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Proportion calculation of the defect parts in track switching equipment

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Abstract. This paper presents a direct calculation of the average value of defective elements of point products. Production process control is considered using the example of the control of point products. The calculation of the first four moments in a binomial distribution of the number of defective elements of point products is made. The standard error of the fraction of defective elements is considered in detail.

1 Introduction

The quality indicator of point products, regardless of the properties of individual products, is the proportion of defect parts in the batch, expressed as a fraction of one or as a percentage. The lower the proportion of defect parts in the batch, the higher the quality of the point system.

When controlling the quality of the point product, the manufacturer is faced with two problems [1, 2]. Firstly, they must organise the control of the production process in such a way that the percentage of unsatisfactory or defect parts is not too high.

Secondly, they must ensure that there is no possibility that batches of products containing too high a percentage of defect parts will leave the factory. When talking about quality control, it is therefore necessary to consider two sides of this task, namely 1) control of the production process and 2) control of the manufacturing process: 1) control of the production process and 2) control of the manufactured point products [3, 4]. At first glance, it may seem that if the production process control is properly organised, there is no reason to worry about the quality of the arrow product when it is marketed or purchased. To a certain extent, this is certainly true: the better the production control is, the lower the costs of organizing the control of the manufactured point products are.

However, even when the production process is satisfactory and the number of defective items in a large batch produced over a long period of time is low, it may occur that individual batches have a high percentage of defects and should be considered unsatisfactory [5, 6].

2 Research methods

The mean value of the distribution of the number of defective arrowheads per sample at k samples of a volume of n equals $\bar{d} = \sum d/k$. Mathematical expectation d, denoted by the

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symbol $E(d)$, is the average number of defective arrowheads in all possible volume samples n, i.e. the mean value for the probability distribution d [1]. This value is obtained by multiplying each possible value d on the probability of its occurrence and the addition of these works. Thus,

$$E\ (d) = \sum [d\boldsymbol{P}(d)] = 0 \cdot 0.216 + 1 \cdot 0.432 + 2 \cdot 0.288 + 3 \cdot 0.064 = 1.2$$

It is also interesting to note the fact that

$$E\ (d) = nP = 3 \cdot 0.4 = 1.2$$

Similarly for the proportion of defect parts:

$$\bar{p} = \sum p/k \text{ and } E\ (d) = \sum [p\boldsymbol{P}(p)] = 0.000 \cdot 0.216 + 0.333 \cdot 0.432 + 0.667 \cdot 0.288 + + 1.000 \cdot 0.064 = 0.4$$

It is interesting to note that $E\ (p) = P = 0.4$. It is clear that since each term in the formula for $E\ (p)$ is exactly one third of the corresponding term in the formula for $E\ (d)$, then the resulting value is also only one-third [2]. Thus, in the general case:

$$E(p) = E(d)/n.$$

The direct calculation of the various moments can be made in the order shown in Table 1 (for the number of defective point guiding components) and Table 2 (for the proportion of defective point guiding components). Note that $\mu_r(d) = n^r \mu_r(p)$ and $\sigma_d^r = n^r \sigma_p^r$, but $\alpha_r(d) = \alpha_r(p)$.

Table 1. Calculating the first four moments α3 and α4 with a binomial distribution of the number of defect parts ($n = 3$, $P = 0.4$)

d	P(d)	d - E(d)	[d-E(d)] P(d)	$[d - E(d)]^2$ P(d)	$[d - E(d)]^3$ P(d)	$[d - E(d)]^4$ P(d)
0	0.216	-1.2	-0.2592	0.3110	-0.3732	0.4478
1	0.432	-0.2	-0.0864	0.0173	-0.0035	0.0007
2	0.288	0.8	0.2304	0.1843	0.1474	0.1179
3	0.064	1.8	0.1152	0.2074	0.3733	0.6719
$\mu_r(d)$			0.0000	0.72	0.144	1.238
σ_d^r			0.8485*)	0.72*)	0.6109	0.5184
$\alpha_r(d)$			-	-	0.236	2.39
*) 0.8485 = $\sqrt{0.72}$ (0.72 is the exact value).						

Table 2. Calculating the first four moments α3 and α4 with a binomial distribution of the proportion of defect parts ($n = 3$, $P = 0.4$)

p	**P**(p)	p - P	[p-P] **P**(p)	$[p\text{-}P]^2$ **P**(p)	$[p\text{-}P]^3$ **P**(p)	$[p\text{-}P]^4$ **P**(p)
0.000	0.216	-0.400	-0.0864	0.03456	-0.01382	0.00553
0.333	0.432	-0.067	-0.0288	0.00192	-0.00013	0.00001
0.667	0.288	0.267	0.0768	0.02048	0.00546	0.00146
1.000	0.064	0.600	0.0384	0.02304	0.01382	0.00829
$\mu_r(p)$			0.0000	0.08	0.00533	0.0153

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σ_p^r	0.2828*)	0.08*)	0.02263	0.0064
$\alpha_r(p)$	-	-	0.236	2.39
*) 0.2828 = $\sqrt{0.08}$ (0.08 is the exact value).				

In order to simplify comparisons, the formulas for the parameters that describe the theoretical distributions of the number and proportion of defective arrowheads are respectively given in the two columns below (Table 3).

The kurtosis coefficients are also of some interest:

$$\mu_4(d) = 3(nPQ)^2 + (1 - 6PQ)(nPQ),$$
$$\mu_4(p) = 3(\frac{PQ}{n})^2 + (1 - 6PQ)\frac{PQ}{n^3},$$
$$\alpha_4(d) = \alpha_4(P) = 3 + \frac{1-6PQ}{nPQ},$$

If P = 0.211325, till $\alpha_4 = 3$
If P < 0.211325, till $\alpha_4 > 3$ and the distribution is island-shaped.
If P > 0.211325, till $\alpha_4 < 3$ and the distribution is flattened.

Table 3. The formulas for the parameters that describe the theoretical distributions of the number and proportion of defective arrowheads

Numerical characteristics of the distribution	Number of defect parts	Proportion of defect parts
Mathematical ob-gyn ….	*E (d) = nP*	*E(p) = P*
Dispersion or second momentum. . .	$\mu_2(d) = \sigma_d^2 = nPQ$	$\mu_2(p) = \sigma_p^2 = \frac{PQ}{n}$
Third point. . .	$\mu_3(d) = Q - P)nPQ$	$\mu_3(p) = (Q - P)\frac{PQ}{n^2}$
Asymmetry coefficient …..	$\alpha_3(d) = \alpha_3(p) = \frac{Q-P}{\sqrt{nPQ}}$	

In the case of arrow products [7, 8], i.e. for the case where n = 3 and P = 0.4, we get the following numerical values:

$$E(d) = 3 \cdot 0.4 = 1.2; \qquad E(p) = 0.4;$$
$$\mu_2(d) = \sigma_d^2 = 3 \cdot 0.4 \cdot 0.6 = 0.72; \qquad \mu_2(p) = \sigma_p^2 = \frac{0.4 \cdot 0.6}{3} = 0.08;$$
$$\sigma_d = \sqrt{0.72} = 0.8485; \qquad \sigma_p = \sqrt{0.08} = 0.2828;$$
$$\mu_3(d) = (0.6 - 0.4) \cdot 0.72 = 0.144; \qquad \mu_3(p) = 0.2\frac{0.4 \cdot 0.6}{9} = 0.00533;$$
$$\alpha_3(d) = \alpha_3(p) = \frac{0.2}{0.8485} = 0.236.$$

The standard error of the proportion of defective arrowheads is such an important characteristic in statistical data processing in general and quality control in particular that it should be considered in more detail [9, 10].

If an infinite general population consists of arrow items that are either good or defective, it can be assumed that the different items are assigned values of either 0 (good) or 1 (defective) [11, 12]. Thus, if the fraction of defective arrow items is 0.4, this distribution can be written as follows:

X	**P** (X)
0	0.60
1	0.40
Total	1.00

The mean and standard deviation for such a population can be calculated in the usual way:

X	**P**(X)	X**P**(X)	X^2**P**(X)
0	0.60	0.00	0.00
1	0.40	0.40	0.40
Total	1.00	E(X)= 0.40	E (X^2)= 0.40
The square of the average value …			$[\mathrm{E}(X)]^2 = 0.16$
Dispersion ……………………			$\sigma_X^2 = 0.24$

Thus, $\sigma_X^2 = 0.24$ and the same value is equal to PQ. That this should indeed be the case is easy to see if you consider the order of calculation σ_X^2:

$$\sigma_X^2 = (0)\cdot Q + (1)\cdot P - P^2 = P - P^2 = P(1-P) = PQ.$$

We can now calculate, in the usual way, the standard error - the variance of the mean of the above distribution:

$$\sigma_{\bar{X}}^2 = \frac{\sigma_X^2}{n}$$

Since $\bar{X} = p$ and $\sigma_X^2 = PQ$, it is obvious that

$$\sigma_{\bar{X}}^2 = \sigma_p^2 = \frac{PQ}{n} \quad \text{and} \quad \sigma_p = \sqrt{\frac{PQ}{n}}$$

Consider the correlation σ_p, from n and P. From the formula above, it follows that σ_p has a maximum value when $P = Q = 0.5$ and varies inversely with $\sqrt{n}$. Nature of dependency σ_p from n and P can be identified more clearly by referring to Table 4, which shows the values σ_p for some specific values n and P. Values P in the table are only up to 0.5, because for the values P, supplementing these values to one, σ_p will have the same value [6].

Table 4. Values σ_p for some values n and P

n	P or Q							
	0.001	0.01	0.05	0.10	0.20	0.30	0.40	0.50
5000	0.00045	0.0010	0.0031	0.0042	0.0057	0.0065	0.0069	0.0071
1000	0.0010	0.0033	0.0069	0.0095	0.013	0.014	0.015	0.016
500	0.0014	0.0044	0.0097	0.013	0.018	0.020	0.022	0.022
50	0.0032	0.010	0.022	0.030	0.040	0.046	0.049	0.050
100	0.0045	0.014	0.031	0.042	0.057	0.065	0.069	0.071
10	0.010	0.031	0.069	0.095	0.126	0.145	0.155	0.158
5	0.014	0.044	0.097	0.134	0.179	0.205	0.219	0.224

Although calculating the value using the formula $\sigma_p = \sqrt{PQ/n}$ and not very difficult, but in some cases, it is necessary to determine approximate values σ_p, with a minimum of effort [13].

To this end, we slightly modify the formula for σ_p:

$$\sigma = \sqrt{\frac{PQ}{n}} = \frac{\sqrt{P(1-P)}}{\sqrt{n}}$$

$$lg\sigma_p = \frac{1}{2}lg[P(1-P)] - \frac{1}{2}lgn$$

In the last formula $lg\sigma_p$ is a linear function *lg* [*P*(1 – P)] and *lg n*. Consequently, it is easy to construct a nomogram to determine from these last two values the numerical value $lg\sigma_p$. If, on the other hand, a nomogram on a logarithmic scale is plotted, it can be used to determine the value directly σ_p by value *P* and *n* [14].

Formula for the asymmetry coefficient of a binomial distribution

$$\alpha_3(d) = \alpha_3(p) = \frac{Q-P}{\sqrt{nPQ}}$$

indicates that asymmetry is positive when $Q > P$ (which is usually the case in quality control), and a negative one, where $P > Q$. In the case of $P = Q = 0.5$, there is no asymmetry [8]. If the value is unchanged *n* asymmetry when the deviation increases *P* from 0.5 is also increasing as $|Q - P|$ becomes greater and the value in the denominator $\sqrt{PQ}$ decreases. If the value is unchanged *P* asymmetry changes inversely $\sqrt{n}$. Thus, if $P < 0.5$, asymmetry increases with a decrease in both *P,* as well as *n.*

Nature of asymmetry coefficient dependence α_3 in a binomial distribution from *n* and *P* can easily be seen when looking at Table 5, which shows the values α_3 at different values *n* and *P.* It is interesting to note that in the case of $n \geq 500$, asymmetry is insignificant, except in cases where *P* or *Q* less 0.05 [15].

Table 5. Absolute values a_3 for a binomial distribution at different values *n* and *P*

n	*P* or *Q*							
	0.001	0.01	0.05	0.10	0.20	0.30	0.40	0.45
5000	0.45	0.14	0.058	0.038	0.021	0.012	0.0058	0.0028
1000	1.00	0.31	0.13	0.084	0.047	0.028	0.013	0.0064
500	1.41	0.44	0.18	0.12	0.067	0.039	0.018	0.0090
100	3.16	0.98	0.41	0.27	0.15	0.087	0.041	0.020
50	4.47	1.39	0.58	0.38	0.21	0.12	0.058	0.028
10	9.99	3.11	1.31	0.84	0.47	0.28	0.13	0.064
5	14.12	4.40	1.85	1.19	0.67	0.39	0.18	0.090

Applying the logarithms of the quantities in the formula for α_3 (*p*), this formula can be converted to the linear form:

$$\alpha_3(p) = \frac{Q-P}{\sqrt{nPQ}} = \frac{1-2P}{\sqrt{P(1-P)}} \cdot \frac{1}{\sqrt{n}}$$

$$lg\,\alpha_3\,(p) = lg(1-2P) - \frac{1}{2}lg[P(1-P)] - \frac{1}{2}lg\,n$$

3 Conclusion

In practice, acceptance inspection plans for a consignment of track switching equipment products are more likely to involve hypothesis testing than parameter estimation. This is mainly because it is easier to classify products as good or defective than to accurately measure them, and because counting the number of defective items is easier than measuring and averaging the population of values. However, these advantages may be negated if a large sample is required to decide on the quality of a batch of arrow products when using quality attributes.

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