Research on Try-before-you-buy Strategy Under Product Fit Uncertainty

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Abstract. With the rapid development of online retail, the drawback of product fit uncertainty in online markets are becoming more and more prominent. In order to alleviate the impact of the product fit uncertainty, online retailers continue to introduce new service strategies. Based on the uncertainty of product matching, this paper constructs and solves the model of direct sales and try-before-you-buy(TBYB) strategy by online retailers under the premise of whether to allow returns. And explore the optimal strategy for the e-retailer in different aiming. The results show that: When the product fit is low, the optimal strategy choice for online retailers is TBYB strategy. When the product fit is high, if products are allowed to be returned, sell the product directly is the optimal strategy choice; if not, both TBYB and direct sales are optimal strategies. When the product fit is moderate, for products that are allowed to be returned, sell products directly when aiming to maximize demand and adopt the TBYB strategy when maximize profits. For products that are not allowed to be returned, Online retailers should sell products directly when aiming to maximize demand. Keywords: Try-Before-You-Buy, Return policies, Hassle cost, Online shopping, Product fit uncertainty.

1 Introduction

With the rapid development of e-commerce, online shopping is gradually integrated into people’s daily life. As of December 2021, China’s online shopping users had reached 842 million up to 59.68 million year on year, accounting for 81.6 percent of the country’s total internet users. While online shopping is booming, the tendency of product fit uncertainty is also becoming more and more obvious. Before receiving purchased products, although the customers are able to access to product information through the supplier’s promotion and the feedback of other consumers, the issues of the lack of direct and real perception for the physical products still exist , for instance, some physical products show distinct size and color to their pictures, and buyers’ comments are inconsistent with the actual situation, so consumers is difficult to assess whether the product match with their preferences accurately, and the product matching uncertainty will cause consumer anxiety before purchase, thus affecting the demand of online retailers and profit level.

Allowing returns is a common way for online retailers to deal with the consumers’ pre-purchase anxiety, such as the strategy of “seven days no reason to return”, and some retailers

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also promise to give consumers longer time to return goods. Allowing returns gives consumers right for trial and error, thus reducing consumers’ pre-purchase anxiety of product fit uncertainty. However, while allowing returns can promote consumer’s purchasing, it does not reduce the uncertainty of product matching.

The try-before-you-buy (TBYB) strategy, as a sales strategy to reduce product fit uncertainty, is adopted by more and more online retailers. It allows the consumers to choose multiple products online at the same time. After placing the order, the retailers will send these products to the consumers together. After trying them out, the consumers will keep the products that best match their own preferences, pay the corresponding fee, and return the unmatched products within a certain period of time. (This strategy has paid off among many online retailers.) In 2013, Warby Parker, an e-commerce company, launched a home-based fitting service, allowing consumers to book five frames at once for five-days trying period, and consumers can keep satisfactory styles and pay for them. Tmall launched TBYB strategy for the users with sesame credit points of more than 650 points. In the first month, about 20,000 items were used TBYB, with more than 134,000 items consumed by about 126,000 consumers, the rate of transaction is 87.2%. JNBY, Buy Together and other enterprises have also opened the TBYB. The TBYB strategy can greatly reduce or even eliminate the uncertainty of consumer product matching, and promote the profit growth of retailers.

On the other side, adopting the TBYB strategy will lead to an increase in the costs. The hassle cost paid by consumers for information searching and delivery processes will also affect the demand and profits of online retailers. In addition, whether the return caused by commodity attributes also affects the optimal decision of online retailers. Therefore, under what conditions should the online retailers adopt the TBYB strategy is the main issue that we need to address. Based on the above research background, this paper provides models of direct selling strategy and TBYB strategy and corresponding solution under the premise of whether to allow returns, and describes the optimal strategy selection for online retailer and the optimal selling strategy selection with the goal of profit and demand maximum.

The remainder of this paper is organized as follows. Section 2 presents a literature review. Section 3 constructs the model of direct sales and the TBYB strategy by online retailers and solves it under the situation of not allowing return and allowing return. Section 4 states the optimal strategy selection for online retailer and makes numerical analysis. Section 5 concludes the paper. All the mathematical proofs are presented in the Appendix.

2 Literature Review

This paper is related to the following literature.

With regard to return policies, Su[1] and Hui[2] examined the impact of different return policies on supply chain performance. Shulman[3] describes how consumers’ purchase decisions and return decisions are influenced by pricing and return cost. Chen[4] compared the two return policies of ‘money-back guarantee’ and ‘hassle-free’ under different conditions. However, the increase in returns has increased costs for online retailers. This requires online retailers to find a balance of demand, price, and cost in their return policy. Zhao et al. discusses the optimal decision of online retailers under three situations: consumer paying the return freight, retailer paying the return freight and paying the return freight together.[5] And some research devised return policies to reduce product fit uncertainty. Davis et al.[6] and Wujin[7] pointed out that providing money back guarantee can alleviate consumers to deal with the risk of product fit uncertainty.

With regard to the try-before-you-buy strategy, Yao et al. pointed out that the sample trial strategy may reduce consumer product fit uncertainty and increase product sales.[8, 9] Gilbert[10] has statistically found that trial samples can encourage consumers to buy. Yang
et al. pointed out that trial activities can reduce the impact of online reviews on sales, while consumers’ messaging behavior during trial activities can reduce the impact of prices and online reviews on sales volume.[11] Li et al. pointed out that the free trial campaign will increase the number of quality reviews, and the greater the uncertainty about product quality, the higher the free trial campaign benefits to online retailers.[12] Davis[13] and Barclay et al.[14] concluded that combining the strategy of pre-payment physical experience with the post-loose return strategy can further reduce consumers’ pre-purchase concerns and the uncertainty of product matching degree. Park[15] studied the maximum number of products allowing consumers to try, and points out that once the system recommendation accuracy exceeds a certain threshold value, the accuracy and the optimal trial number are inverted U-type relationship. Li[16] notes that the adoption of the TBYB strategy depends largely on operating costs, product value and the size of the member market segments.

We also reviewed the literature on information disclosure. Bruyn and Lilien divided the purchasing decision-making process of consumers into three stages, they found that in different stages, consumers have different perceived usefulness to online reviews, and high-quality online reviews play an important role.[17] But online retailers is hard to controlling online reviews, and can even get fake reviews from real customers with poor experiences and from competitors.[18] In addition, online retailers gradually adopt the mode of online exhibition hall to provide more product information and reduce product fit uncertainty, thus affecting the formulation of retailers’ sales strategies. Gao and Su believe that online virtual exhibition halls can reduce the uncertainty of product matching by screening out consumers who are not satisfied with the product before purchase.[19]

To sum up, the above studies respectively discussed the TBYB strategy, information disclosure and the return policies. However, with the rapid development of online retail in recent years, some problems and deficiencies have been exposed in the existing research: (1) online reviews are difficult to control and guide, and the existence of negative word-of-mouth will make consumers’ purchase decisions more complicated and affect the sales volume of online retailers. Although AR, VR and other technologies try to restore the real feeling of consumers in real shopping, the feeling in virtual environment is still not as rich in information as physical experience brings to consumers. (2) The return polices can only alleviate consumers’ concerns before purchase to a certain extent, but cannot reduce or solve the uncertainty of product matching degree from the former perspective. Moreover, a high return rate will also affect the reputation of online retailers and the sales volume of products. (3) Online trial strategy has only been applied in product sales of online retailers in recent years, and there are few relevant studies. In particular, most of the current theoretical research on the TBYB strategy is described from an empirical perspective, or consider the TBYB strategy separately for modeling and analysis to study the operating mechanism of the strategy, and rarely consider the decision problems of TBYB and return polices at the same time.

In this study, both TBYB strategy and direct sales are taken into consideration, and the inherent characteristics of goods are taken into account. The effects of optimal strategy and product matching degree of online retailers on profit and demand are discussed from the perspectives of permissible returns and non-permissible returns. This is different from the previous research innovation, but also more in line with the reality.

3 Model

3.1 Problem Description and Research Hypothesis

This paper illustrates a sales system consisted of online retailers and consumers, the online retailers decide the product price and whether to adopt the TBYB strategy. When online
retailers adopt the TBYB strategy, consumers can still choose to buy directly. The TBYB strategy refers to that consumers can select multiple products with different colors, flavors and functions of the same category online, when the return is not allowed, online retailers will send the consumers’ selected products and the samples to consumers together. When received all the samples and corresponding products, consumers can try out all of the samples, and keep the best match and pay the fee and then send the other products back to the online retailers; When the return is allowed, online retailers will send the products to consumers without samples, consumers will keep the products that match them best and pay for them, and afterwards Send the other products back to the online retailer. In both scenarios, consumers will pay a certain hassle cost for the process of sending back the product.

Therefore, there will be four situations discussed in this paper. (1) The NN case. Online retailers sell products directly, and are not allowed to return products. (2) The TN case. Online retailers adopt the TBYB strategy, but not allow the customers to return products. (3) The NR case. Online retailers sell their products directly and allow the customers to return them. (4) The TR case. Online retailers adopt the TBYB strategy and allow returns.

This paper makes the following assumptions: (1) standardize the production cost of the product to 0, and there is no depreciation loss of the returned product; (2) consumers will not keep the useless product if it is allowed to be returned and there is no consumer who maliciously use the first payment service, that is, only when the consumers tend to buy the product will use the TBYB service; (3) consumers will buy at most one product in a single online purchase process.

The notation and terminologies used in this paper are as follows:

- $p$: Price of one unit of product
- $b$: Product matching degree, the probability that a product matches a consumer’s preference
- $v$: Consumer valuation $v$ follows a uniform distribution on $[0, a]$, $a$ is the highest, willingness of consumers to pay
- $h$: Hassle cost of return
- $L$: Delivery costs
- $s$: Unit cost of return
- $r$: Return compensation of unit product, $r \leq S$
- $t$: Unit cost of purchasing freight insurance, $t < r$

### 3.2 The NN Case

Online retailers to adopt a direct sales strategy, selling a class of products that not allowed to return, the price of the products is $p^{NN}$. Some of the products match the consumer with probability $\beta$, and if the product matches the needs of the consumer, the utility of the customer for the product is $v - p^{NN}$. If the consumer is dissatisfied with the product, the utility obtained by the consumer is $0 - p^{NN}$ because it cannot be returned.

The expected utility of consumers buying products directly is:

$$E(U^{NN}) = \beta (v - p^{NN}) + (1 - \beta) (0 - p^{NN}).$$

(1)

Consumers will choose to buy the product if and only if $E(U^{NN}) \geq 0$, namely $v \geq v_{NN} = \frac{p^{NN}}{\beta}$, the demand of the product is:

$$D^{NN} = \int_{p_{NN}}^{a} dv = a - v_{NN} = a - \frac{p^{NN}}{\beta}.$$  

(2)
The expected profit of the online retailers is:

$$\pi^{NN} = p^{NN}D^N = p^{NN}\left(a - \frac{p^{NN}}{\beta}\right).$$

Eq. (1.2) and Eq. (1.3) can calculate the purchase decisions of consumers in the NN case, as well as the optimal product pricing, maximum demand and optimal profit of online retailers, and the results are shown in Proposition 1.

Proposition 1: The optimal purchase decision of consumers is that: all consumers valued at $v \in \left[0, \frac{h}{2}\right]$ will not buy the product, and all consumers valued at $v \in \left[\frac{h}{2}, d\right]$ will buy the product directly. The optimal price of online retailers is $p^{NN} = \frac{\alpha \beta}{2}$, the maximum demand is $D^{NN} = \frac{a}{2}$, and the best profit is $\pi^{NN} = \frac{\alpha \beta}{4}$.

### 3.3 The TN Case

Considers that online retailers should adopt the strategy of TBYB on the basis of direct sales, selling a class of products that are not allowed to return, the price of the products is $p^{TN}$. The decision process is divided into three stages, and the decision process is analyzed through backward induction. In the third stage, consumers will eliminate the uncertainty of the product matching degree after trying multiple specifications of products, so when consumers work $v - p^{TN} - h > 0$, consumers will buy the product by the TBYB strategy. In the second stage, consumers make purchase decisions: choose TBYB strategy with a certain hassle cost $h$ buying the product directly or abandon. In the first stage, online retailers decide on product pricing based on consumers’ purchase decisions.

The expected utility of a customer for the product is:

$$\left\{ \begin{array}{l} E\left(U^{TN}\right) = \max\{v - p^{TN}, 0\} - h, \quad -p^{TN} \leq \bar{p} - p^{TN}, p^{TN} \leq \bar{p}, \\
E\left(U^{NN}\right) = \beta\left(v - p^{TN}\right) + \left(1 - \beta\right)\left(0 - p^{TN}\right). \end{array} \right.$$  

Among them, $E\left(U^{TN}\right)$ indicates the expected utility of consumers buying products through the TBYB strategy, and $E\left(U^{NN}\right)$ indicates the expected utility of consumers buying products directly. Let $E\left(U^{TN}\right) = E\left(U^{NN}\right)$, it can get the threshold $\bar{v} = \frac{h}{1 - \beta}, \bar{p} = \frac{\alpha \beta}{1 - \beta}$, and then can get the product demand $D^{TN}_1$ for TBYB strategy and the product demand $D^{TN}_2$ for buying products directly:

$$D^{TN}_1 = \left\{ \begin{array}{l} \int_0^{a} dv = a - \bar{v} = a - \frac{h}{1 - \beta} \quad , p^{TN} \leq \bar{p}, \\
\int_{\bar{v}}^{a} dv = a - v_{TN} = a - p^{TN} - h, \quad p^{TN} > \bar{p}. \end{array} \right.$$  

$$D^{TN}_2 = \left\{ \begin{array}{l} \int_{\bar{v}}^{\bar{v}} dv = \bar{v} - v_{NN} = \frac{h}{1 - \beta} - \frac{p^{TN}}{\beta} \quad , p^{TN} \leq \bar{p}, \\
0 \quad , p^{TN} > \bar{p}. \end{array} \right.$$  

Assume that the average delivery cost of a unit product sold by an online retailer using TBYB strategy is $L$, and when $p^{TN} > \bar{p}$, the expected profit of the online retailer is:

$$\pi^{TN} = \left(p^{TN} - L\right)\left(a - v_{TN}\right) = \left(p^{TN} - L\right)\left(a - p^{TN} - h\right).$$  

Derive $\pi^{TN}\left(p^{TN}\right)$ with respect to $p^{TN}$, let $\frac{\partial \pi^{TN}\left(p^{TN}\right)}{\partial p^{TN}} = 0$, and then can get the optimal price $p^{TN}_* = \max\left\{ \frac{a + L - h}{2}, \bar{p}\right\}$. When $p^{TN} \leq \bar{p}$, the expected profit of online retailers is:
\begin{equation}
π^{TN} = a(p^{TN} - L) + \frac{hL}{1 - \beta} \left(\frac{p^{TN}}{\beta}\right).
\end{equation}

Derive \(π^{TN}\) with respect to \(p^{TN}\), let \(\frac{∂π^{TN}}{∂p^{TN}} = 0\), and then can get the optimal price \(p^{TN^*} = \min\left\{\frac{a\beta}{2}, p\right\}\).

The optimal product pricing, optimal demand and optimal profit of the online retailer in the TN case are shown in proposition 2.

Proposition 2: When online retailers to the TBYB strategy, consumers have different purchase decisions for products with different matching degrees. According to the characteristics of the second partial derivative, it is known that online retailers also have corresponding optimal pricing, maximum demand and optimal profit.

Let \(A = \sqrt{a^2 + (a + L - h)^2 + 4hL}^2 - 4a^2(a + L - h)^2\), there are:

(1) When \(0 \leq \beta \leq \frac{a^2 + (a + L - h)^2 + 4hL - A}{2a^2}\), all consumers valued at \(v \in [0, \frac{a + L - h}{2}]\) will abandon, and all consumers valued at \(v \in [\frac{a + L - h}{2}, a]\) will buy the product through the TBYB strategy. The optimal price of online retailers is \(p^{TN^*} = \frac{a + L - h}{2}\), the maximum demand is \(D^{TN^*} = \frac{a + L - h}{2}\), and the optimal profit is \(π^{TN^*} = \left(\frac{a - L - h}{2}\right)^2\).

(2) When \(\frac{a^2 + (a + L - h)^2 + 4hL - A}{2a^2} < \beta \leq 1 - \frac{h}{a}\), all consumers valued at \(v \in \left[\frac{1}{2}, \frac{h}{1 - \beta}\right]\) will buy the product directly, and all consumers valued at \(v \in \left[\frac{a + L - h}{2}, a]\right\) will buy the product through the TBYB strategy. The optimal price of online retailers is \(p^{TN^*} = \frac{a\beta}{2}\), the maximum demand is \(D^{TN^*} = \frac{a}{2}\), and the demand of the TBYB channel and the direct purchase channel are respectively: \(D_1^{TN^*} = a - \frac{h}{1 - \beta}, D_2^{TN^*} = \frac{h}{1 - \beta} - \frac{a}{2}\). And the best profit is \(π^{TN^*} = \frac{a^2\beta}{4} - aL + \frac{hL}{1 - \beta}\).

(3) When \(1 - \frac{h}{a} < \beta \leq 1\), all consumers valued at \(v \in \left[\frac{1}{2}, \frac{a}{1 - \beta}\right]\) will abandon, all consumers valued at \(v \in \left[\frac{a + L - h}{2}, a]\right\) will buy the product through the TBYB strategy. The optimal price of online retailers is \(p^{TN^*} = \frac{a\beta}{2}\), the maximum demand is \(D^{TN^*} = \frac{a}{2}\), and the best profit is \(π^{TN^*} = \frac{a^2\beta}{4}\).

### 3.4 The NR Case

Online retailers sell a class of products priced at \(p_{NR}\) and allow consumers to return mismatched products. After receiving the product, the consumer is satisfied with the product and retains it with probability \(\beta\), dissatisfied with the product and chooses to return with probability \(1 - \beta\). The consumer will decide whether to buy the product according to its desired utility.

When returning the goods, consumers need to pay the return fee \(s\). After the online retailer confirms that receiving the returned goods, the third-party insurance company will pay the return compensation to the buyer as \(r\), \(r < s\). Return freight insurance is provided by online retailers, with the unit cost of \(i\) and \(t < r\). In this case, when the product matches the consumer preference, the valuation is \(v\) and the consumer can get utility, while when the consumer applies for a return of the product and receives a full refund from the online retailer, the consumer can obtain utility \(r - s\).

The expected utility of consumers buying products directly is:

\begin{equation}
E\left(U^{NR}\right) = \beta\left(v - p^{NR}\right) + (1 - \beta)(r - s).
\end{equation}
Let $E(U^{NR}) \geq 0$, it means $v \geq v_{NR} = \frac{\beta p^{NR} + (1 - \beta)(s - r)}{\beta}$, all consumers valued at $v \in [v_{NR}, a]$ will buy the product, and all consumers valued at $v \in [0, v_{NR})$ will abandon. The demand of the product is:

$$D^{NR} = \int_{v_{NR}}^{a} dv = a - v_{NR} = a - \frac{\beta p^{NR} + (1 - \beta)(s - r)}{\beta}. \quad (10)$$

Considering the cost of return freight insurance, the average profit of online retailers selling unit products is $\beta p^{NR} - t$, so the expected profit of online retailers is:

$$\pi^{NR} = (\beta p^{NR} - t) D^{NR} = \left(\beta p^{NR} - t\right) \left[a - \frac{\beta p^{NR} + (1 - \beta)(s - r)}{\beta}\right]. \quad (11)$$

The Formula (10) and Formula (11) can calculate the optimal product pricing and the optimal demand and the optimal profit of the online retailers in this case, and the results are shown in Proposition 3.

Proposition 3: When online retailers sell their products directly, all consumers valued at $v \in \left[\frac{(\beta a + r) + (1 - \beta)(s - r)}{2\beta}, a\right]$ will buy the product, and all consumers valued at $v \in [0, \frac{(\beta a + r) + (1 - \beta)(s - r)}{2\beta}]$ will abandon. The optimal price of online retailers is $p^{NR*} = \frac{(\beta a - t) - (1 - \beta)(s - r)}{2\beta}$, the maximum demand is $D^{NR*} = \frac{(\beta a - t) - (1 - \beta)(s - r)}{2\beta}$, and the best profit is $\pi^{NR*} = \frac{\beta p^{NR*} - (1 - \beta)(s - r)}{2\beta}$.

To ensure that the demand is non-negative, We define $\beta \geq \frac{t + s - r}{a - r + s}$.

### 3.5 The TR Case

Considers that online retailers should adopt the TBYB strategy on the basis of direct sales, selling a class of products that are allowed to return, the price of the products is $p^{TR}$. Similar to the previous paragraph, the decision process is divided into three stages. In the third stage, consumers will eliminate the uncertainty of the product matching degree after trying multiple specifications of products, so when consumers work $v = p^{TR} = h > 0$, consumers will buy the product by TBYB strategy, and consumers who buy directly need to decide whether to return the goods according to the matching degree of the product and themselves. In the second stage, consumers make purchase decisions: choose TBYB strategy with a certain hassle cost $h$ buying the product directly or abandon. In the first stage, online retailers decide on product pricing based on consumers’ purchase decisions.

The expected utility of the product is:

$$E(U^{TR}) = \max \left\{ E(U^{NR}) = E \left(U^{TR} \right) = \begin{cases} v - p^{TR}, 0 & h \\ \beta \left(v - p^{TR}\right) + (1 - \beta)(r - s). \end{cases} \right\}. \quad (12)$$

Let $E(U^{TR}) = \frac{E(U^{NR}), it can get the threshold $\bar{v}_2 = h \frac{1}{1 - \beta} + r - s + p^{TR}$, $h = \frac{(1 - \beta)(s - r)}{\beta}$, and then can get the product demand $D^{1R}_1$: for TBYB strategy and the product demand $D^{2R}_2$: for buying products directly:

$$D^{1R}_1 = \begin{cases} \int_{v_{NR}}^{a} dv = a - v_{NR} = a - p^{TR} - h & h < \bar{h} \\ \int_{\bar{v}_2}^{a} dv = a - \bar{v}_2 = a - \frac{h}{1 - \beta} - s - r - p^{TR} & h \geq \bar{h}. \end{cases} \quad (13)$$

$$D^{2R}_2 = \begin{cases} 0 & h < \bar{h} \\ \int_{v_{NR}}^{\bar{v}_2} dv = \bar{v}_2 - v_{NR} = \frac{h}{1 - \beta} - \frac{s - r}{\beta} & h \geq \bar{h}. \end{cases} \quad (14)$$
When \( h < \bar{h} \), the expected profit of online retailers is:

\[
\pi^{TR} = (p^{TR} - L)(a - v_{TR}) = (p^{TR} - L)(a - p^{TR} - h).
\]

(15)

Derive \( \pi^{TR} \) with respect to \( p \), let \( \frac{\partial \pi^{TR}}{\partial p} = 0 \), and then can get the optimal price \( p^{TR} = \frac{a + L - h}{2}, \) and the optimal profit \( \pi^{TR} = \left(\frac{a - L - h}{2}\right)^2. \)

When \( h \geq \bar{h} \), the expected profit of online retailers is:

\[
\pi^{TR} = (p^{TR} - L)\left(\frac{h}{1 - \beta} + s - r - p^{TR}\right) + \left(\beta p^{TR} - t\right)\left(\frac{h}{1 - \beta} - s - r\right).
\]

(16)

Derive \( \pi^{TR} \) with respect to \( p^{TR} \), let \( \frac{\partial \pi^{TR}}{\partial p^{TR}} = 0 \), and then can get the optimal price \( p^{TR} = \frac{a + L - h}{2}, \) and the optimal profit \( \pi^{TR} = \left(\frac{a - L - h}{2}\right)^2 + (\beta L - t)\left(\frac{h}{1 - \beta} - \frac{s - r}{\beta}\right). \)

The optimal product pricing, optimal demand and optimal profit of the online retailer in the TR case are shown in proposition 4.

Proposition 4: When online retailers consider adopting the TBYB strategy, consumers have different purchase decisions for products with different product matching degrees. According to the second partial derivative characteristics, it can be seen that online retailers also have corresponding to the optimal product pricing, maximum demand and optimal profit.

1) When \( h < \bar{h} \), namely \( 0 \leq \beta \leq \frac{s - r}{h + s - r} \), all consumers valued at \( v \in \left[0, \frac{a + L - h}{2}\right] \) will abandon, and all consumers valued at \( v \in \left[\frac{a + L - h}{2}, \alpha\right] \) will buy the product through the TBYB strategy. The optimal price of online retailers is \( p^{TR} = \frac{a + L - h}{2}, \) the maximum demand is \( D^{TR} = \frac{a - L - h}{2}, \) and the optimal profit is \( \pi^{TR} = \left(\frac{a - L - h}{2}\right)^2. \)

2) When \( h \geq \bar{h} \), namely \( \frac{s - r}{h + s - r} < \beta \leq 1 - \frac{2h}{a + L + h + 2(s - r)}, \) all consumers valued at will abandon, all consumers valued at \( v \in \left[0, \frac{a + L - h}{2} + \left(\frac{1 - \beta(s - r)}{\beta}\right)\right] \) will buy the product directly, and all consumers valued at \( v \in \left[\frac{a + L - h}{2} + \frac{h}{1 - \beta} + r - s, \alpha\right] \) will buy the product through the TBYB strategy. The optimal price of online retailers is \( p^{TR} = \frac{a + L - h}{2}, \) the maximum demand is \( D^{TR} = \frac{a - L - h}{2} - \left(\frac{1 - \beta(s - r)}{\beta}\right), \) and the demand of the TBYB channel and the direct purchase channel are respectively:

\[
D^{TR}_1 = \frac{a + L - h}{2} - \frac{h}{1 - \beta} + s - r, \quad D^{TR}_2 = \frac{h}{1 - \beta} - \frac{s - r}{\beta}.
\]

And the best profit is

\[
\pi^{TR} = \frac{(a - L - h)^2}{4} + (\beta L - t)\left(\frac{h}{1 - \beta} - \frac{s - r}{\beta}\right).
\]

3) When \( 1 - \frac{2h}{a + L + h + 2(s - r)} < \beta \leq 1 \), all consumers valued at \( v \in \left[0, \frac{a + L - h}{2} + \left(\frac{1 - \beta(s - r)}{\beta}\right)\right] \) will abandon, all consumers valued at \( v \in \left[\frac{a + L - h}{2} + \frac{h}{1 - \beta} + r - s, \alpha\right] \) will buy the product directly. The optimal price of online retailers is \( p^{TR} = \frac{a + L - h}{2}, \) the maximum demand is \( D^{TR} = \frac{a - L - h}{2} - \left(\frac{1 - \beta(s - r)}{\beta}\right), \) and the best profit is \( \pi^{TR} = \frac{\beta(a + L - h - 2r)}{2} \left(\frac{a - L - h}{2} - \frac{(1 - \beta)(s - r)}{\beta}\right). \)

Proposition 4 shows that under the condition that online retailers use the TBYB strategy, when the product matching degree is low, the optimal purchase method for all consumers is to use TBYB strategy to purchase. When the product matching degree is in the middle, both purchase methods have a certain demand. At this time, consumers with high valuation are more inclined to buy by using TBYB strategy. Consumers with moderate valuation will buy directly when they are allowed to return, while consumers with low valuation will abandon. When the product matching degree is high, the optimal purchase method for all consumers is direct purchase.
The optimal profit maximization problem can be formulated as:

\[ \pi^N = \frac{a^2}{4} \quad \text{if} \quad 0 \leq \beta \leq \frac{a^2}{2(a^2)} \]

\[ \pi^T = \frac{a^2}{4} - aL + \frac{hL}{1-\beta} \quad \text{if} \quad \frac{a^2}{2(a^2)} + 4hL - A < \beta \leq 1 - \frac{h}{a} \]

\[ \pi^{NR} = \frac{\{4(a-L-h) + (\beta - t)(\frac{h}{1-\beta} - \frac{s-r}{\beta})\}^2}{4\beta} \quad \text{if} \quad 0 \leq \beta \leq \frac{s-r}{h+s-r} \]

\[ \pi^{TR} = \frac{\{4(a-L-h) + (\beta - t)(\frac{h}{1-\beta} - \frac{s-r}{\beta})\}^2}{4\beta} \quad \text{if} \quad \frac{s-r}{h+s-r} < \beta \leq 1 - \frac{2h}{aL+h+2(s-r)} \]

4 The Optimal Strategy Selection for Online Retailers

4.1 The Optimal Strategy Selection Targeting Profit Maximization

The optimal profit function of the retailer under the circumstances obtained in Chapter III is as follows:

Proposition 5 reflects the change of retailer optimal strategy with product uncertainty when targeting maximum profit.

Proposition 5: (1) In the case of no allowed to return, when \( 0 \leq \beta \leq \frac{a^2}{2(a^2)} \), online retailers should adopt the TBYB strategy; and when \( 1 - \frac{h}{a} \leq \beta \leq 1 \), both the TBYB strategy and direct sales strategy can be adopted.

(2) In the case of allowing returns, when \( 0 \leq \beta < \frac{a^2}{2(a^2)} + 2(s-r)(a+s-r)+B \) , online retailers should adopt the TBYB strategy to obtain the optimal profit. Among \( B = 4(a-L-h)^2 + 4(a-L-h)^2(t+s-r)(a+s-r) \).

Proposition 5 indicates that when allowing returns, online retailers can get more revenue by using the TBYB strategy than direct sales, and both strategies receive equal returns when the product matching degree is high. In the case of return is not allowed, when the product matching degree is low, online retailers can get higher returns by using TBYB strategy than adopting direct sales strategy. When the product matching degree is high, online retailers can get higher returns only by direct sales.

Based on the consideration of returns, there exists a threshold \( \bar{\beta}_1 = \min \left\{ \frac{a^2}{2(a^2)} + 4hL - A, \frac{a^2}{2(a^2)} \right\} \) and \( \bar{\beta}_2 = (\frac{a^2}{2(a^2)} + 2(s-r)(a+s-r)+B) \) for adopting the TBYB strategy, which determines the application scope of the direct sales strategy. It can be found that with the increase of the delivery cost of online retailers or the hassle cost, the overall region adopting the TBYB strategy shows a shrinking trend. Only when the product matching degree is low enough, can we ensure that online retailers have a large space to adopt the TBYB strategy.

4.2 The Optimal Strategy Selection Targeting Demand Maximization

The maximum demand function of the retailer under all circumstances is as follows:

Proposition 6: (1) Without allowing returns, online retailers aiming to maximize demand should adopt a direct sales strategy. When and only then, the TBYB strategy is only equal to the direct sales strategy.

(2) When allowing returns, when any of the following conditions are met, the demand-targeted online retailers should adopt the TBYB strategy:

(1) \( \frac{s-r}{h+s-r} < \frac{1}{L} \) and \( 0 \leq \beta < \frac{a^2}{2(a^2)} + 2(s-r)(a+s-r)+B \)

(2) \( \frac{s-r}{L-h-s+r} < \frac{1}{L} \) and \( 0 \leq \beta < \frac{a^2}{2(a^2)} + 2(s-r)(a+s-r)+B \)

(3) \( \frac{1}{L} < \frac{s-r}{h+s-r} \) and \( \beta < \frac{a^2}{2(a^2)} + 2(s-r)(a+s-r)+B \).
circumstances | The optimal profit
--- | ---
NN | \( D^{\text{NN}} = \frac{a}{2} \)
TN | \( D^{\text{TN}} = \begin{cases} \frac{a-L-h}{2}, & \text{if } 0 \leq \beta \leq \frac{a^2+(a+L-h)^2+4hL-A}{2a^2} \\ \frac{a}{2}, & \text{if } \frac{a^2+(a+L-h)^2+4hL-A}{2a^2} < \beta \leq 1 \end{cases} \)
NR | \( D^{\text{NR}} = \begin{cases} \frac{2\beta}{7}(\beta s-r)-(1-\beta)(s-r), & \beta \geq \frac{t(s-r)}{a+r+s} \\ \frac{2\beta}{7}(\beta s-r)-(1-\beta)(s-r), & \beta < \frac{t(s-r)}{a+r+s} \end{cases} \)
TR | \( D^{\text{TR}} = \begin{cases} \frac{a-L-h}{2}, & \text{if } 0 \leq \beta \leq \frac{s-r}{h+s-r} \\ \frac{a-L+h}{2} - \frac{(1-\beta)(s-r)}{\beta}, & \text{if } \frac{s-r}{h+s-r} < \beta \leq 1 \end{cases} \)

Proposition 6 shows that when returns are not allowed, online retailers with the primary goal of demand maximization should adopt direct sales, and the TBYB strategy can only be used when the product matching degree is above a certain threshold. In the case of allowing returns, it can be found that when the product matching degree is low, online retailers can use the best profit and use the TBYB strategy to achieve the maximum demand. When the product matching degree is moderate, online retailers can obtain the optimal profit by using the TBYB strategy, and directly selling products can achieve the maximum demand. When the product matching degree is high, the online retailers can get the optimal profit and achieve the maximum demand by selling the products directly.

4.3 Numerical Analysis

By setting reasonable values for parameters in the model and ensuring that all parameters are within the effective range, simulation experiments are conducted to verify the conclusion, the selected parameters are as follows: consumer highest payment willingness \( a = 1 \), online retailers delivery cost \( L = 0.01 \), hassle cost \( h = 0.03 \), consumer return fees \( s = 0.08 \), return compensation of unit product \( r = 0.05 \), and unit cost of purchasing freight insurance \( t = 0.02 \).

Figure 1. The impact of product matching when return is not allowed

Figure 1 (a) depicts the effect of product matching degree on the optimal profit of online retailers when they adopt the direct sales strategy or TBYB strategy. It can be found that when the product matching degree below a certain level, because the TBYB strategy can reduce consumer product matching uncertainty, lower risk, so online retailers use the TBYB strategy can get the optimal profit, the optimal purchase decision of consumers is TBYB strategy in this scenario. When the product matching degree is higher than a certain level, the
profit of directly selling products is higher than that of the TBYB strategy. When the product matching degree is extremely high, the optimal purchase way for consumers in both cases is direct purchase. Therefore, only when the product matching degree is below a certain threshold value, it is more advantageous for online retailers to adopt the TBYB strategy. Figure 1 (b) depicts the impact of the product matching degree on the maximum demand. It can be found that when the product matching degree is lower than a certain level, the demand for direct sales is higher than that of the TBYB strategy. When the product matching degree is extremely high, there demands are both the same. Therefore, no matter how the degree of product matching, the direct sales strategy is optimal.

As can be seen from figure 2 (a), in the case of allowing returns, when the product matching degree is lower than a certain level, the TBYB strategy is more advantageous, while when the product matching degree is higher, direct sales can always bring higher profits to online retailers. It is worth noting that when the product matching degree is high, although the optimal purchase method of all consumers under the TBYB strategy is to directly buy products, it is very close to and not equal to the optimal profit in direct sales, and the profit of direct sales is always slightly higher than the TBYB strategy. As shown in figure 2 (b), when the product matching degree is lower than a certain level, using the TBYB strategy is more advantageous, because consumers tend to choose it to avoid risks. When the product matching degree is high, direct sales can bring greater demand to online retailers.

Moreover, parameters such as delivery cost, and hassle cost may also have an influence on the optimal decision choice. If targeting maximum profit. When returns are not allowed, in figure 3 (a), when the product matching degree is very low level($\beta = 0.01$), TBYB as the optimal strategy only occurs when the delivery cost L and hassle cost h are low or high; in figure 3 (b), when the TBYB strategy occurs as the optimal strategy, the delivery cost and hassle cost at a low level. In addition, we noticed that the area of equal profit between TBYB and direct sales appeared, and gradually expanded from high hassle cost area to low area with the growth of product matching degree $\beta$. If the product matching degree $\beta = 1$, the profit of both TBYB and direct sales will be exactly the same.

When returns are allowed. As shown in figure 4 (a), the TBYB strategy as the optimal strategy only occurs, both the delivery costs L and the hassle costs h are at a low level. However, the TBYB strategy will gradually lose its advantage with the growth of product matching degree $\beta$, until direct sales will completely replace it as the optimal strategy. And
we have noted that with the increase of delivery costs and the decrease of hassle costs, the profit that can be obtained from the TBYB strategy will gradually decrease.

If targeting maximum demand. When returns are not allowed, as shown in figure 5, the TBYB strategy shows an advantage only at the extremely high level of product matching degree $\beta$ (i.e., 0.94 1), but this is only equal to the demand of the direct sales strategy. This means that, from a demand perspective, if returns are not allowed, then online retailers should not adopt the TBYB strategy, unless the product is highly matched.

When return is allowed. The TBYB strategy tend to be the most effective choice for suppliers. As shown in figure 6 (a), the direct sales strategy is optimal when the freight insurance is extremely low and $\mu$ (i.e. $s-r$) below a threshold. While the area of direct sales as an optimal strategy expands with the increased product matching degree, it is also a very small area. And we note that with the increase of freight insurance $t$, the profit of direct sales strategy decreases. While the profit of the TBYB strategy is not significantly affected by the freight insurance $t$ and $\mu$, at this time the online retailer does not have to focus on reducing them.
5 Conclusions

This paper starts with the situation of not allowing return and allowing return, and considers to maximize demand or profit of online retailers when using the TBYB strategy or the direct sales strategy, respectively. Further, this paper discussed the optimal choice for online retailers under different product matching degree with the goal of demand or profit maximization. Finally, it was studied that how can product matching degree, consumer hassle cost, distribution cost, return cost and return compensation differences influence the optimal strategy of online retailers through the numerical simulation from which, the following conclusions were made:

1. For the products that are not allowed to return and the online retailer aims profit maximization, if the product matching degree is relatively low, the TBYB strategy should be adopted; if the product matching degree is higher than a certain threshold, the TBYB strategy or direct sales is the optimal choice;
2. For the products that are not allowed to return and online retailers aim at demand maximization, the direct sales strategy should be adopted. If the product matching degree is above a certain threshold, both the TBYB strategy and the direct sales strategy are the optimal choice;
3. If the product matching degree is low, the online
retailer can realize the maximum profit, and when the product matching degree is high, direct sales can not only achieve the maximum demand, but also obtain the optimal profit. (4) The profit of the TBYB strategy is not significantly affected by the freight insurance \( t \) and \( \mu \), the online retailer does not have to focus on reducing them. In the future, we can focus on irrational consumers retaining multiple products, further to study the impact of retention quantity on consumer and corporate decisions, and make up for the deficiency of this paper assuming that consumers will only retain a single product.

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References