On the radiometric resolution of SAR signals

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Abstract. In accordance with the provisions of the theory of statistical decisions, an expression for the error probability of discriminating the intensities of two signals as a function of the radiometric resolution of a synthetic aperture radar is obtained. The result of observing fluctuations at the input of the processing channel over a finite interval is represented by a sequence of complex samples that are the sum of samples of uncorrelated normal noise and signal samples. This sequence is considered as a normal complex random vector with a specified normalized correlation matrix, while the processing algorithm is assumed to be given. The proposed approach is compared with known methods for determining the radiometric resolution: the heuristic method and the differential radio contrast method. The advantages of the proposed method are demonstrated.

1 Introduction

Radiometric resolution is a measure of a system's ability to discriminate or resolve the regions with different scattering properties. Currently, there is no strict definition of the concept of radiometric resolution of synthetic aperture radar (SAR). In a broad sense, radiometric resolution (the contrast sensitivity) is commonly understood as the ratio of the increment in the intensity of the signal reflected by the resolution element (the increment in the specific effective scattering area) to the average intensity after synthesis (in decibels), which makes it possible to detect this contrast with sufficient reliability [1]. However, there is no generally accepted quantitative measure of this reliability. Several approaches to determining the radiometric resolution are considered below.

2 The heuristic method

The heuristic method for calculating the radiometric resolution [2] is the most widely used one. According to it, the absolute increment of the signal power $\Delta P$ above the average level $P_0$ is equal to the dispersion of the fluctuations $\sigma_f^2$ of the processed signal with the average power $P_0$:

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\[ \Delta P = \sigma_f^2. \]

With a single synthesis, the processing result has a probability density \( w(z) \) corresponding to the \( \chi^2 \)-distribution with two degrees of freedom [3]:

\[
w(z) = \exp\left(-z/2\sigma_0^2\right)/2\sigma_0^2, \quad z \geq 0, \tag{1}\]

where \( \sigma_0^2 \) is the dispersion of the real and imaginary parts of the normal signal at the output of the synthesis channel. At that

\[
\sigma_f^2 = 2\sigma_0^2 = P_0 + P_n,
\]

where \( P_n \) is the power of the noise samples.

When summing up \( N \) independent processing results, the value of \( \sigma_f^2 \) decreases by \( N \) times. Hence, one gets the well-known formula for radiometric resolution \( \delta_c \) [4]:

\[
\delta_c = 10 \log\left[(P_0 + \Delta P)/P_0\right] = 10 \log\left[1 + \left(1 + 1/q^2\right)/\sqrt{N}\right], \tag{2}\]

where

\[
q^2 = P_0/P_n \tag{3}\]

is the ratio of the average signal power to the noise power at the output of the processing channel (output power signal-to-noise ratio (SNR)).

The expression (2) determines the radiometric resolution through the root-mean-square deviation of radio brightness fluctuations of a radar image of a fragment of a homogeneous statistically flat surface (the background) obtained using incoherent accumulations.

The value of the radiometric resolution is limited by the residual level of fluctuations of the processed signal. In turn, the residual fluctuations are caused by the random nature of reflections from the element as an extended target (the so-called “speckle noise”) as well as by the uncorrelated noise at the output of the receiving device (the energy characteristics of the radio channel). The possibility of reducing the level of fluctuations during processing is associated with the use of several independent radio images \( N \) (also referred to as looks) obtained during the time when the resolution element was in the irradiation zone. In most cases, these independent images are obtained during inter-period processing of the received signal.

According to [5, 6], the probability of correct contrast detection, given by the parameter \( \delta_c \) according to formula (2), is \( P_d = 0.65 \).

In Fig. 1, one can see the dependence of the radiometric resolution \( \delta_c \) on the SNR \( q^2 \) at the output of the processing channel calculated by the formula (2) for \( N \) incoherent accumulations, its values being 1, 2, 3, 5, 10, 20. Fig. 2 demonstrates the dependence of the radiometric resolution on the number of incoherent accumulations \( N \) taking the values 1, \ldots, 20 when the values of SNR at the output of the processing channel are \( q^2 = 0, 10 \) and 20 dB.
Fig. 1. The dependence of the radiometric resolution on SNR.

Fig. 2. The dependence of the radiometric resolution on the number of incoherent accumulations.

The diagrams in Fig. 1 and Fig. 2 prove that the energy characteristics of the radio channel during radio observation by means of SAR specifically affect the efficiency of radio observation. An increase in SNR above $q^2 \sim 0$ dB has practically no effect on the value of the contrast sensitivity.

3 The differential radio contrast method

To reveal the relationship between the radiometric resolution and the probability of detecting objects in a radar image, it is proposed in [7] to use the method of differential radio contrast [8]. This allows us to define and evaluate the radiometric resolution of SAR based on the probability of detecting radio contrast in a radar image of a diffusely scattering homogeneous surface in the presence of intrinsic noise. In this case, the probability of correct detection of the radio contrast of the radar image elements in terms of power is determined by the formula

$$P_d = \frac{1}{(N-1)!} \left[ \frac{1 + C_b \rho_{b2}}{2 + (1 + C_b) \rho_{b2}} \right]^{N-1} \frac{(N-1+k)!}{k!} \left[ \frac{1 + C_b \rho_{b2}}{2 + (1 + C_b) \rho_{b2}} \right]^k,$$

(4)

where $C_b = \rho_{b1}/\rho_{b2}$ is the radio contrast of two background elements; $\rho_{b} = B_b / B_n$ is the background-to-noise ratio; $B_b$ is the radio brightness of the background element, which is proportional to the specific effective scattering surface; $B_n$ is the radio brightness of the SAR intrinsic noise; $N$ is the number of incoherent accumulations.

Within the framework of the method of differential radio contrast that is performed by SAR radiometric resolution in terms of the background $C_b^*$, the radio contrast $C_b$ of two background elements is assumed to be, determined by the probability of correct detection corresponding to the noise equivalent ($P_d = 0.67$). In Fig. 3, one can see the dependence of the probability of correct detection $P_d$ of the difference in radio brightness of the radar image background elements on the radio contrast $C_b$. It is calculated by formula (4) for the number of incoherent accumulations $N = 1, 2, 3, 5, 10, 20$ and two values of the background-
to-noise ratio $\rho_b = 0$ dB (Fig. 3a) and $\rho_b = 20$ dB (Fig. 3b). And in Fig. 4, the dependences of the SAR radiometric resolution in terms of the background $C_b^*$ (in dB) on the number of incoherent accumulations $N = 1, 10$ and $20$ are presented for the probabilities of correct detection of radio contrast $P_d = 0.65$ (Fig. 4a) and $P_d = 0.67$ (Fig. 4b).

**Fig. 3.** The probability of a correct detection of the radio contrast of the two radar image background elements: a) $\rho_b = 0$ dB; b) $\rho_b = 20$ dB.

**Fig. 4.** The dependence of the radiometric resolution on the number of incoherent accumulations: a) $P_d = 0.65$; b) $P_d = 0.67$.

The differential radio contrast method is focused on evaluating the SAR radiometric resolution of radar images after secondary processing [8] during calibration and validation procedures, as well as during SAR ground and flight tests [7]. However, it does not reflect any quantitative estimates of efficiency of radio observation, taking into account the SAR received signal processing procedure. Therefore, it is useful to consider another approach to determining the radiometric resolution, the one that makes such estimates possible.

### 4 The likelihood method

The radiometric resolution can be defined as the ratio of the intensities of the two SAR received signals, at which they can be discriminated at the output of the processing channel.
with a specified probability $p$. In this case, $P_0$ (3) will be considered as the averaged value of the signal power, relative to which the increments $\Delta P$ are introduced, so that

$$P_{s1} = P_0 + \Delta P_1, \quad P_{s2} = P_0 - \Delta P_2.$$  

(5)

In this case, when discriminating, the two hypotheses are tested: $H_1$ – that the signal $s_1$ with the power $P_{s1}$ is observed, and $H_2$ – that the signal $s_2$ with the power $P_{s2}$ is observed. The most likelihood hypothesis is taken as the decision.

The result of observing the fluctuation $y(t)$ over a finite interval is represented by a sequence of complex samples $y_i$, $i = 1, \ldots, M$ that are equal to the sum of the samples of the uncorrelated normal noise $n$ and the signal $s$. Signal samples are assumed to be a normal complex random vector with a specified normalized correlation matrix

$$R = ss^*/\sigma_s^2,$$

(6)

where $\sigma_s^2 = P_s$ is the dispersion (the power) of the signal samples, the overline is the operation of statistical averaging, and the symbol “$*$” means transposition with complex conjugation.

If the dispersion of noise samples is $\sigma_n^2 = P_n$, then, in accordance with [9], the optimal result of processing samples $y$, providing the estimate of the signal intensity $s$, is

$$\hat{\sigma}_s^2 = \sum_{m=1}^{M} \mu_m \left( \sum_{i=1}^{M} v_{mi} y_i \right)^2,$$

(7)

where

$$\mu_m = \sigma_s^2 \lambda_m / \left( \sigma_n^2 + \sigma_s^2 \lambda_m \right)^2,$$

(8)

$\lambda_m$, $v_m$ are the eigenvalues and eigenvectors of the matrix $R$ (the size of the correlation matrix is $M \times M$).

It follows from the relation (8) that signal $y$ processing should consist of multichannel linear filtering with the weight functions $v_m$ ($M$ channels), calculating the power of the output signals of linear filters, and then summing the results with the weight coefficients $\lambda_m$.

In practice, only $N$ eigenvalues $\lambda_m$ are significantly different from zero, and the number $N$ can approximately be taken equal to the ratio of the observation interval to the signal correlation interval. Considering these values to be the same and taking into account that the scale of the estimate can be arbitrary, without significant loss of generality, it can be taken that:

$$\hat{\sigma}_s^2 = \sum_{m=1}^{M} \left| \sum_{i=1}^{M} v_{mi} y_i \right|^2.$$  

(9)

The expression under the module sign corresponds to the weighted summation of the samples of the input oscillation to obtain single images, and the subsequent summation reflects the operation of incoherent accumulation. It is important to note that in order to obtain $N$ single images in (9) during coherent processing, the entire sequence of complex
samples $y_i$ is used, which are summed with different weight coefficients $v_{mi}$.

Otherwise, the relation (9) can be written as [10]:

$$\hat{\sigma}_y^2 = \sum_{m=1}^{N} \left[ \left( \sum_{i=1}^{M} v_m \text{Re} \ y_i \right)^2 + \left( \sum_{i=1}^{M} v_m \text{Im} \ y_i \right)^2 \right],$$

where $\text{Re} \ y_i$, $\text{Im} \ y_i$ are the real and imaginary parts of the samples $y_i$.

Each of the partial sums is a normal random variable with the dispersion

$$\sigma_m^2 = \sigma_n^2 + \sigma_s^2 \lambda_m.$$ 

Since the sum of the eigenvalues $\lambda_m$ is equal to the trace of the matrix $R$ [11]:

$$\sum_{m=1}^{N} \lambda_m = \text{tr} R = M,$$

then it can be taken:

$$\lambda_m = \lambda = M/N = L,$$

$$\sigma_m^2 = \sigma_n^2 = \sigma_s^2 + L \sigma_s^2.$$ 

Thus, the probability density of random processing results (9) corresponds to $\chi^2$-distribution with $2N$ degrees of freedom [12]:

$$w(x) = \frac{1}{2^N (N-1)! \sigma^2} \left( \frac{x}{\sigma^2} \right)^{N-1} \exp \left( -\frac{x^2}{2\sigma^2} \right), \quad x \geq 0.$$ 

The likelihood ratio $\Lambda$ under the hypotheses $H_1$ and $H_2$ is defined as the ratio of the conditional probability densities $w(y|H_1)$ and $w(y|H_2)$:

$$\Lambda = \frac{w(y|H_1)}{w(y|H_2)}.$$ 

(11)

If $\Lambda > 1$, then the hypothesis $H_1$ is accepted, while if $\Lambda < 1$, then the hypothesis $H_2$ is accepted.

In both cases, erroneous decisions can be made due to the random nature of both signal and noise. Therefore, it is necessary to determine the probability of correct decisions for the cases when the signal power is $P_{s1}$ and $P_{s2}$, respectively.

The conditional probability densities in (11) are [13]:

$$w(y|H_1) = \frac{1}{(2\pi)^M \det R_1} \exp \left[ -\text{Re} \left( \frac{1}{2\sigma_1^2} y^+ R^{-1} y \right) \right],$$

$$w(y|H_2) = \frac{1}{(2\pi)^M \det R_2} \exp \left[ -\text{Re} \left( \frac{1}{2\sigma_2^2} y^+ R^{-1} y \right) \right],$$

where $\sigma_1^2$, $\sigma_2^2$ are the dispersions of the real and imaginary components of the samples $y$ at the signal powers $P_{s1}$ and $P_{s2}$, respectively:
The condition \( \Lambda > 1 \) is met by
\[
\mathbf{y}^* \mathbf{R}^{-1} \mathbf{y} / 2\sigma_1^2 + \ln \det \mathbf{R}_1 < \mathbf{y}^* \mathbf{R}^{-1} \mathbf{y} / 2\sigma_2^2 + \ln \det \mathbf{R}_2
\]
or, after some simple transformations, one gets
\[
\left(\sigma_1^2 - \sigma_2^2\right) \mathbf{y}^* \mathbf{R}^{-1} \mathbf{y} / 2\sigma_1^2 \sigma_2^2 > \ln \left(\det \mathbf{R}_1 / \det \mathbf{R}_2\right).
\]

Taking into account the conditions (6), (7), (9), the expression (12) can be represented as:
\[
\Re \left(\mathbf{y}^* \mathbf{R}^{-1} \mathbf{y}\right) > 2N \left[\sigma_n^2 + L\sigma_{s1}^2\right] \left[\sigma_n^2 + L\sigma_{s2}^2\right] \ln \left(\sigma_n^2 + L\sigma_{s1}^2\right)
\]
\[
\quad / \left[\sigma_n^2 - L\sigma_{s2}^2\right] \left(\sigma_n^2 - L\sigma_{s2}^2\right).
\]

Thus, the probability of fulfilling the condition \( \Lambda > 1 \) at \( P_s = P_{s1} \) (the correct decision) is equal to
\[
P_{pl} = \int_{x_1}^{\infty} w\left(x|\sigma_1^2\right)dx,
\]
where
\[
x_1 = 2N \left[\frac{1 + 1/q^2 - \delta P_2}{\delta P_1 + \delta P_2}\right] \ln \left(\frac{1 + 1/q^2 + \delta P_1}{1 + 1/q^2 - \delta P_2}\right),
\]

where \( q^2 = LP_0/P_n \), \( \delta P_1 = \Delta P_1/P_0 \), \( \delta P_2 = \Delta P_2/P_0 \), and the probability of the erroneous decision is
\[
P_{e1} = 1 - P_{pl}.
\]

Accordingly, when \( P_s = P_{s2} \), the probability that \( \Lambda > 1 \) (the erroneous decision) is:
\[
P_{e2} = \int_{x_2}^{\infty} w\left(x|\sigma_2^2\right)dx,
\]
where
\[
x_2 = 2N \left[\frac{1 + 1/q^2 + \delta P_1}{\delta P_1 + \delta P_2}\right] \ln \left(\frac{1 + 1/q^2 - \delta P_1}{1 + 1/q^2 - \delta P_2}\right),
\]
while the probability of the correct decision is
\[
P_{p2} = 1 - P_{e2}.
\]

If the hypotheses \( H_1 \) and \( H_2 \) are equiprobable, then the probability of the discrimination error is
\[
P_e = \left(P_{e1} + P_{e2}\right)/2.
\]

Taking into account (10), for the conditional probability density of random processing results (9), it can be written
\[ w(x|\sigma^2) = x^{N-1} \exp(-x/2)/2^N(N-1)!, \quad x \geq 0. \]

The integrals in (13), (15) have analytical expressions [14]:

\[
\int_b^\infty w(x|\sigma^2) \, dx = \frac{\exp(-b/2)}{2^{N-1}(N-1)!} \left[ b^{N-1} + \sum_{k=1}^{N-1} 2^k b^{N-k-1}(N-1)(N-2)\ldots(N-k) \right].
\]

When calculating the integral (18), the lower limit is \( b = x_1 \) taken from the formula (14) or \( b = x_2 \) taken from the formula (16).

In determining

\[ \delta P = (P_0 + \Delta P)/P_0 = P_0/(P_0 - \Delta P_2) \]

the dependence of the discrimination error probability \( p_e \) (17) on the increment \( \delta P \) can be presented as it is shown in Fig. 5. Here \( N = 1, 2, 3, 5, 10, 20 \) is the number of incoherent accumulations, while SNR \( q^2 = 0 \) dB (Fig. 5a) and \( q^2 = 20 \) dB (Fig. 5b).

In Fig. 6a and Fig. 6b, one can see the diagrams of the dependence of the radiometric resolution \( \delta_r = \delta P \) (in dB) on SNR \( q^2 \) varying from 0 to 20 dB calculated by the formula (17) for the same values of \( N \) and the error probabilities \( p_e = 0.3 \) (thus, the probability of a correct decision is 0.7) and \( p_e = 0.33 \) (thus, the probability of a correct solution is 0.67), respectively.

The diagrams in Fig. 5 and Fig. 6 make it possible to determine the required values of \( q^2 \) and \( N \) based on the specified values of the radiometric resolution and the error probability.

**Fig. 5.** The dependence of the error probability on the signal power ratio: a) \( q^2 = 0 \) dB; b) \( q^2 = 20 \) dB.
Fig. 6. The dependence of the radiometric resolution on SNR: a) $p_e = 0.3$; b) $p_e = 0.33$.

5 Discussion of the results

Comparison of the methods for determining the radiometric resolution are carried out based on the diagrams in Fig. 7, where, for each of the methods, the dependences of the radiometric resolution on the number of incoherent accumulations are demonstrated for three values of the signal (background)-to-noise ratio.

The curves are constructed in such a way that at $N = 1$ (no incoherent accumulation present) the value of the radiometric resolution for the signal (background)-to-noise ratio $q^2 = 0$ dB takes the same value of 4.8 dB. This is achieved at different values of the probability of a radio contrast correct detection:

- $P_d = 0.65$ for the heuristic method;
- $P_d = 0.666$ for the differential radio contrast method;
- $P_d = 0.695$ for the likelihood method.
Fig. 7. The dependence of the radiometric resolution on SNR and the probability of a correct radio contrast detection: a) the heuristic method; b) the differential radio contrast method; c) the likelihood method.

It follows from Fig. 7 that under the indicated observation conditions, the likelihood method gives a higher probability of correct radio contrast detection. It is by ~7% higher than the one provided by the heuristic method, and by ~4% higher compared with the differential radio contrast method provided probability. With an increase in the signal (background)-to-noise ratio up to $q^2 = 20$ dB, the trend is preserved and the radiometric resolution is ~3.1 dB in the case of the differential radio contrast method, while the likelihood method shows a significant improvement in the probability of radio contrast correct detection, it is reaches the values up to 0.74 (more than 10% improvement), due to the processing algorithm used.

6 Conclusion

The most common heuristic method for calculating the radiometric resolution, in which the absolute increase in the signal intensity above the average level is equated to the root mean square value of the fluctuations of the processed signal with an average intensity, makes it possible to reveal the dependence of the radiometric resolution on the number of incoherently accumulated radar images and SNR value. In this case, it is implicitly assumed that the probability of correct detection of the contrast of the specific effective scattering surface, specified by the parameter $\delta_c$ according to the formula (2), is about 0.65. However, this method does not reflect any quantitative estimates of the efficiency of radio observation and gives an underestimated value of the radiometric resolution compared to other methods.

The differential radio contrast method, based on determining the probability of correct detection of the radio brightness difference of the two radar image elements, makes it possible to obtain an estimate of the radiometric resolution in the case of incoherent accumulation, taking into account the statistical features of the process of detecting radio contrast in the radar image. This method is focused on evaluating the radiometric characteristics of SAR radar images after secondary processing during calibration and validation procedures, as well as during SAR ground and flight tests.

According to the likelihood method, the radiometric resolution is defined as the ratio of the intensities of two signals, at which their difference can be detected with a specified
probability. Thus method provides an estimate of the radiometric resolution, taking into account the SAR signal processing algorithm that it is important at the SAR design stage, because any processing procedure can be represented by a quadratic form.

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