Mathematical Modelling of Solar Still and its Comparison Analysis with Experimental Investigation through Parametric Analysis

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Abstract. Solar distillation is very much effective for availing drinkable pot water, especially in desert areas and salt-producing areas. A Solar Still is a device that can be used to convert impure (saline) water into fresh drinking water using the process of Distillation. The energy source used in a Solar Still is sunlight thus it uses renewable energy. This paper describes a set of equations that can be used to find the distillate yield of a solar still by taking input parameters such as Solar Insolation and the basin area. A solar still is used for producing fresh drinking water from impure water and the equations help in estimating the yield. It also determines the efficiency of the solar still.

1. Introduction

Looking at the current trends of population growth, the need for fresh water is immense and only about 3% of the world’s water is fresh water.[1][2] Lack of fresh water has created a variety of problems across the globe and also affects the health of humans in many regions. This Solar Still aims to produce fresh water from impure water using solar insolation to try and increase the supply of fresh water. Since it uses a renewable energy source, it solves many issues, especially in countries that receive a lot of sunlight. This Solar still consists of an insulated basin covered by a glass plate.[3] The solar insolation falls on the basin heating and evaporating the water, which then becomes and water vapor condenses on the cooler glass plate. This condensed water is then collected by a pipe and poured into a vessel, which is the distillate, which is pure water.

A phase difference exists between the saline water and air above the water surface. Due to the heating of water, the immediately over the water surface will get saturated at that temperature of the water. Now, when solar radiation increases the temperature of saline water at that time corresponding air's saturation pressure also increases.[3][4] But near the glass surface, its water vapor's partial pressure will be less because of saline water temperature is higher than the temperature of the inner side of the glass cover. Now, this difference in partial pressure of water vapor is the main factor in the transfer of water vapor from the water level to the glass surface. Condensation of this water vapor occurs at the inner side of the glass cover.

The influence of solar insolation can be seen on distillate yield, but this mathematical model helps accurately determine the yield and also efficiency and thus does not require experiments to be carried out time and again.[5]
2. Experimental Setup

The setup on which experiments were conducted was a single-slope solar still. It has a basin of 1 m² area which is painted black and is insulated.[3][6] It has been painted black to maximize the absorption of solar radiation which is falling upon it. It is covered by a glass plate. The connections have been made air-tight silicon Rubber is mostly used because of its good elasticity characteristic. So no water vapor may leak out of it. At the bottom of the glass plate, a pipe is running which has been cut in half. This pipe collects the condensed water on the glass cover and deposits it outside the solar still into a vessel. The calibrated Thermocouples are attached in the various places of the solar still to measure the temperature of the same. These thermocouples are connected to the temperature recorder channel and inserted into the solar still through the holes at the side of the solar. These thermocouples are used to measure the temperature of saline water, glass cover, basin, air-vapor cavity, and ambient atmosphere. [7]

![Single slope solar still with inner black surface](image)

This vessel contains the distillate yield and has markings on its outer surface to accurately measure the distillate yield. The glass plate is at an angle of 23 degrees to the horizontal. In Ahmedabad, where the setup is located, in the May month one can assume useful solar radiation to be incoming from 8:00 AM to 5:00 PM. The average daily solar radiation in May month is found to be 6 kWh/sq. meter. [8][9] The thickness of the glass plate is 4 mm.

3. Mathematical Modelling

In the mathematical modeling of solar still, we use the energy conservation law at every stage where solar intensity is applied. Through this we get certain equations say 2 or 3. Now we modify this equation so that we can make them the function of all the general parameters which affect the solar intensity. [10] Our main motto is to find the solar yield and efficiency of our solar still model. So that we can compare these yield and efficiency values to experimental efficiency values. From that, we can find how much error is there in our experimental model.[11]

Now, before we start to derive the model of solar still, we have to state some general assumptions which should be considered in the derivation of the mathematical model.

These assumptions are as follows:[3][12]

1. There should be no vapor leakage in the whole model.
2. The whole system should be in a quasi-steady state condition.
3. Over the whole thickness of the glass cover and water column, no temperature gradient is available.
4. All the heat transfer coefficients should be linear with the temperature at every time interval.
5. The heat capacity of all the insulators is negligible.
6. We take a time interval of 60 seconds for this model.
7. Glass cover and water physical properties are constant at the operating temperature.

Now, we start writing the energy balance equations from the glass cover because that's where the first solar intensity comes in contact. After we apply it to the saline water. We didn't use any heat storage material otherwise we have to derive energy balance for that too.

Balancing Of Glass Cover:[13][14]

\[ M_g \alpha_g \frac{dT_g}{dt} = (I(t) \times \alpha_g) + Q_{evap} + Q_{r,w-g} - Q_{r,g-sky} + Q_{c,w-g} \]

Where,
\( I(t) = \) Solar insolation
\( \alpha_g = \) Absorptivity of the glass cover
\( Q_{evap} = \) Evaporative heat transfer, water to the glass
\( Q_{r,w-g} = \) Radiative heat transfer, water to the glass
\( Q_{r,g-sky} = \) Radiative heat transfer, glass to the sky
\( Q_{c,w-g} = \) Convective heat transfer, water to the glass

Balancing of saline water: [13][14]

\[ M_w C_w \frac{dT_w}{dt} = (I(t) \times \tau_g) - Q_{evap} - Q_{r,w-g} - (m \times C_w \times T_w) - Q_{c,w-g} \]

Where,
\( \tau_g = \) transmittance of the glass cover
\( m = \) instantaneous water production

Calculation of heat transfer coefficients: [13][14]

\[ Q_{r,w-g} = h_{r,w-g}(T_w - T_g) \]

\[ h_{r,w-g} = \varepsilon_{eff} \sigma [((T_w + 273)^2 + (T_g + 273)^2) \times (T_w + T_g + 546)] \]

\[ \varepsilon_{eff} = \frac{1}{\frac{1}{\varepsilon_w} + \frac{1}{\varepsilon_g} - 1} \]

\[ Q_{r,g-sky} = \sigma \varepsilon A(T^4 - T^4_1) \]

\[ Q_{c,w-g} = h_{c,w-g} A \omega (T_w - T_g) \]

\[ h_{c,w-g} = 0.884 X \left( \frac{p_w - p_g}{2.723 \times 10^4 - p_w} \right)^{1/3} \]

\[ p_w = \exp(25.3170 - \frac{5144}{T_w + 273}) \]
\[ P_g = \exp\left(25.3170 - \frac{5144}{T_g + 273}\right) \]

The yield of solar still is given by this equation: [13][14]

\[ m_w = \frac{Q_{e,w-g}}{h_f g} = \frac{h_{e,w-g} A_w (T_w - T_g)}{h_f g} * 3600 \]

Where,

\[ h_{e,w-g} = 16.273 * 10^{-3} * h_{e,w-g} * \left(\frac{p_w - p_g}{T_w - T_g}\right) \]

The efficiency of solar still is given by this equation:[13][14][15]

\[ \eta = \frac{\text{Evaporative Heat Flux}}{\text{solar flux}} = \frac{Q_{e,w-g}}{I(t) * A_s} \]

4. Experimental Analysis

Now, as we derived these equations for finding the yield of solar still. To solve this equation, we prepare the computer program to solve these non-linear equations. We use the design and climate parameters as input parameters which were already known to us.[16][17]

We start the calculations by taking all the temperature values as ambient temperature values at time \( t = 0 \) sec. after that, we use the values of known parameters to find the values of all the heat transfer coefficients. From that, we find the values of \( T_g \) and \( T_w \) for the initial time interval \( t \) as 1 min. the hourly yield of solar still can be found from these values. [18][19][20] Now, we again repeat this same process for the next time interval and find the different values of hourly yield at different time intervals.

Table 1. Experimental Data of Solar Still with a water depth of 20 mm. [21]

<table>
<thead>
<tr>
<th>TIME</th>
<th>SOLAR RADIATION (W/m²)</th>
<th>WATER TEMPERATURE (°C)</th>
<th>GLASS TEMPERATURE (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00 AM</td>
<td>457</td>
<td>31.6</td>
<td>32.1</td>
</tr>
<tr>
<td>10:00 AM</td>
<td>673</td>
<td>39.4</td>
<td>37.4</td>
</tr>
<tr>
<td>11:00 AM</td>
<td>809</td>
<td>50.4</td>
<td>45.1</td>
</tr>
<tr>
<td>12:00 PM</td>
<td>893</td>
<td>60.9</td>
<td>48.7</td>
</tr>
<tr>
<td>1:00 PM</td>
<td>879</td>
<td>65.1</td>
<td>54.4</td>
</tr>
<tr>
<td>2:00 PM</td>
<td>828</td>
<td>66.8</td>
<td>56.3</td>
</tr>
<tr>
<td>3:00 PM</td>
<td>664</td>
<td>65.1</td>
<td>49.8</td>
</tr>
<tr>
<td>4:00 PM</td>
<td>472</td>
<td>63.6</td>
<td>54</td>
</tr>
<tr>
<td>5:00 PM</td>
<td>194</td>
<td>56.2</td>
<td>46.1</td>
</tr>
<tr>
<td>6:00 PM</td>
<td>40</td>
<td>46.1</td>
<td>38.4</td>
</tr>
<tr>
<td>9:00 PM</td>
<td>443</td>
<td>28.7</td>
<td>26.2</td>
</tr>
</tbody>
</table>

This is the Experimental data of solar still in the month of march-2019. The depth of saline water in the solar still was kept at 20 mm. The experiments were carried out from 9:00 AM to 9:00 PM.[22][23]
5. Results and Discussion

This data includes the experimentally found yield of solar still. From this data, we used the values of solar intensity, water, and glass temperature values in the derived mathematical model. We assumed that plate temperature and water temperature are equal (T/p = T_w). We have also calculated these values:

- Volume(V) = L * W * X * H = 1.086 * 1 * 0.02 = 0.02172 m³
- M_w = ρ * V * X = 1000 * 0.02172 = 21.72 Kg
- C_p = 4.2 J/g°C = 4200 J/Kg°C
- Solar insulation = I*α
- A = 1.7376 m²

These values are used in the calculation of this values of solar still yield and efficiency values. These values are given in this table. These values are used in the calculation of the below values:

Area exposed = 1.086 m²
Latent heat of evaporation = 2260kJ/kg

In this work, efforts have been made to find the yield value between the time intervals of 1 hour. So for that, we take an average of solar intensity value for the calculation of Yield. From the literature, we may calculate the heat transfer coefficient values. Similarly, we may calculate the partial pressure values of water and glass cover.

Table 2. Mathematical model-based theoretical and Experimental values of Yield and Efficiency of solar still.[21]

<table>
<thead>
<tr>
<th>THEORETICAL YIELD (Kg)</th>
<th>EXPERIMENTAL YIELD (Kg)</th>
<th>THEORETICAL EFFICIENCY (%)</th>
<th>EXPERIMENTAL EFFICIENCY (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>46.63</td>
<td>46</td>
<td>6</td>
<td>4.3</td>
</tr>
<tr>
<td>192.61</td>
<td>190</td>
<td>13.89</td>
<td>9.95</td>
</tr>
<tr>
<td>444.038</td>
<td>438</td>
<td>11.93</td>
<td>8.55</td>
</tr>
<tr>
<td>701.219</td>
<td>690</td>
<td>34.98</td>
<td>25.05</td>
</tr>
<tr>
<td>977.291</td>
<td>964</td>
<td>23.54</td>
<td>16.86</td>
</tr>
<tr>
<td>1222.63</td>
<td>1206</td>
<td>37.13</td>
<td>26.59</td>
</tr>
<tr>
<td>1433.51</td>
<td>1414</td>
<td>66.14</td>
<td>47.37</td>
</tr>
</tbody>
</table>

\[ ∑=1555.15 \quad ∑ = 1534 \quad \text{Avg } \eta = \frac{530.258/9}{58.91 \%} \quad \text{Avg } \eta = \frac{379.718/9}{42.19 \%} \]

We then plot the different graphs based on the above calculation. This graph includes the comparison of theoretical Vs experimental Efficiencies.[3][24] Also, comparison of solar still yield and water temperature variation with time. These graphs are shown below:

In Fig. 2 it can be observed that experimentally available yield is more than that of theoretical available yield at every point of the day. It may be due to a discrepancy in the meddling of the efficiency equation and considering the losses inside the solar distillation system. Fig. 3 indicated that the temperature of the water inside the solar still increases continually from the morning at 9.00 am to the afternoon at 3.00 pm approximately. Then after due to the sun starts to set and solar radiation decreases. As a result, the water...
temperature decreases. In Fig. 4 we may observe that the efficiency calculated experimentally is a little bit higher as compared to theoretical estimated efficiency. It is due to the higher amount of yield available through experiments throughout all the days.

**Figure 2.** Comparison Between the Theoretical and Experimental solar still Yield Values for the water depth of 20 mm

**Figure 3.** Water Temperature variation with an increase in the solar intensity values
Figure 4. Comparison of theoretical and experimental efficiency values for single slope solar still for a water depth of 20 mm.

6. Conclusion
In this paper, we derived the theoretical yield and theoretical efficiency of solar still at an optimum water depth of 20 mm. From these calculations, we found that in the yield of solar still, the error between the theoretical and practical model is 36%. we also found the error in the efficiency of solar still between theoretical and experimental is around 17%. To improve the yield we can use solar reflectors to increase solar intensity. We can also use heat storage material to increase the evaporation of saline water.

References
14. S. Nanda Kumar, P.P. Shantharaman, Fabrication of solar water distillation system, IJCEM Vol. 2, Issue 1, April 2015, ISSN: 2348 9510.