

Determination of the temperature field of the thermoelectric element when it cools down

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Abstract. In this paper, the question of finding the temperature field of a rectangular thermoelectric element under boundary conditions of the third kind with an adiabatically-isolated side is considered. Based on the Fourier method, the distribution of the temperature field of the thermoelectric element is obtained. The solution itself is obtained in the form of a series containing trigonometric and exponential functions. The reliability of the obtained result is confirmed by the fact that one of the special cases leads the problem to a problem with boundary conditions of the first kind, when the surface temperature is constant. The presented analysis of the main characteristics of thermoelectric converters makes it possible to determine the criteria for selecting materials and temperature conditions in order to increase the efficiency of such converters.

1 Introduction

To date, many works have been devoted to the study of heat transfer processes. In particular, non-stationary processes and methods of temperature measurement are of particular scientific interest. A feature of these processes is the continuous change in the heat content of bodies and their associated heating or cooling. Nonstationary thermal processes are heat exchange processes occurring in a time-varying temperature field. A feature of these processes is the continuous change in the heat content of bodies and their associated heating or cooling. Such processes are often found during heating and cooling of processed preparations and products in technological processes [1-8].

In the automation of technological processes, it is very often necessary to take indicators of temperature changes in order to load them into control systems for further processing. This requires high precision, low-inertia sensors capable of withstanding high temperature loads in a certain measurement range. As a thermoelectric converter, thermocouples are widely used – differential devices that convert thermal energy into electrical energy.

The devices are also a simple and convenient temperature sensor for a thermoelectric thermometer designed to make accurate measurements within wide temperature ranges. In particular, the control automation of gas boilers and other heating systems is triggered by an electrical signal coming from a sensor based on a thermocouple. The sensor designs provide the necessary measurement accuracy in the selected temperature range.

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The thermocouple structurally consists of two wires, each of which is made of different alloys. The ends of these conductors form a contact (hot junction) made by twisting, using a narrow welding seam or butt welding. The free ends of the thermocouple are closed by means of compensation wires to the contacts of the measuring device or connected to an automatic control device. At the junction points, another so-called cold junction is formed.

The principle of operation is based on the thermoelectric effect. When the circuit is closed, for example, with a millivolt meter thermo-EMF occurs at the points of adhesions. But if the contacts of the electrodes are at the same temperature, then these EMFs compensate for each other and the current does not arise. However, it is necessary to heat the place of hot soldering with a burner, then according to the Seebeck effect there will be a potential difference that supports the existence of an electric current in the circuit.

The thermophysical properties of a thermoelectric element are the most important physical characteristics that determine the patterns of their behaviour under different operating conditions. Thus, the experimental study of the thermophysical properties of such converters is of considerable scientific and applied interest.

The paper considers the case of an adiabatically isolated plasticity wall. Which leads to the fact that the task is asymmetric. Several papers have been devoted to the issues of the distribution of temperature fields in the presence of adiabatic isolation [9-11].

2 Main part

The main task of this work is to find the temperature distribution of a semi-bounded thermoelectric element in the form of a plate with a thickness δ under boundary conditions of the third kind (fig. 1)

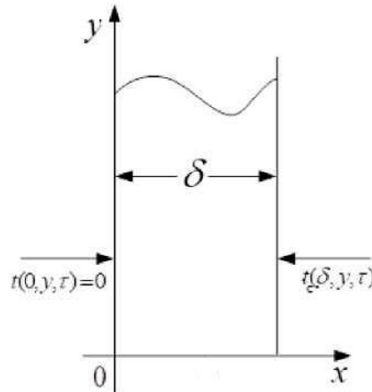


Fig. 1. Semi-limited thermoelectric element in the form of a plate.

To determine the temperature field of a thermocouple, it is necessary to solve a one-dimensional differential equation of thermal conductivity

$$\frac{\partial T}{\partial t} = \frac{\lambda}{c\rho} \frac{\partial^2 T}{\partial x^2} \quad (1)$$

Equation (1) includes the complex

$$a = \frac{\lambda}{c\rho} \quad (2)$$

where: a is the coefficient of thermal conductivity, which characterizes the rate of temperature change in the material during non-stationary thermal processes. The coefficient of thermal conductivity is the density of the heat flux at a single temperature gradient, related to the density of the substance and its heat capacity. It is proportional to the rate of change of the temperature or the rate of propagation of the isothermal surface in the body. All other things being equal, the body that has more a is more likely to heat up or cool down.

Formula (2) reflects the relationship of thermal conductivity λ and thermal conductivity. The thermal conductivity of a material is a measure of the ability of that material to conduct heat through it. The thermal conductivity of a material, on the other hand, is the thermal inertia of this material. The thermal conductivity of a material is the thermal inertia of that material. It can be understood as the ability of a material to conduct heat relative to the heat accumulated per unit volume.

Equation (1) itself must satisfy the following conditions:

$$T|_{t=0} = T_0 \quad (3)$$

and boundary: there is a heat exchange on the right

$$\frac{\partial T}{\partial x} + hT|_{x=\delta} = 0 \quad (4)$$

but not on the left

$$\frac{\partial T}{\partial x}|_{x=0} = 0 \quad (5)$$

The solution is found by introducing a new variable

$$\tau = at \quad (6)$$

In this case, equation (1) is simplified

$$\frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial x^2} \quad (7)$$

Let's imagine the temperature of a thermocouple as a product of two functions: one of which is $X(x)$ – a function of the coordinate, the other is $Y(\tau)$ – time

$$T(x, \tau) = X(x)Y(\tau) \quad (8)$$

To find a solution, we will use the method of separating variables. Applying the Fourier method using the graphical method, we obtain the desired solution in the form of trigonometric and exponential functions

$$T(x, \tau) = \sum_n T_0 \frac{2 \sin \mu_n}{\mu_n + \sin \mu_n \cos \mu_n} \cos\left(\frac{\mu_n x}{\delta}\right) e^{-\left(\frac{\mu_n}{a}\right)^2 \tau} \quad (9)$$

Next, we introduce the notation

$$Bi = \delta h = \frac{\alpha \delta}{\lambda} \quad (10)$$

where: Bi is a dimensionless number of Bio. In dimensionless form, equation (9) is written as

$$\theta(x, F_0) = \frac{T}{T_0} = \sum_n \frac{2 \sin \mu_n}{\mu_n + \sin \mu_n \cos \mu_n} \cos\left(\frac{\mu_n x}{\delta}\right) e^{-\mu_n^2 F_0} \quad (11)$$

where

$$F_0 = \frac{a \tau}{\delta^2} \quad (12)$$

is the Fourier criterion.

Since $\cos(\mu_n x / \delta)$ is a limited quantity, and $\exp(-\mu_n^2 F_0)$ is a rapidly decreasing quantity, in the so-called region with a regular thermal regime (at $F_0 \geq 0.25$), the series becomes rapidly converging and can be replaced only by the first term. In this case, expression (11) will take

$$\theta = N_n \cos\left(\frac{\mu_1 x}{\delta}\right) e^{-\mu_1^2 F_0} \quad (13)$$

We will conduct a study of the behaviour of the temperature field of a thermocouple at different values of Bi . First, consider the case of small values of the number Bi . For small μ , the characteristic equation will take the form

$$\mu^2 = Bi \quad (14)$$

In this case, the temperature of the thermoelectric element is evenly distributed over the thickness and in a dimensionless form it can be determined by the formula

$$\theta = \cos\left(\sqrt{Bi} \frac{x}{\delta}\right) e^{-Bi F_0} \quad (15)$$

In the limit ($Bi \rightarrow 0$), the cosine value is 1, then the temperature field will take the form

$$\theta = e^{-Bi F_0} \quad (16)$$

Whence it follows that the expression (16) does not depend on X . This means that the temperature field changes in time according to an exponential law.

In the case when the number Bi tends to infinity, it means that the intensity of the external heat exchange is infinitely large. Which leads to the fact that the surface temperature of the thermocouple is equal to the ambient temperature. In this case, we obtain a problem with boundary conditions of the first kind when the surface temperature is constant.

The temperature distribution of the thermoelectric element at different times at different values of the Bi number is shown in figure 2.

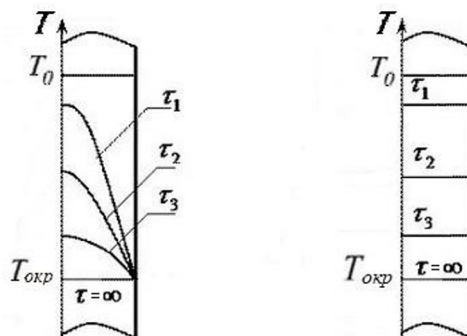


Fig. 2. The change in the temperature field of the thermoelectric element during cooling under the conditions $B_i \rightarrow \infty$ and $B_i \rightarrow 0$.

3 Conclusion

Thus, in this paper, an analytical expression was obtained for determining the temperature field of a thermoelectric element in the form of a plate with an adiabatically isolated side under boundary conditions of the third kind. The solution is obtained in the form of a series containing trigonometric and exponential functions. Special cases were also considered. For this purpose, the obtained solution was investigated at small and large values of the Bio number. The reliability of the results is confirmed by the fact that one of the special cases leads the problem to a problem with boundary conditions of the first kind, when the surface temperature is constant.

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