Evaluation of compressive strength of concrete in bipolar fuzzy environment

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Abstract. Concrete is the most widely used building material. Numerous varieties of concrete are available. They are resolved by the formation of binders and the types of aggregate used to suit the application of the engineered material. The quality of the concrete is based on the specific proportions of different materials added to it such as cement, slag, ash, water, superplasticizer, coarse aggregate and fine aggregate. The compressive strength of concrete hangs on many factors such as water-cement ratio, cement strength, quality of concrete material during making of concrete which is determined using MCDM methods. In this paper COPRAS and WASPAS methods are used to determine the ranking order of various set of concrete mixtures produced in the laboratory.

1 Introduction


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mixtures. Based on the above concepts COPRAS and WASPAS bipolar fuzzy methods are developed to determine the compressive strength of concrete materials.

2 COPRAS method for BF set

COPRAS stands for Complex Proportional Assessment. Utility degree of the alternatives under the influence of conflicting criteria is determined. The main advantages of COPRAS method is the use of utility degree to illustrate which one is better and worse. Bipolar COPRAS method begins with the decision matrix (DM):

$$
BFM = \begin{bmatrix}
BF_1 & (\delta_{11}^-, \delta_{11}^+) & (\delta_{12}^-, \delta_{12}^+) & \cdots & (\delta_{1q}^-, \delta_{1q}^+)
\\
BF_2 & (\delta_{21}^-, \delta_{21}^+) & (\delta_{22}^-, \delta_{22}^+) & \cdots & (\delta_{2q}^-, \delta_{2q}^+)
\\
\vdots & \vdots & \vdots & \ddots & \vdots
\\
BF_p & (\delta_{p1}^-, \delta_{p1}^+) & (\delta_{p2}^-, \delta_{p2}^+) & \cdots & (\delta_{pq}^-, \delta_{pq}^+)
\end{bmatrix}
$$

where \( p \) denotes number of alternatives, \( q \) number of criteria and \((\delta_{pq}^-, \delta_{pq}^+)** is the performance of the \( p^{th} \) alternative with respect to \( q^{th} \) criterion.

**Definition 2.1.** Maximizing index is determined for the criteria to be maximized.

$$
A_p = \sum_{q=1}^{s} b_{pq}
$$

where \( b_{pq} \) denotes beneficial criteria.

**Definition 2.2.** Minimizing index is determined for the criteria to be minimized.

$$
B_p = \sum_{q=1}^{s} nb_{pq}
$$

where \( nb_{pq} \) denotes non-beneficial criteria.

**Definition 2.3.** The relative weight for each alternative is determined by

$$
RW_p = A_p + \frac{B_{p\min} \sum_{p=1}^{r} B_p}{B_p \sum_{p=1}^{r} \frac{B_{p\min}}{B_p}}
$$

**Definition 2.4.** Utility degree assists in responses ranking which is calculated on evaluation of preferences on all responses with most effective response.

$$
U_p = \frac{RW_p}{\max(RW_p)} \times 100\%
$$

3 Procedure of COPRAS method

**Step 1.** All the informations are represented in the form of matrix as DM is constructed.

**Step 2.** Normalization of DM with respect to beneficial and non-beneficial criteria.

**Step 3.** For each criteria, the weight value is determined.

**Step 4.** Determine the weighted normalized DM with positive and negative membership.

**Step 5.** Aggregate the membership values of weighted normalized DM.

**Step 6.** Summation over the beneficial criteria using Definition 2.1.

**Step 7.** Summation over the non-beneficial criteria using Definition 2.2.
Step 8. Determine relative weight over each alternative using Definition 2.3.
Step 9. Estimate the utility degree which gives ranking of the responses. The flowchart of BF COPRAS and WASPAS is represented in Fig 1.

![Flowchart of COPRAS and WASPAS method](image)

Fig. 1. Flowchart of COPRAS and WASPAS method

4 Application

Concrete is a suitable material for construction composed of cement and aggregates, mixed with water which hardens with time. So it does not exist as an independent construction material. Because of its strength, durability, reflectivity and versatility it is globally used for the construction of foundations, columns, beams, slabs and other load bearing elements shown in Fig. 2.
Concrete can be produced in various shapes and sizes. These outstanding properties have made concrete a trustworthy and enduring option for construction companies for both commercial and domestic types of constructions. Materials are mixed in specific proportions to obtain the appropriate compressive strength which is the ability of a structure to carry the loads on its surface without any crack or deflection. Its SI unit is mega pascals (MPa). The compressive strength of concrete is a measure of the concrete’s ability to resist loads which tends to compress it. Design engineers use the specified concrete mixture to design structural elements. Under a given specific age (days), engineers determined the actual concrete compressive strength of different mixtures of concrete from the laboratory which is given in Table 1.

<table>
<thead>
<tr>
<th>Cement</th>
<th>Slag</th>
<th>Ash</th>
<th>Water</th>
<th>Plasticizer</th>
<th>C-agree</th>
<th>F-agree</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>439</td>
<td>177</td>
<td>0</td>
<td>186</td>
<td>11</td>
<td>885</td>
<td>708</td>
<td>91</td>
</tr>
<tr>
<td>315</td>
<td>137</td>
<td>0</td>
<td>145</td>
<td>6</td>
<td>1130</td>
<td>745</td>
<td>28</td>
</tr>
<tr>
<td>390</td>
<td>189</td>
<td>0</td>
<td>146</td>
<td>22</td>
<td>945</td>
<td>756</td>
<td>56</td>
</tr>
<tr>
<td>214</td>
<td>98</td>
<td>25</td>
<td>182</td>
<td>7</td>
<td>1066</td>
<td>786</td>
<td>28</td>
</tr>
<tr>
<td>149</td>
<td>153</td>
<td>194</td>
<td>192</td>
<td>8</td>
<td>935</td>
<td>623</td>
<td>28</td>
</tr>
</tbody>
</table>

Let the Bipolar fuzzy sets $BF_1, BF_2, BF_3, BF_4, BF_5$ represent five concrete mixtures which are evaluated based on the parameters such as cement, slag, ash, water, superplasticizer, coarse aggregate, fine aggregate and ages (Fig. 3).
Step 1: Construct DM

\[
\begin{align*}
BF_1 & = (0.01, 1) \quad (-0.87, 0.11) \quad (-0.05, 0.93) \quad (0.01, 1) \quad (-0.36, 0.62) \quad (-0.26, 0.72) \quad (-0.10, 0.88) \quad (0.01, 1) \\
BF_2 & = (-0.07, 0.91) \quad (-0.88, 0.10) \quad (-0.32, 0.66) \quad (-0.05, 0.93) \quad (0.01, 1) \quad (-0.15, 0.83) \quad (0.01, 1) \quad (-0.49, 0.5) \\
BF_3 & = (-0.14, 0.84) \quad (-0.88, 0.10) \quad (0.01, 1) \quad (-0.07, 0.91) \quad (-0.22, 0.76) \quad (-0.15, 0.83) \quad (-0.04, 0.94) \quad (-0.49, 0.5) \\
BF_4 & = (-0.13, 0.85) \quad (0.01, 1) \quad (-0.98, 0.00) \quad (-0.007, 0.98) \quad (-0.15, 0.83) \quad (-0.15, 0.83) \quad (-0.09, 0.89) \quad (0.01, 1) \\
BF_5 & = (-0.24, 0.74) \quad (-0.26, 0.72) \quad (-0.98, 0.00) \quad (-0.12, 0.86) \quad (-0.57, 0.41) \quad (0.01, 1) \quad (-0.10, 0.88) \quad (-0.49, 0.5)
\end{align*}
\]

Step 2: Normalization of DM

\[
\begin{align*}
BF_1 & = (1, 1) \quad (-0.01, 0.86) \quad (-0.18, 0.0007) \quad (1, 1) \quad (-0.02, 0.67) \quad (-0.03, 0.72) \quad (-0.09, 0.88) \quad (1, 1) \\
BF_2 & = (-0.12, 0.91) \quad (-0.01, 0.95) \quad (-0.03, 0.001) \quad (-0.18, 0.93) \quad (1, 0.41) \quad (-0.06, 0.83) \quad (1, 1) \quad (-0.02, 0.5) \\
BF_3 & = (-0.06, 0.84) \quad (-0.01, 1) \quad (1, 0.007) \quad (-0.13, 0.91) \quad (-0.04, 0.55) \quad (-0.06, 0.83) \quad (-0.23, 0.94) \quad (-0.02, 0.5) \\
BF_4 & = (-0.07, 0.85) \quad (1, 1.00) \quad (-0.01, 0.5) \quad (-1.2, 0.98) \quad (-0.06, 0.5) \quad (-0.06, 0.83) \quad (-0.10, 0.89) \quad (1, 1) \\
BF_5 & = (-0.04, 0.74) \quad (-0.03, 0.13) \quad (-0.01, 1) \quad (-0.07, 0.86) \quad (-0.01, 1) \quad (1, 1) \quad (-0.09, 0.88) \quad (-0.02, 0.5)
\end{align*}
\]

Step 3: For each criteria, the weights are

\[\begin{align*}
wg_1 & = 0.1296, \quad wg_2 = 0.1385, \quad wg_3 = 0.0966, \quad wg_4 = 0.1894 \\
wg_5 & = 0.1221, \quad wg_6 = 0.1220, \quad wg_7 = 0.1463, \quad wg_8 = 0.0551.
\end{align*}\]

Step 4: Weighted normalized DM

\[
\begin{align*}
BFM & = (0.12, 0.12) \quad (-0.00, 0.11) \quad (-0.01, 0.0007) \quad (0.18, 0.18) \quad (-0.003, 0.08) \quad (-0.004, 0.08) \quad (-0.01, 0.12) \quad (0.05, 0.05) \\
& \quad (-0.01, 0.11) \quad (-0.00, 0.13) \quad (-0.002, 0.001) \quad (-0.03, 0.17) \quad (0.12, 0.05) \quad (-0.007, 0.10) \quad (0.14, 0.14) \quad (-0.001, 0.02)
\end{align*}
\]

Step 5: Aggregated weighted normalized DM
\[
BFM = \begin{pmatrix}
0.12 & 0.05 & -0.001 & 0.18 & 0.03 & 0.04 & 0.05 & 0.05 \\
0.05 & 0.06 & -0.001 & 0.07 & 0.08 & 0.04 & 0.14 & 0.01 \\
0.05 & 0.06 & 0.04 & 0.07 & 0.03 & 0.04 & 0.05 & 0.01 \\
0.05 & 0.07 & 0.02 & -0.02 & 0.02 & 0.04 & 0.05 & 0.05 \\
0.04 & 0.006 & 0.04 & 0.07 & 0.06 & 0.12 & 0.05 & 0.01 \\
\end{pmatrix}
\]

Step 6: \( A_1 = 0.4740, A_2 = 0.3286, A_3 = 0.2364, A_4 = 0.1831, A_5 = 0.3128 \).

Step 7: \( B_1 = 0.0896, B_2 = 0.1502, B_3 = 0.1478, B_4 = 0.1264, B_5 = 0.1148 \).

Step 8:
\( RW_1 = 0.6444, RW_2 = 0.4302, RW_3 = 0.3397, RW_4 = 0.3039, RW_5 = 0.4457 \).

Step 9: \( U_1 = 100\%, U_2 = 66\%, U_3 = 52\%, U_4 = 47\%, U_5 = 69\% \).

5 Conclusion

In this paper the compressive strength of concrete materials is evaluated using both COPRAS and WASPAS MCDM methods. The ranking order by COPRAS method is 1-3-4-5-2 whereas by WASPAS method is 1-2-4-5-3 (Fig. 4). Though the best alternative is \( BF_1 \) in both the methods the ranking of the other alternatives vary.

![Comparison of COPRAS and WASPAS](image.png)

References


