Comparative analysis of problem of thermal shock of elastic elements of small satellite in one-dimensional and two-dimensional formulations

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Abstract. The article demonstrates a comparative analysis of one-dimensional and two-dimensional problems of thermoelectricity for thermal shock of a plate. Similar resolutions of a one-dimensional problem in the framework of a two-dimensional problem are identified. Recommendations for such resolutions are given. The results of the research can be used in modeling the thermal shock of the elastic elements of a small satellite.

1 Introduction

Elastic elements in the composition of the satellite appeared at the initial stage of the development of space technology. Extended solar panels were required for ensuring a high power-to-weight ratio of the satellite [1-3]. They generated a sufficient amount of electricity for the regular operation of all target and supporting systems of the satellite. A number of spacecraft such as Bion satellite [4-6] and Photon satellite [7-9] did not contain large elastic elements and used the energy of batteries. However, in this case, the period of their active existence was significantly limited [10]. Thus, the appearance of solar panels at Foton-M No. 4 allowed more than three times (up to 60 days) to increase the period of active existence of the spacecraft.

While operating a satellite in near-Earth orbit, it will periodically be in the shadow of the Earth [11-13]. It is impossible to avoid shadow areas of the orbit during a long service life of a satellite. While being in the Earth's shadow, a thermal shock occurs due to the absence of solar radiation in the shadow area of the orbit [14, 15]. The same phenomenon is also observed when the satellite enters the area illuminated by the Sun. Thermal shock generates deformations of large elastic elements of the satellite, primarily solar panels [16-18]. These deformations are the source of disturbances in the motion of the satellite [19–21].

It should be noted that for spacecraft of different classes, the significance of this disturbances is significantly different. For example, for spacecraft with a large mass (the class of orbital space stations or the middle class), the effect of thermal shock on the orbital motion of these spacecraft is negligibly small compared to other disturbing factors [22–24]. Therefore, the problem of thermal shock of large elastic elements was not considered in the

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context of research and control of the motion of such spacecraft. However, the miniaturization of space technology and the widespread use of small satellites for solving a wide range of tasks have made the problem of thermal shock a topical one. The ratio of the mass of the elastic and rigid parts for small satellite is much higher than for other spacecraft. Therefore, the significance of disturbances associated with the relative motion of elastic elements for small satellite is also significantly higher than for other spacecraft. Thus, for a small satelllite, it has not yet been possible to create favorable conditions for the implementation of gravitationally sensitive processes due to violation of the requirements for microaccelerations [25–27]. Therefore, nowadays there is not a single implemented technological project of a small satellite, despite the obvious advantages of small satellite in the field of space technology [28-30].

The usage of small satellite for remote sensing of the Earth also led to the need to study and solve the problem of thermal shock [31-33]. To meet the requirements of orientation while solving problems of remote sensing of the Earth from space, it is necessary to effectively control the rotational motion of a small satellite, taking into account all the main disturbances, including thermal shock [34-36].

Nowadays an important problem is the problem of cleaning the near-Earth space from human-made space debris [37-39]. Now, numerous methods have been developed for cleaning up space debris [40–43]. To clean up bulky debris, a number of researchers suggest using a tug spacecraft and a tether system [44–46]. At the same time, to solve this problem effectively, it is necessary to estimate the inertial-mass parameters of space debris [47-49] and take into account the thermal shock. Otherwise, the tether connection between the tug spacecraft and space debris may be broken [50-52].

Thus, it can be argued that the problem of studying the influence of thermal fluctuations on the dynamics of a small satellite is topical in order to effectively control it.

2 Comparative analysis of models

The spacecraft solar panel is a complex structure [53-55]. The panel frame can be made from both classical [56-58] and composite [59-61] metal materials. Recently, composite materials have been most widely used. To obtain an approximate solution, a significant simplification of the solar panel model is necessary. Otherwise, mathematical models of thermal conductivity are complex. The purposes of creating these models may also be different. The initial purpose may be to estimate the maximum impact of thermal shock on the satellite dynamics [62, 63]. In research [63], for such an estimate, the model of a homogeneous beam is considered. Also, to achieve this goal, the homogeneous plate model is often used [64–66]. This is entirely justified, because in many cases the influence of thermal shock is not an important disturbing factor. Therefore, before analyzing a more complex model, one should make sure that the effect of thermal shock cannot be neglected. In the research [62] was considered that for the maximum effect of a thermal shock, it is sufficient to consider a one-dimensional thermoelasticity problem with a simplified representation of a solar panel in the form of a plate [64–66] or a beam [63].

However, for a correct assessment, it is necessary to choose the boundary conditions for the one-dimensional heat equation, which ensures the maximum impact of thermal shock. Thus, in researches [67, 68] it is assumed that at the moment of thermal shock the plate has the shape of a plane that is perpendicular to the incident heat flux (Fig. 1).

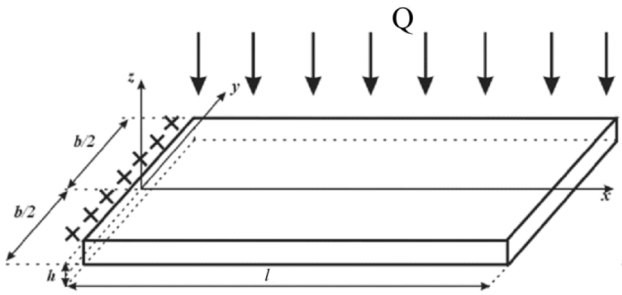

Fig. 1. The shape of the plate and the incident heat flux at the moment of thermal shock (according to the research [64]).

This situation contributes to the maximum heating of the plate by the incident heat flux. That is why the effect of thermal shock will be maximum. Any other position of the plate relative to the heat flow will cause its less intense heating, and, consequently, smaller temperature deformations.

The one-dimensional model of thermal conductivity assumes that the temperature field depends only on the z coordinate and time: T = T (z; t) [12, 64]. This significantly simplifies the heat equation [66, 68]:

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial z^2}, \ 0 < z < h, \ t > 0 \tag{1}$$

where a – coefficient of thermal diffusivity; h – the plate thickness (Fig. 1).

Equation (1) demonstrates that each point of the plate layer $z = const$ will have the same temperature with other points of this layer. In combination with the perpendicularity of the initial flat shape of the plate to the incident flow, this provides maximum heating. For the maximum influence on the motion of a small satellite, it is necessary to introduce the appropriate boundary conditions for the problem of thermoelasticity. They are expressed in the presence of a rigid seal of one of the edges of the plate (Fig.1). Geometric boundary conditions for $x = 0$ [64, 66]:

$$\begin{cases} u_z = 0, \ x = 0, \ t > 0; \\ \dfrac{\partial u_z}{\partial x} = 0, \ x = 0, \ t > 0. \end{cases} \tag{2}$$

provide maximum transfer of disturbing forces and moments to the body of a small satellite.

In (2), u_z is the deflection of the points of the plate (the component of the displacement vector in the direction of the z axis (Fig. 1).

Simultaneous fulfillment of the conditions for maximum heating of the plate and maximum transmission of disturbances through the mount to the body of the small satellite provides the maximum effect of thermal shock on the movement of the small satellite. This formulation makes sense to solve the problem of the effect of thermal shock when answering the question of the need to take it into account on the motion of a small satellite along with other perturbing factors.

However, it should be considered that the simultaneous fulfillment of such conditions in reality is extremely unlikely. Thus, in research [22] the requirement (2) is weakened and a hinged model of fastening a solar battery panel with a spring is considered. Therefore, on the one hand, the flat shape of the plate is perpendicular to the incident heat flux at the moment

of thermal shock. On the other hand, the rigid fastening of the solar panel to the body of a small satellite. This creates an overestimation of the real impact of thermal shock on the motion of a small satellite.

While taking into account the thermal shock, the model can be refined and a two-dimensional formulation can be used [69, 70].

$$\frac{\partial T}{\partial t} = a^2\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right),\ 0 \le x \le l,\ 0 < z < h,\ t > 0 \tag{3}$$

where l – the length of the plate.

It should be considered that the two-dimensional problem differs significantly from the one-dimensional problem when constructing a numerical model. In a one-dimensional formulation, only one situation was possible, where the plate has a flat shape, and the heat flux falls perpendicular to this plane. In a two-dimensional setting, an infinite number of simulated situations is possible. Therefore, it is worthwhile to indicate which of them is considered. An effective and correct comparison of one-dimensional and two-dimensional models will be the situation when, in some of the limiting cases, the two-dimensional model degenerates into a one-dimensional one. Therefore, in this article, within the framework of a two-dimensional model, it is proposed to consider the following situation. The plate has a curved shape at the moment of thermal shock, described by some function $u_z0=u_z0(x,0)$ (Fig. 2), and the incident flow is perpendicular to the plate surface near the seal.

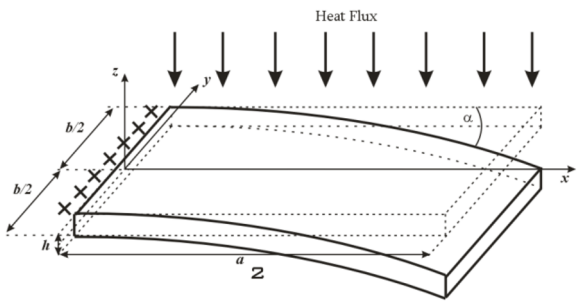

Fig. 2. The shape of the plate and the incident heat flux at the moment of thermal shock (according to the research [62]).

Then, in the limiting case $u_z0=u_z0(x,0)\equiv 0$ the two-dimensional model will degenerate into a one-dimensional one. In general, research [71] demonstrates the possibility of loss of plate stability in the framework of a two-dimensional formulation. Therefore, the complexity of the two-dimensional model is associated not only with an infinite number of initial forms of the elastic element at the moment of thermal shock.

However, let us return to checking the correspondence between one-dimensional and two-dimensional problems in a static formulation. In the limiting case $u_z0=u_z0(x,0)\equiv 0$ the approximate solution of the one-dimensional problem will completely correspond to the two-dimensional problem. The conformity research in other cases will be carried out in the direction of changing the initial deflection of the plate, i.e. change of the function $u_z0=u_z0(x,0)$ with respect to the indicated limiting case.

First of all, it should be consedered that when constructing an approximate analytical solution of the thermoelasticity problem within the framework of a two-dimensional model of thermal conductivity, the temperature representation is subject to revision. Instead of T = T(z; t) we have T = T(x, z; t). The absence of temperature dependence on the x coordinate (Fig. 1) made the heating of the surface layer uniform (Fig. 3, curve 2).

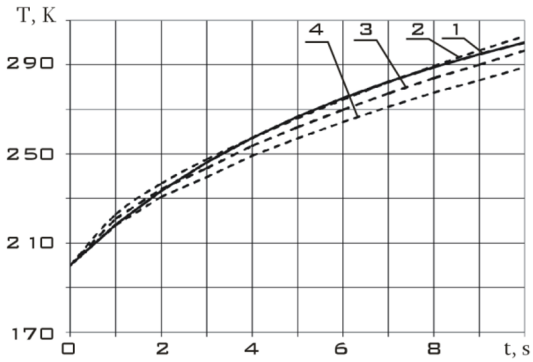

Fig. 3. Temperature dynamics of the surface layer of the plate at the initial deflection $u_{z0} = -0{,}1\, x^2/l^2$ (according to the research [70]): 1 – according to the formula (3) at $C = 200$ *K/m*, $a = 1$ *s*; 2 –near the termination ($x \approx 0$); 3 – in the middle of the plate ($x \approx l/2$); 4 – on the free edge of the plate ($x \approx l$).

The dynamics of this heating was approximated in researches [64, 66] by the following expression:

$$T(z,\, t) = Cz\frac{t}{t+a} + T_0,\ 0 \le z \le h,\ \ t > 0 \tag{4}$$

where C and a – constants.

Curve (3) at $C = 200$ K/m, $a = 1$ s is demonstrated in Fig. 3 (curve 1). However, for a two-dimensional model of thermal conductivity, this is not correct. Fig. 3 demonstrates the temperatures of the plate points near the embed ($x \approx 0$), in the middle of the plate ($x \approx l/2$), and also near the free edge ($x \approx l$). In this case, the dependence of the initial deflection on the x coordinate was approximated by the following function: $u_z0(x,\ 0) = -0{,}1\, x_2/l_2$. The curves have significant differences and are not explained by expression (3). Obviously, with an increase in the initial deflection, the functions presented in Fig. 3 will diverge even more significantly.

In researches [69-71], the following replacement option (3) is proposed:

$$T(x,\, z,\, t) = Cz\frac{t}{t+a} + f_2(x,\, t) + T_0,\ 0 \le x \le l,\ 0 \le z \le h,\ \ t > 0 \tag{5}$$

In this case, the partial derivative with respect to the z-coordinate for functions (3) and (4) will be identical. Let us use the description of the deflection dynamics using the Sophie Germain equation [72] for a one-dimensional model [12, 64]:

$$D\frac{\partial^4 u_z}{\partial x^4} + \rho h\frac{\partial^2 u_z}{\partial t^2} = -2\mu\alpha\int_{-h/2}^{h/2}\left[2\frac{\partial T}{\partial z} + z\frac{\partial^2 T}{\partial z^2}\right]dz,\ 0 \le x \le l,\ t > 0 \tag{6}$$

where D – cylindrical bending stiffness of the plate; μ – Lame coefficient; α – linear expansion coefficient of the plate material.

Then the integrals on the right side of equations (5) coincide when expressions (3) and (4) are substituted into the integrals. Therefore, the solution of the one-dimensional problem for deflection [64] is:

$$u_z = \frac{At}{t+a}\left(x^4 - 4lx^3 + 6l^2x^2\right),\ 0 \le x \le l,\ \ t > 0 \tag{7}$$

(where A – *constant*) can also be used in a two-dimensional setting. However, expression (6) must be corrected taking into account the initial deflection [69, 70]:

$$u_z = \frac{At}{t+a}\left(x^4 - 4lx^3 + 6l^2x^2\right) + u_{z0},\ 0 \le x \le l,\ \ t > 0 \tag{8}$$

Thus, it can be argued that the solution for the one-dimensional problem can be used in the framework of the two-dimensional formulation, provided that expression (4) for the temperature field is valid

3 Conclusion

As part of this research, an analysis was made of the correspondence between the two-dimensional and one-dimensional formulations of the thermal shock problems. The possible use of the solution of a one-dimensional problem for a two-dimensional problem in the case of a static formulation (taking into account only the initial deflection without taking into account the dynamics of natural vibrations) is established. In this case, it is necessary to take into account the initial deflection in the expression for deflections (7) and use the approximation of the temperature field (4). Thus, under certain conditions, it is possible to achieve not significant complication of the solution of the problem compared to the general formulation.

The results of the work can be used in modeling the thermal shock of the elastic elements of a small satellite.

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