Energetically optimal dimensions of parallelepiped-shaped industrial structures

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Abstract. Technical structures with a parallelepiped form are frequently used in various areas of industry. For example, as fluid containers, ice and minerals piles, underground and surface storages and cooling chambers. A task of determining the optimum dimensions of the structure to minimize the energy expenses to support a normative temperature in the structure arises. The minimum surface area of the structure at a given volume is accepted as a criterion of optimality. A problem of unconstrained optimization for a structure with a parallelepiped form was formulated and solved. Equations to determine the optimum dimensions of width, height and length of the structure were obtained. It was determined that a cube is the most optimal form.

1 Introduction

Saving energy resources in various areas of industry is a current task to which significant attention of the scientific and engineering community is devoted [1- 6]. Technical structures with a parallelepiped shape are widely used in industry. Among them are, for example, fluid containers, piles of minerals or ice, surface or underground storage chambers and coolers. As a rule, the main requirement for such structures is to have the minimum surface area with maximum capacity. This requirement reduces the amount of resources needed for construction and additional materials for thermal insulation and decreases the heat loss with the external environment. The heat stream towards or away from the structure is directly proportional to its surface area.

Therefore, to save energy resources, the heat exchange surface area should be minimized while maintaining the capacity. For example, water tanks need to be heated over the winter or thermally insulated [7]. Storing ice all year for the needs of rail transport also requires thermal insulation [8]. Leaching of gold-containing rocks in the winter period requires isolation from the environment through creation of a heated cupola. This process is very energy-intensive, especially in the regions of the far north where the main gold deposits are located [9, 10]. The choice of an optimal form of the structure as a way of reducing energy and economic expenses to support the normative thermal conditions is especially important

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when building underground and surface storages and cooling chambers [11-14]. That is, the task is applicable for many types of structures regardless of their purpose.

The aim of the research was to determine the optimal geometrical dimensions (width, height and length) of a rectangular parallelepiped structure with limitations of one of the properties for technical or technological reasons.

2 Materials and methods

An objective function is constructed and its extremes surveyed. That is, an optimization problem without conditions will be solved. The objective function has the following form:

$$S = 2(ka\text{^}2 + V(1+k)/ka),\ k = h/a,\ V = ahl \tag{1}$$

Where a is width, m, h is height, m, l is length, m, V is volume, m^3, S is surface area, m^2.

The conditions of optimal parameters for a minimum surface area can be found from the expression:

$$\frac{dS}{dk} = 0 \text{ and } \frac{d}{dk}\left(\frac{dS}{dk}\right) > 0 \text{ at } k = k_{opt} \tag{2}$$

At the point $k = k_{\text{opt}}$, the minimum of the objective function (1) is located. From the equation (2) it follows that

$$k_{opt} = (\sqrt{Va})/a \tag{3}$$

The optimum length and height at the set width and volume can be found from expressions:

$$l = V/(ak) \text{ and } h = ka \tag{4}$$

Where $k = k_{\text{opt}}$ is determined using the equation (3). At set volume and width of the parallelepiped, the optimum height and length, minimizing its surface area, are obtained using the equations:

$$h = \sqrt{V/a} \text{ and } l = \sqrt{V/a} \tag{5}$$

This means that at set volume of the object and one fixed geometric parameter, the other two parameters must be equal to and determined using the equation (5). Any other values of the height and width will cause an increase in total surface area and thus an increase in material and energy expenses to maintain the required thermal regime in the structure. A specific case of a parallelepiped will be considered, with properties $k = 1$, $h = a$ (square face). In this case, the objective function has the form:

$$S = 2(a\text{^}2 + 2V/a) \tag{6}$$

A minimum analysis of the function leads to the expression:

$$a = \sqrt[3]{V} \tag{7}$$

The optimum length in this case will be determined as:

$$l = V/a^2 = V/\sqrt[3]{V^2} = \sqrt[3]{V} \tag{8}$$

3 Results and discussion

The comparison of the equations (7) and (8) shows that the optimal geometric parameters minimizing the surface area is the equality $h = l = a$. This means that a cube is the most optimal shape of the structure according to the criterion of minimum surface area at a given volume. If, for some reason, it is not possible to build the structure in the form of a cube, the optimal geometric properties should be determined using the formula (5). To illustrate the obtained results, calculations with variating properties were done and are presented as charts in the figures 1, 2 and 3. The figure 1 shows a 3D chart of change in the objective function depending on the parameters *k* and *a*.

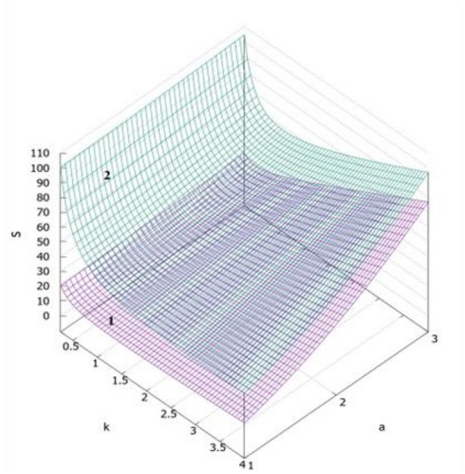

Fig. 1. Change in surface area of the object (S, m^3) at varying width depending on the coefficient of width to height (*k*) for various volumes of the object: 1 - 1.0 m^3, 2 - 5.0 m^3.

The charts show that the dependence of the surface area on the parameter k is not linear and at a given point there is a minimum of the objective function. This is typical for structures with different volumes (see surfaces 1 and 2). This regularity is visible on the figure 2. The figure 2 shows curves characterizing the change in the surface area of an object with a width of 3 m depending on the coefficient of width to height (*k*) for various volumes of the object.

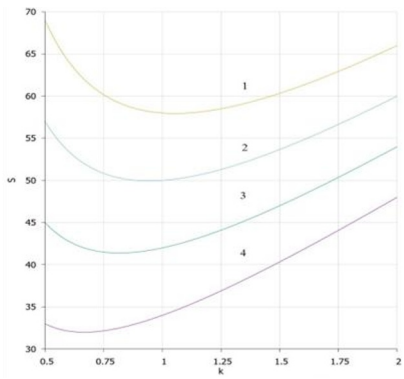

Fig. 2. Change in surface area of an object (S, m^3) with a width of 3 m depending on the coefficient of width to height (k) for various volumes of the object. 1 - 5.0 m^3, 2 - 4.0 m^3, 3 - 3.0 m^3, 4 - 2.0 m^3.

The curves in the picture show that the minimum of the objective function at a given width of the object significantly depends on the volume of the object. The minimum value of k increases with the object volume. For example, for an object with the volume of 2.0 m^3

(curve 4), the minimum value of the objective function is within the interval of k of 0.5 to 0.75. For an object with a volume of 5.0 m^3 (curve 1), the minimum value of the objective function is within the values of k of 1.0 to 1.25. The charts in figure 3 reflect the change in optimum value of the coefficient of width to height that determines the minimum of the objective function (k_{opt}) depending on the volume of the object at different values of fixed width.

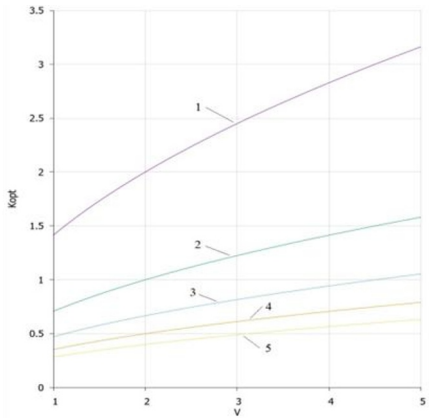

Fig. 3. The optimum value of the coefficient of width to height (k_{opt}) depending on the volume of the object (V, m^3) at different values of width (a, m). 1 - 1.0 m, 2 - 3.0 m, 4 - 4.0 m, 5 - 5.0 m.

An analysis of the chart shows that the dependence of the coefficient k_{opt} on the volume of the object is stronger the smaller is the set width. For example, when the volume of the object changes from 1.0 m^3 to 3.0 m^3 at a fixed width of 5.0 m, the optimum coefficient increases by 1.3x. At a fixed width of 1.0 m, it increases by 1.7x. That is, the greater is the fixed width of the object, the less the k_{opt} coefficient depends on its volume. A gently sloping character of the curves in the figure 3 as the width of the object increases shows this.

4 Conclusion

A task of determining the optimum dimensions of an industrial structure with a parallelepiped shape with the aim of minimizing energy expenses (heat losses) on supporting the thermal regime was solved. Equations to determine the geometric parameters of the structure at a given volume were obtained. It was found that given a volume and one fixed geometric parameter, the other two parameters have to be equal and determined using the obtained equation. Any other geometric dimensions will lead to an increase in the surface area of the structure and thus towards increase in material and energy expenses to support its thermal regime. It was demonstrated that the most optimal shape of the structure is a cube. Further research should be directed towards comparative calculations of the energy and economic expenses to support a normative thermal regime in structures with various geometric parameters.

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