both a local change in wave energy and a change in the senior cumulants of sea surface elevations occur [12, 13]. The purpose of this work is to analyze the short-period variability of the cumulants of sea waves of the second, third and fourth orders.

## 2 Materials and methods

### 2.1 Data for simulation

The problem of comparing the results of remote sensing and contact wave measurements in situ is that in the first case, the characteristics of the sea surface are determined, averaged over space, in the second, averaged over time. We will assume that the space and time averaging are equivalent. Spatial and temporal scales can be linked using the dispersion relation for surface waves

$$\omega^2 = gk,$$

(3)

where $\omega$ and $k$ are the circular frequency and the wavenumber of the surface wave, $g$ is the gravitational acceleration.

To determine the spatial and temporal scales of the simulated area, consider the characteristics of the spacecraft SeaSat-1 [14]. The diameter of the area on the surface illuminated by the radar at an orbit altitude of 800 km and a beam width of 1.6° is ~22 km. With a length of dominant waves of 60 m (characteristic long waves in the Black Sea), their phase velocity is $\omega/k \approx 9.7$ m/s. In this case, the wave characteristics averaged over 40 min approximately correspond to the characteristics obtained by averaging over space.

For numerical analysis, we will use wave measurement data obtained on the stationary oceanographic platform of the Marine Hydrophysical Institute. Measurements were carried out in 2005 and 2006 in the summer and autumn periods. For measurements, a wave-recording gauge was used, the sensor of which is a vertically stretched string [15].

## 2.2 Statistical distribution of sea surface elevations

When modeling the sea surface, the probability density function is usually constructed on the basis of specified statistical moments (or cumulants). Statistical moments of order $n$ are given by the equations

$$\mu_n = \overline{\xi^n}.$$

(4)

If a normalization of a random variable by its root-mean-square value is introduced, then $\mu_3$ is skewness, $\mu_4 - 3$ is excess kurtosis.

Two types of models are used to approximate the probability density function of sea surface elevations. The first type is models constructed on the basis of a series expansion by Chebyshev-Hermite polynomials [16]. Approximations are usually used in the form [8]

$$P_{GC}(\xi) = \frac{\exp\left(-\dfrac{\xi^2}{2D_\xi}\right)}{\sqrt{2\pi\mu_2}}\left[1 + \frac{\mu_3}{6}H_3\left(\frac{\xi}{\sqrt{\mu_2}}\right) + \frac{\mu_4 - 3}{24}H_4\left(\frac{\xi}{\sqrt{\mu_2}}\right)\right],$$

(5)

where $H_n(x)$ are Chebyshev-Hermite polynomials of degree n. Chebyshev-Hermite polynomials of the third and fourth degree are described by the equations

$$H_3(x) = x^3 - 3x,$$

(6)

$$H_4(x) = x^4 - 6x^2 + 3.$$

(7)

The second type of models describing the statistics of sea surface elevation is the Gaussian mixture. The probability density function is given as [17]

$$P_S(\xi) = \frac{\alpha_1}{\sqrt{2\pi}\sigma_1}\exp\left(-\frac{(\xi - m_1)^2}{2\sigma_1^2}\right) + \frac{\alpha_2}{\sqrt{2\pi}\sigma_2}\exp\left(-\frac{(\xi - m_2)^2}{2\sigma_2^2}\right),$$

(8)

where the model parameters are determined by a system of equations

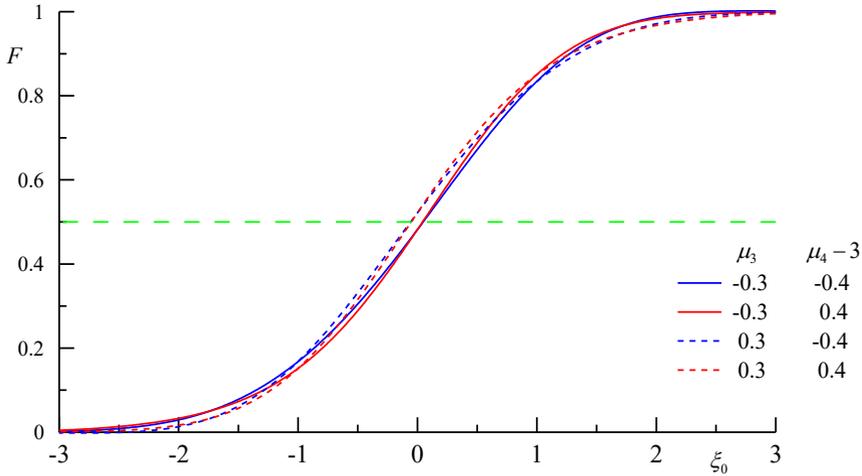$$\begin{cases} \alpha_1 m_1 + (1 - \alpha_1)m_2 = 0 \\[2mm] \alpha_1\left(m_1^2 + \sigma_1^2\right) + (1 - \alpha_1)\left(m_2^2 + \sigma_2^2\right) = 0 \\[2mm] \alpha_1\left(m_1^3 + 3m_1\sigma_1^2 - \mu_3\right) + (1 - \alpha_1)\left(m_2^3 + 3m_2\sigma_2^2 - \mu_3\right) = 0 \\[2mm] \alpha_1\left(m_1^4 + 6m_1^2\sigma_1^2 + 3\sigma_1^4 - \mu_4\right) + (1 - \alpha_1)\left(m_2^4 + 6m_2^2\sigma_2^2 + 3\sigma_2^4 - \mu_4\right) = 0 \end{cases}$$

(9)

The procedure for calculating the model parameters is described in [18].

To estimate the median of the statistical distribution offset, consider the distribution function $F(\xi_0)$, which is the probability that the random variable $\xi$ does not exceed $\xi_0$

$$F(\xi_0) = \int_{-\infty}^{\xi_0} P(\xi)\,d\xi .$$

(10)

Calculated for several values $\mu_3$ and $\mu_4$ distribution functions are shown in Fig. 1.



**Fig. 1.** Distribution function $F(\xi_0)$

The median of the statistical distribution $\xi_M$ is determined from the condition

$$\int_{-\infty}^{\xi_M} P(\xi)\,d\xi = 0.5 .$$

(11)

The heights of sea waves are usually characterized by a significant wave height $H_S$. A significant wave height is the average height of one third of the heights of the highest waves. With the second statistical moment $H_S$ is related by the relation
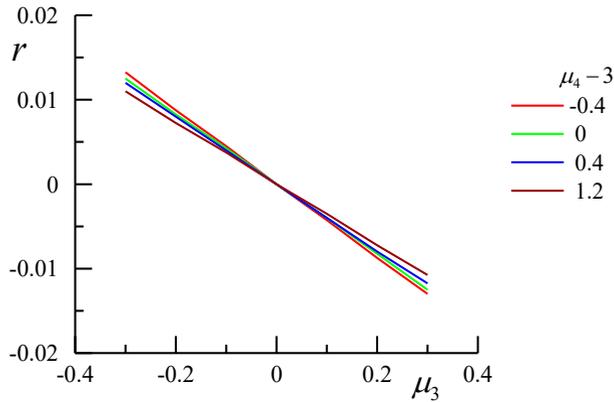
$$H_S = 4\sqrt{\mu_2} .$$

(12)

It is advisable to determine the displacement $\xi_M$ as a part of the significant wave height

$$\xi_M = rH ,$$

(13)

where r is a dimensionless parameter that defines the displacement of the median as part of a significant wave height.

The main factor determining the median bias is the skewness. However, as can be seen from Fig. 2., the excess kurtosis also affects the displacement. In case of $\mu_3 \neq 0$, the median of the distribution is shifted relative to the zero. This shift is shown in Fig. 2. Since both positive and negative values $\mu_3$ are observed in field conditions [19, 20, 21], the

median shift is also alternating in sign. Note that the values $\mu_3 < 0$ were also obtained by modeling waves in laboratory conditions [22].



**Fig.2.** Displacements of the distribution median of sea surface elevations

## 3 Discussion

### 3.1 Short-period variability of sea waves

Typical changes in the statistical characteristics of surface waves obtained by averaging over 5 minutes with a total duration of the measurement session of 40 minutes are shown in Fig. 3. Relative to its average value, the parameter $\sqrt{\mu_2}$ and, accordingly, $H_S$, as a rule, changes by less than 10%. This makes it possible to consider the wave energy stationary at a given time interval.

At the same time, noticeable changes in skewness and kurtosis are observed. Within 40 min, both skewness and kurtosis can change sign. The parameter $r$ can take both positive and negative values, which corresponds to a change in the sign of skewness. With this $\overline{H}_S = 136$ cm these changes in the statistical characteristics of surface waves lead to shifts in the median distribution of the about of one centimeter. Here, the line at the top means a forty-minute averaging.
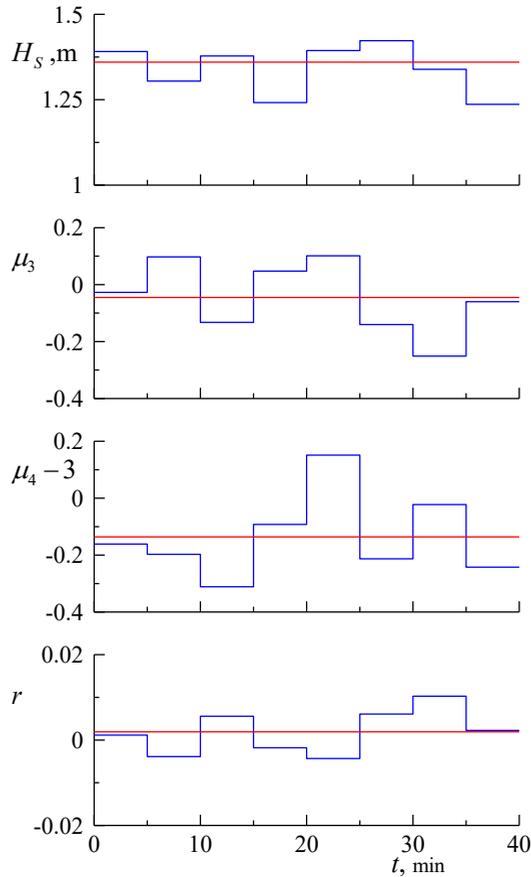
### 3.2 Electromagnetic bias

Note that spatial inhomogeneities of surface waves affect another component of SSB, it affects the electromagnetic bias. This component arises as a result of hydrodynamic modulation of short waves by a long wave. As a result wave troughs being better radar reflectors than wave crests. The mean scattering level does not coincide with the distribution median of surface elevations. It is shifted towards the wave troughs.

In most cases, the skewness of surface waves is positive. In these cases, the signs skewness bias and electromagnetic bias coincide. SSB depends on the significant wave height, if $H_S$ increases, then SSB also increases

$$\text{SSB} = \alpha H_S,$$

<div align="right">(14)</div>

where the values of the dimensionless coefficient $\alpha$ lie within 0.02-0.05 [23].



**Fig. 3.** Local changes in the characteristics of sea waves: blue line – averaging 5 min; red line – averaging 40 min

## 4 Conclusion

The variability of the state of the sea surface is the main factor determining the error of measuring sea level. Usually, when analyzing this error, estimates of wave characteristics determined over a sufficiently long period of time are used. The surface height statistics area assumed to be constant over the total area illuminated by the radar.

The characteristics of real sea waves are heterogeneous in space and time. It is shown that the displacement of the median of the surface elevation distribution calculated for short time intervals (5 min), which determines the skewness bias, is alternating. The absolute value of the median shift reaches two percent of the significant wave height. It was also found that in most cases the signs of skewness bias and electromagnetic bias coincide.

## 5 Acknowledgments

research of processes of the marine environment formation and evolution and on mathematical modeling using data of remote and contact measurements".

## References

1. S.A. Lebedev, I.V. Gusev, Sovremennye problemy distantsionnogo zondirovaniya Zemli iz kosmosa, 18(2), 18-35 (2021)

2. Y. Cheng, Q. Xu, L. Gao, X. Li, B. Zou, T. Liu, Remote Sensing, **11(10)**, 1176 (2019)

3. S.I. Badulin, V.G. Grigorieva, P.A. Shabanov, V.D. Sharmar I.O. Karpov, Advances in Space Research **68(2)**, 978-988 (2021)

4. A.S. Zapevalov, Izvestiya, Atmospheric and Oceanic Physics **48(2)**, 200–206 (2012)

5. N. Pires, M. Fernandes, C. Gommenginger, R. Scharroo, Remote Sensing. **8(7)**, 576 (2016)

6. G.S. Brown, IEEE Trans. Antennas Propagat **AP-25**, 67-74 (1977)

7. K.V. Pokazeev, A.S. Zapevalov, V.V. Pustovoytenko, Moscow University Physics Bulletin **68(5)**, 420-425 (2013)

8. V.M. Burdyugov, Processes in GeoMedia **3(33)**, 1724-1729 (2022)

9. G.S. Hayne, IEEE Transactions on Antennas and Propagation. **AP-28,** 687-692 (1980)

10. J.R.Medina, R.T. Hudspeth, Coast. Eng. **14(6)**, 515–542, 1990

11. W. Huang, S. Dong, Ocean Eng. **225,** 108745 (2021)

12. C. Guedes Soares, Z. Cherneva, E.M.Antão, Ocean Engineering **31,** 1147–1167 (2004).

13. V.A. Ivanov, V.A. Dulov, S.Yu. Kuznetsov, S.F. Dotsenko, M.V. Shokurov, Ya.V. Saprykina, V.V. Malinovsky, V.G. Polnikov, Geography, Environment, Sustainability **5(1)**, 84–111 (2012)

14. G.S.Hayne, IEEE Trans. Antennas Propagat. **AP-28,** 687-692 (1980)

15. A.S. Zapevalov, A.N. Bol'shakov, V.E. Smolov, Oceanology **51(3)**, 406–413 (2011)

16. M.J. Kendall, A. Stewart, The Advanced Theory of Statistics. **I**. Distribution theory. (1958)

17. N.N. Aprausheva, S.V. Sorokin, Pattern Recognition and Image Analysis **23(3)**, 341-347 (2013)

18. A.S. Zapevalov, A.S. Knyazkov, Physical Oceanography **29(4)**, 395-403 (2022)

19. E. Bilyay, B.O. Ozbahceci, A.C. Yalciner, Nat. Hazards Earth Syst. Sci. **11(3)**, 659–666 (2011)

20. A.K. Jha, S.R. Winterstein, Proceedings of the 19th International Offshore Mechanics and Arctic Engineering Symposium. ASME. Paper No. OMAE 00-6125 (2000)

21. A.S. Zapevalov, A.V. Garmashov, Izvestiya, Atmospheric and Oceanic Physics **58(3)**, pp 263-269 (2022)

22. N.E. Huang, S.R. Long, J. Fluid Mech. 101(1), 179–200 (1980)

23. C. Gommenginger, M. Srokosz, C. Bellingham, H. Snaith, N. Pires, M.J. Fernandes, N. Tran, D. Vandemark, T. Moreau, S. Labroue, R. Scharroo, Sea state bias: 25 years on, Presentation at and Abstract in: 25 years of progress in radar altimetry, 24-29 September 2018, Ponta Delgada, Sao Miguel Island Azores Archipelago, Portugal. (2018)