Modeling of double standards and soft power in cellular automata competition systems

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Abstract. We consider a discrete analog of the classical A. Lotka–V. Volterra competition model by cellular automata. In the classical model, we know that the type of its time evolution depends on the double standards coefficients being at certain ranges of their possible values. The paper shows that the same situation holds for the discrete model either. We can see there is a soft power effect for the classical model. The classical competition model turns into a cooperative positional differential game, the limitations of which are the original system of competition equations by A. Lotka–V. Volterra. The controls are the coefficients of double standards when considering it concerning social systems. The effect of soft power is that the parties tend to compare the competitive pressure on them by the rival population with the one within the domestic population and may take the less stress of the opponent for his favorable attitude towards them, and more extensive—for the hostile manifestation. Whereas comparing the external competitive pressure with the internal pressure in this game does not give us any information—everything depends exclusively on the coefficients of double standards. Simulation experiments with the discrete competition model implemented in the cellular automata show that the effect of soft power also takes place in this case.

1 Introduction

This work is devoted to two problems: firstly, the search for ways to formalize descriptive sciences by means of computer science and mathematics, and secondly, the study of the relationships arising between system-dynamic and agent descriptions of one subject area.

Academician A.A. Dorodnitsyn wrote about the first of these tasks [1]: "I consider the task of introducing computer science methods into descriptive sciences to be one of the most important, perhaps the most important problem of the near future!" Indeed, at present (which is the very close future of the quote) we see many interesting approaches to such formalization. Several such studies are cited in [2, 3].

The subject of the study of this work will be intercultural interaction, studied using competition equations dating back to the works of A. Lotka and V. Volterra [4], taking into account the fact that when applied to social systems, the mathematical model turns from...
ordinary differential equations into a much more complex object – differential game. Analysis of this game allows us to conclude that the type of behavior of the model primarily depends on the double standards coefficients, setting the difference in the power of competition between "native" and "aliens," as well as to reveal a rather paradoxical effect of "soft power," the essence of which is that one of the populations seeks to displace the other from the system, without causing a negative reaction on its part, since the competitive pressure of the first population on the second is less, than competitive pressure within the second population, but, fundamentally, greater than within the first [2].

This paper discusses agent analogues of the model [2] implemented by cellular automata. And here we move on to the second of the above tasks. Cellular automata are quite easy to implement and therefore a very popular medium of agent modeling. In this environment, models are implemented from a wide variety of subject areas, for example, work [2-5]. At the end of the last century, A. Dewdney noted the analogy between the cell-automatic game WaTor [5] invented by him and the continuous predator-victim model of A. Lotka and V. Volterra [4]. How can such an analogy be expanded? - Discrete and continuous models differ fundamentally and the apparatus of research of the former is significantly weaker than the latter.

In this work, we will be convinced using simulation modeling that in a cellular-automatic model the role of double standards is the same as in continuous, as well as in the existence of a "soft power" effect in it.

While the analysis of continuous models can be carried out primarily analytically, the main approach for analyzing the discrete model in question is to conduct simulation experiments for different values of model parameters. For these purposes, a software implementation of a cell-automatic multivariate competition model implemented in C++, was used.

### 2 Multidimensional model of competition

In the work [14], an n-dimensional model of competition was considered:

\[
\frac{dx_i}{dt} = \alpha_i x_i \left( 1 - \sum_{j=1}^{n} \frac{m_{i,j}}{x_j} \right) \quad m_{i,j} = \| m_{i,j} \| = n
\]

Here, \( n \) is the number of populations; \( x_i \) - \( i \)-th population size; \( \alpha_i \) - malthusian coefficient of the \( i \)-th population, i.e. a parameter reflecting the reproduction rate of this population; \( x_j^+ \) - the medium capacity of the \( i \)-th population, i.e. the maximum population size for which the resource is still enough.

Finally, \( A = \| m_{i,j} \| \) is a \( n \times n \) size competition matrix consisting of double standard coefficients and reflecting the relationships between populations. More precisely, the value of \( m_{i,j} \) shows how many times the competition of population \( j \) with population \( i \) is stronger (or lesser) than the competition within the population \( j \) itself; the main diagonal of the matrix is thus filled with 1. The values of the coefficient above 1 mean intolerance, and below - tolerance.

The work [2] investigated the behavior of the system for a two-dimensional case (i.e., for two populations), namely, it was determined that it significantly depends (with the exception of a number of particular unlikely cases) only on the hit of double-standard coefficients \( m_{i,j} \) to one of the ranges:

1. \(-\infty, 0]\) - super tolerance;
2. Incident - tolerance;

3. 1 is an attitude without prejudice nor preference;

4. Incident - intolerance.

At the same time, four key cases arise. The first, mutual tolerance, occurs when the double standard ratios for both populations turn out to be less than one. In this situation, the trajectory of the system on the phase plane in the limit comes to a stable node with non-zero coordinates, which means that these populations can coexist at infinity.

The second is mutual intolerance – a situation where both coefficients are greater than one. Here, the trajectory in the main case leads to one of the points $x_1^*, x_2^*$, meaning survival within the limit of just one of the populations.

The third case occurs when one of the coefficients is higher than one and the second is lower. In this situation, the tolerate population lacks the chances of survival and the system in the limit comes to the survival of another population that fills the entire capacity of the environment.

Finally, the fourth case is the ratio without prejudice and preference - the case of equality of both coefficients to unity. In this case, both populations retain non-zero numbers at infinity.

As can be seen from the above, the result of modeling is determined by a large number based only on the values of the coefficients of double standards, and other parameters (for example, the capacity of the environment) play a secondary role. It is the calculation of this that the effect of "soft power" arises.

The essence of the "soft power" is that when several cultures interact (in the considered case - two), representatives of one culture can influence representatives of other cultures in order to voluntarily involve the latter in actions beneficial for the first culture.

For the considered model, it becomes possible to talk about soft power when, instead of a system of differential equations, a positional differential game with controls in the form of Malthusian coefficients, environment capacities and double-standard coefficients for each of the populations is considered, and the final populations are considered to be the prizes.

In this situation, one of the populations can underestimate the visible degree of impact on the competitor population, despite the fact that the actual impact will be significant.

In fact, consider the situation from the point of view of a representative of one of the populations, for example, at number 1. For example, values may be available for observation $\frac{x_1}{x_2^*}$ and $m \frac{x_1}{x_2}$ that is, the force of pressure from the own population and the alien population, respectively.

It seems natural that these values can be compared with the purpose of determining which of the populations is more "dangerous": namely, a greater value of the value corresponds to a greater apparent danger. Thus, when the visible pressure from the alien population is lower than the pressure from the own, the representative of the first population may conclude that the second population is not dangerous.

It is clear that this conclusion will be incorrect: after all, as emphasized earlier, the comparison of these values is not related to the result of modeling, which is determined as based on the coefficients of double standards only. At the same time, these coefficients themselves are associated with the internal interaction in the population and therefore may not be observed by representatives of other populations.

Based on the above, one population can apply "soft force" in relation to another: for this it is enough for it to change its environment capacity and double standard coefficient in this way so that, firstly, the double standard coefficient turns out to be higher than one, and,
secondly, the visible competitive pressure on the alien population turned out to be less than the visible pressure within the second population itself. If at the same time another population really turns out to be "convinced" that the first population is not dangerous and will be tolerant to it, that is, it will set its double standard ratio less than one, this will allow the first population to displace the competitor over time and fill the entire capacity of the environment, thus maximizing its prize.

3 Cell-automatic multidimensional competition model

Let us give a description of agent analogue of the cell-automatic multidimensional competition model, adding a small modification: namely, we will enter an additional parameter for each population $P_j$ - competitive pressure force of the $j$-th population.
to competition equal to 3 (for both populations), a vector of forces of competitive pressure $\begin{pmatrix} 0.5; 1 \end{pmatrix}$ and double standard coefficient matrix $\begin{pmatrix} 1 & 0.9 \\ 1.5 & 1 \end{pmatrix}$.

With the values considered, the value of $2,1_D$ the visible pressure of the first population on the second is equal to $1.5 \times 0.5 = 0.75$, and the value of $2,2_D$ the internal apparent pressure within the second population is $1 \times 1 = 1$. Thus, the representatives of the second population may seem that the first population does not pose a significant threat to them.

Moreover, even the value of $1,2_D = 0.9$ turns out to be less than $2,1_D$, meaning that the apparent pressure of the second population on the first is higher than the apparent pressure of the first on the second.

4 Results of numerical experiments

The figure below (Fig. 1) on the left side shows the field of the cellular automaton and the magnitude of competitive pressure forces, and on the right side - a graph of the number of populations at various points in time and coefficients of double standards, 1.5 - competition with "strangers" is 1.5 times more than with "own," 0.9 - competition with "strangers" is 90% of intra-population competition.

Fig. 1. Demonstration of soft power effect

As you can see from the experiment, the first population easily displaces the second, as it could be expected from the analysis of the continuous model, since the coefficients of double standards clearly speak "in favor" of the first population.

Consider the results of several more experiments. In all the figures on the left is the playing field at the final moment of the game, when only one population has already remained. Dark cells are occupied by its representatives, light cells are empty. Empty cells remain on the field due to intra-population competition.
On the right are graphs of population numbers. Along the abscissa axis - time (in modeling steps). Along the axis of ordinate - the number of populations in pieces. Graphs can begin with a positive time due to the peculiarity of the implementation of the simulation program: the graph field is considered as a kind of window, on the right side of which new current populations appear all the time. Gradually, the graph moves from right to left, how much it fits in the window. Therefore, if you stop the experiment before the initial populations reach the left side of the window (for example, due to the fact that one of the populations has already disappeared and nothing more interesting will happen), it seems that the experiment began with positive time (which is insignificant, since the start of the time is always relative).

Fig. 2. Delayed reproduction of the first population

Figure 2 shows the results of an experiment in which the initial numbers of both populations are 20, resistance to competition is 2 for both populations, and the mating age of the first population is twice that of the second: 4 versus 2 (the strength of competitive pressure and the double standard table are shown in the screenshot). This experiment demonstrates that an even faster reproduction rate is not a guarantee of survival for the second population. Despite the fact that the first population reproduces more slowly, and its visible pressure on the second population is less than the visible pressure within the second population, it is the first population that displaces the second over the time – even if at the beginning the second population has a short-term advantage in size. This demonstrates the possibility of using "soft power" even in the case of an unfavorable value of the marriage age parameter.

Fig. 3 shows the results of modeling at initial numbers of populations equal to 20, mating age equal to 3 for both populations; resistance to competition at the same time for the first population is 2, and for the second – 3. As can be seen from the results, higher resistance to competition is also not a key to survival. Over the time, the first population again displaces the second (and at the same time, its visible pressure on the second population turns out to be less than the visible pressure within the second population).

Fig. 4 shows the results of an experiment in which the mating ages of both populations are 3, the resistance to competition for both populations is 3, and the initial numbers differ: 200 in the second population and 20 in the first. Despite a tenfold initial lag.
In numbers, the first population manages to displace the second in a very short time period. At the same time, its visible pressure is still relatively small.

Fig. 3. Increased persistence in the second population

Fig. 4. Increased initial population size of the second population

Fig. 5 shows that, however, for a constructed agent model, it is not possible to derive an unambiguous inference about the result of a simulation based on a matrix of double standard coefficients, as it was possible in the case of a system-dynamic description. The experiment shown in the screenshot repeats the conditions of the experiment with Figure 3, with the exception of double standard coefficients (the intolerance of the first population and to less of the second are reduced, and the competitive pressure of the first on the second is still four times lower than the internal pressure in the second population. Here, despite the coefficients of double standards favorable for the first population, as a result, representatives of the second population survive (therefore, the “living” cells in the working field are gray, and not black as in the previous and the subsequent figures).
Fig. 5. Increased persistence of the second population; double standard coefficient matrix changed.

Fig. 6. Experiment with three populations. Finally, Fig. 6 shows the results of the experiment for the case of three populations. For all three populations, the same values of initial numbers were chosen - at 20, marriage age - at 3 and resistance to competition - at 3. In this example, the second and third populations observe high apparent pressures not only within their population, but also relative to each other. Despite this, it is the first population that poses a real danger to them: as a result, it is she who displaces both the second and third populations and completely captures the entire playing field (internal competition leaves empty cells).

The schedules of endangered populations are almost the same. Note that in the three-dimensional case, the entire competition matrix is shown under the graph of population dynamics.
5 Conclusion

The paper considers a system-dynamic and agent description of the competition model, an attempt is made to transfer forecasts of the behavior of a continuous model to a discrete model. The simulation experiments carried out both show the possibility of applying the forecasts obtained in one model to another, and demonstrate the presence of restrictions in such transitions.

The possibility of using “soft power” in the cellular-automatic model of competition is shown, including in the case of different values of model parameters for different populations, as well as in the multidimensional case (with the number of populations greater than 2). Nevertheless, in the fifth experiment (Fig. 5) there was a “misfire,” why?

It should be noted that even the game WaTor turns out to be much more capricious than the continuous predator-victim model. Unlike the latter, it does not oscillate under any positive initial conditions and parameter values. If the parameters and initial conditions are poorly selected, situations are quite possible when all sharks die of hunger, and fish occupy the entire working field. Or vice versa—sharks eat all fish, after which they die of hunger themselves. Something similar happens in our fifth experiment. Intolerance of the intolerant side and tolerance of the tolerant side are not very pronounced, while the strength of the internal competitive pressure of the tolerant side is significantly (4 times) higher than the pressure on it of the intolerant side. The discrete model does not cope with such parameter values, as happens in the WaTor game. However, in a fairly wide range of more balanced model parameters (experiments 1-4 and 6), the “soft power” effect takes place.

References


