Heat exchange between the heating element and its shell under the boundary condition of the fourth kind

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Abstract. In this article, the question of finding the temperature field of a heating element made in the form of a cylinder enclosed in a shell of cylindrical shape is considered. It is assumed that the thermal contact between the solid–coating systems is ideal. In this case, heat exchange occurs under boundary conditions of the third and fourth kind. In this paper, based on the methods of differentiation and integration, a solution was obtained to the problem of the distribution of the temperature field in both bodies. The resulting solution has an analytical form containing quadratic and logarithmic functions.

1 Introduction

The energy use of heat is based on processes that convert heat into mechanical work. The technological use of heat is based on the realization of heat for the purposeful change of physic-chemical properties during the implementation of various technological processes. Devices in which direct heat supply is used for technological purposes include various furnaces, dryers, heaters, heaters, etc. The science that studies the patterns of heat exchange between bodies is called the theory of heat transfer [1-7]. In the mathematical theory of thermal conductivity, an important place is occupied by studies of heat transfer processes in solids with a cylindrical channel, the surface of which is subject to a given mode of thermal action. Separately, heating elements made in the form of a cylinder can be distinguished. Such elements have great mechanical strength and are widely used in heat exchangers [8]. They can be both electrical and fuel-generating elements of a nuclear reactor [9-15].

There are two types of heating elements. Open heating elements (spiral) – it is assumed that the spiral through which the electric current passes is in direct contact with water without any additional insulators. The spiral heats up and thereby heats the water. This type of heating element is very simple in design, functional in operation, allows the manufacturer to save money. To ensure electrical safety, the tubes inside which the spiral is located have to be made of plastic. Such a design has a huge drawback – once an air bubble gets inside the device, the spiral almost instantly burns out, the device fails. As a rule, such heating elements

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are used in all three-phase flowing water heaters (due to their small dimensions and due to good heat transfer).

Closed-type heating elements (TEN). The electric spiral is located inside a kind of sealed "package" and has no direct contact with the heated medium. An example of a closed-type heating element is the good old Soviet boiler. If you look at the section of such an element, we will see that there is a copper or brass hollow tube outside (it is she who contacts the flowing water), inside of which there is a spiral filled and baked with a special composition. The contact of the spiral and the tube is not allowed, respectively, the presence of an electric current in the water flowing through the device is excluded. Elements of such a design are considered more electrically safe. Such heating elements are also called TEN. The areas of application of TEN are huge – these are kettles, various electric heaters, technological and industrial equipment.

2 Main part

The main task of this work is to find the distribution of the temperature field of a heating element having the form of a cylinder, as well as the shell field, also cylindrical, surrounding the element under boundary conditions of the fourth kind (fig. 1). In this case, heat exchange on the outer side of the shell occurs at α = const.

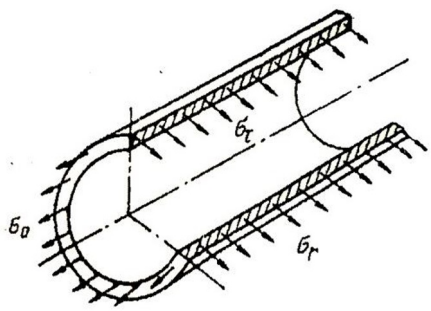

Fig. 1. The heating element in the shell.

To determine the temperature field of a cylindrical heat-releasing element, we solve the equation of thermal conductivity. Let the same heat source with a specific power q_v operate inside. In this case, the two-dimensional differential equation of thermal conductivity will take the form

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{q_v}{\lambda_1} \tag{1}$$

where: λ_1 is the coefficient of thermal conductivity of the material from which the heating element is made. In addition, due to the symmetry, there is no heat flow in the center

$$\left.\frac{dT}{dr}\right|_{r=0}=0 \tag{2}$$

The temperature field of the shell is described by the following equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_2 r\frac{\partial T}{\partial r}\right)=0 \tag{3}$$

where: λ_2 is the coefficient of thermal conductivity of the material from which the shell is made. At the same time, during heat exchange, the boundary conditions are boundary conditions of the fourth kind. In this case, the temperatures of the touching surfaces are the same

$$T_1\big|_{r=R_1} = T_2\big|_{r=R_1} \tag{4}$$

where T_1 and T_2 are the temperatures of the heating element and the shell, respectively. At the same time, the heat flows at their interface should be the same

$$-\lambda_1 \frac{dT_1}{dr}\bigg|_{r=R_1} = -\lambda_2 \frac{dT_2}{dr}\bigg|_{r=R_1} \tag{5}$$

The solution of equation (1) is

$$T_1 = -\frac{q_v r^2}{4\lambda_1} - C_1 \ln r + C_2 \tag{6}$$

Constant C_1 =0 due to the final value of the temperature in the center. Because of this

$$T_1 = -\frac{q_v r^2}{4\lambda_1} + C_2 \tag{7}$$

The solution of equation (3) is

$$T_2 = \frac{C_3}{\lambda} \ln r + C_4 \tag{8}$$

We find the constant C_3 from the boundary condition of the fourth kind (5)

$$-2q_v R_1 = \frac{C_3}{R_1} \tag{9}$$

from where the constant C_3 is equal to

$$C_3 = -2q_v R_1^{\,2} \tag{10}$$

From where the temperature T_2 is equal to

$$T_2 = \frac{-2q_v R_1^{\,2}}{\lambda_2} \ln r + C_4 \tag{11}$$

Next, we will use the boundary condition (4)

$$-\frac{q_v R_1^{\,2}}{4\lambda_1} + C_2 = \frac{-2q_v R_1^{\,2}}{\lambda_2} \ln R_1 + C_4 \tag{12}$$

where from

$$C_2 = q_v R_1^{\,2}\left(\frac{1}{4\lambda_1} - \frac{2\ln R_1}{\lambda_2}\right) + C_4 \tag{13}$$

We find the constant C_4 from the boundary condition of the third kind: convective heat exchange with the air medium occurs on the surface of the shell

$$-\lambda_2 \frac{dT_2}{dr}\bigg|_{r=R_2} = \alpha(T_2 - T_0) \tag{14}$$

where T_0 is the ambient temperature. Substituting expression (11) into (14), we obtain

$$4q_v R_2 = \alpha\left(C_4 - T_0 - \frac{2q_v R_1^2}{\lambda_2}\ln R_2\right) \tag{15}$$

from where the constant C_4 is equal to

$$C_4 = T_0 + \frac{2q_v R_1^2}{\lambda_2}\ln R_2 + \frac{4q_v R_2}{\alpha} \tag{16}$$

where T_0 is the ambient temperature. Substituting expression (11) into (14), we obtain

$$4q_v R_2 = \alpha\left(C_4 - T_0 - \frac{2q_v R_1^2}{\lambda_2}\ln R_2\right) \tag{17}$$

When substituting this expression into equation (13), we find the constant C_2

$$C_2 = T_0 + q_v R_1^2\left[\frac{1}{4\lambda_1} + \frac{2\ln\left(R_2/R_1\right)}{\lambda_2}\right] + \frac{4q_v R_2}{\alpha} \tag{18}$$

Then the expression for determining the temperature field of the heating element will take the form

$$T_1 = T_0 + \frac{q_v(R_1^2 - r^2)}{4\lambda_1} + \frac{2q_v R_1^2}{\lambda_2}\ln\left(R_2/R_1\right) + \frac{4q_v R_2}{\alpha} \tag{19}$$

and for the shell

$$T_2 = T_0 + \frac{2q_v R_1^2}{\lambda_2}\ln\left(R_2/r\right) + \frac{4q_v R_2}{\alpha} \tag{20}$$

3 Conclusion

In this paper, the problem of finding the temperature field of a heating element made in the form of a cylinder and the temperature field of the shell itself, enclosing the element under boundary conditions of the fourth kind, was solved. An analytical expression was obtained for finding the temperature field of the heating element and its enclosing shell in the form of an expression containing quadratic and logarithmic functions.

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