A Study of Two-level Inventory Allocation for E-commerce Pre-sales Based on Two Return Methods

Chenxu Du[1], *Fumin* Deng[1], *Chaozhu* Wang[2], *Peng* Luo[1], and *Xuedong* Liang[1,*]

[1]Business School, Sichuan University, Chengdu 610065, People's Republic of China
[2]CCECC Pakistan Branch, Islamabad 44000, Pakistan

Abstract. In recent years, e-commerce shopping festivals based on the pre-sale model have become increasingly popular. Under the pre-sale model, some e-commerce companies have used inventory front-loading methods to give consumers higher time satisfaction for alleviating the pressure caused by order piling. At the same time a new return method has been generated, but existing studies have only considered the traditional return method. In this paper, a dual-objective decision model based on NSGA-II algorithm is developed for maximizing enterprise profit and maximizing consumer time satisfaction under the premise of considering both return methods, and the optimal two-level inventory allocation decision scheme is calculated for e-commerce enterprises. The results show that the existence of two return methods will have a more significant impact on enterprise profit, and therefore is more practical for e-commerce enterprises to make two-level inventory allocation decisions. **Keywords:** E-commerce pre-sale, Returns, Inventory front-loading, Inventory control, Two-level inventory configuration.

1 Introduction

E-commerce and online platform sales have become the choice of more and more people, according to the "China E-commerce Report (2021)" released by the Ministry of Commerce of the People's Republic of China, the total e-commerce transactions in China reached 42.3 trillion yuan in 2021, with a year-on-year growth rate of 19.6%, and the national online retail sales reached 13.09 trillion yuan, with a year-on-year growth rate of 14.1%, accounting for the national e-commerce transactions With 30.9% of the total amount, China has become the largest country in the world in terms of e-commerce transactions. In this context, Chinese e-commerce platforms have launched pre-sale shopping festivals to respond to market demand in order to increase the number of orders to a greater extent [1]. The pre-sale shopping festival breaks the conventional pattern of payment-order processing-delivery by dividing the payment behavior into two stages: the pre-sale stage of paying a deposit and waiting for the final payment is the first stage; in the second stage, consumers who have paid a deposit can pay the final payment, and those who have not paid a deposit can buy in full [2]. Hereinafter referred to as the first stage and the second stage. At the same time, consumers who paid a deposit in the first stage can get a deposit inflation to offset part of the final payment [3].

*e-mail: liangxuedong@scu.edu.cn

In the e-commerce product inventory front model, in order to enable consumers to receive the goods faster, e-commerce companies in the first stage will be based on the collected product order information to send the goods, transported to the distribution center and the terminal distribution station two nodes, the terminal distribution station inventory will be used to meet the first stage of consumers who have paid a deposit [4]. When the inventory of terminal distribution station is not enough to meet the demand of consumers in the first stage, the distribution center will ship the goods to meet the remaining demand of consumers; when the inventory of terminal distribution station is still surplus to meet the demand of consumers in the first stage, the remaining inventory will be used to meet the demand of consumers in the second stage; when the inventory of terminal distribution station is still surplus to meet the demand of consumers in both stages, the surplus goods will be returned to the distribution center. When the inventory of the terminal distribution station still has inventory surplus after meeting the consumer demand in both stages, the excess goods will be returned to the distribution center, while incurring reverse logistics cost [5]. Although the larger amount of inventory forward will enable more consumers to receive the goods, it is also possible to generate more reverse logistics costs [6]. Therefore, for e-commerce enterprises, the inventory quantity decision of distribution center and terminal distribution station will have a great impact on the profits of e-commerce enterprises [7].

In the existing studies on refund behavior due to consumers' impulsive consumption regret, most of the studies only consider the behavior of refunded consumers who directly give up payment at the final payment stage [8], i.e., the deposit at the pre-sale stage becomes the profit of e-commerce enterprises. However, in practice, more consumers often choose to pay the final payment first and then apply for a refund in order to be able to return all the money in order to enable them to avoid losing the deposit, and this refund behavior will have an impact on the profit of the e-commerce enterprise as well as the pre-sale inventory.

Based on the existing research on e-commerce pre-sales-returns, this paper takes into consideration the impact of the refund method of paying the full price and then returning the goods on the profit of e-commerce enterprises, and innovatively proposes to consider the two return methods of giving up the payment and returning the goods after paying the full price, and in the context of considering these two return methods, this paper establishes a dual-objective decision model based on NSGA-II algorithm to maximize the enterprise profit and consumer time satisfaction The NSGA-II algorithm is used to solve the optimal two-level inventory allocation decision scheme for e-commerce enterprises. In the last part, a case study of a brand of basketball shoes on "Double Eleven" is selected for this decision scheme. The analysis shows that this two-level inventory allocation decision model has better fitting results for the actual return situation of e-commerce companies considering two types of returns, which is of certain practical significance and provides new ideas for subsequent research on two-level inventory returns of e-commerce companies.

2 Literature Review

With the booming of global online shopping, various kinds of e-commerce discount methods appear to the public. In this paper, a dual-objective decision model is constructed with the inventory decision objective of maximizing corporate profit and maximizing consumer time satisfaction. Many experts and scholars have systematically studied the pre-sale model. Ma et al. based on the deposit pre-sale model, solved the inventory to obtain the maximum expected revenue by building the expected revenue model [2]. Demirag OC analyzed three refund models-zero refund, partial refund, and full refund-by studying seasonal products and full refunds on retailers' profits using the pre-sale model [9]. In a study of Alibaba executives, Wu analyzes the entry costs of e-commerce companies participating in e-commerce promotions and refines the cost structure on this basis to analyze the costs and benefits associated with companies participating in the e-commerce pre-sale model [10]. Zhang's dual-channel production and sales network optimization strategy based on the pre-sale model and considers the order decoupling point due to the impact of changes in the production and sales model on apparel sales due to differences in order decoupling points [11]. Cho and Tang propose that retailers update their demand for the normal sales phase based on real-time data from pre-sales and argue that there is a relationship between the demand for the two phases [12]. Li et al. develop a decision model for valuing overconfident consumers and analyze retailers' pre-sales strategies in a two-phase environment [13]. Prasad et al. investigate whether retailers use pre-sales under different market parameters in the pre-sale and sale periods. The distribution pattern of demand under the presale model was also considered, assuming that the number of consumers under both sales models obeys a normal distribution [14]. In addition to traditional supply and demand, other social factors can also influence firms' choice of pre-sale mode, for example, Zhang et al. examined three pre-sale strategies for manufacturers who produce seasonal products and sell them to retailers, constructing for each pre-sale mode with the objectives of profit maximization, service level, and optimal pricing decisions [15].

A three-stage decision making approach (TSDA) has been proposed by Gao and Le to solve the problems of dynamic joint pricing and seat inventory control in formulation and resolution [16]. In addition to this, Cai et al. presented a method to assist in the formulation of inventory management policies and optimization of expected profit. The model considers a two-cycle scenario in which retailers place orders based on presale forecasts and then have the opportunity to update orders prior to peak season based on the latest forecasts and current inventory levels [17]. Based on this study, Madadi proposes a multi-level environment consisting of supplier-warehouse-retailer in which a decentralized ordering model and a centralized ordering model verify that multi-level inventory is effective for suppliers in terms of cost savings and risk reduction. Of course, the analysis can also be done from the consumer's perspective [18]. Li et al. investigated the retailer's ordering strategy and inventory allocation strategy to reduce risk in terms of potential consumer opportunistic gains. There is also a need for related research when considering the impact on inventory management strategies when considering the delivery and return of goods [19]. Shao et al. developed a theoretical model of the impact of lenient return policies on consumers' purchase intentions in a cross-border e-commerce environment and explored the impact of consumers' perceptions of risk under different return policies adopted by cross-border e-retailers. The e-commerce return approach can also be considered from a consumer perspective [20]. Wang et al. investigated the relationship between customers' willingness to repurchase and their perceptions of various aspects of the product return process for e-retailers [21]. JD Hess and GE Mayhew, in their study of returns for direct sellers of apparel, assumed that consumers' return times obeyed an exponential distribution, and given the proportion of returns, developed a split-adjusted risk model as an alternative to the simple regression of return time in their study [22]. On the other hand, Davis assumed that returns and sales are linearly related and the ratio between the two is used as the return rate and can be a constant value [23]. Anderson et al. studied that the optimal return strategy needs to balance demand and cost effects and argued that the return rate can obey the Poisson distribution when considering the link of consumer returns this hypothesis [24]. In the context of new retailing combined with pre-sales, Liu et al. develop a seller order model with and without reference price dependence. The model considers consumer order cancellation and delayed purchase behavior, and investigates the impact of limited consumer rationality on the number of seller orders and profitability during pre-sales and spot periods [**?**].

At the same time, many experts and scholars have also conducted research related to the pre-sale model and customer time satisfaction from the perspective of consumers. Shugan and Xie divided consumers into two categories: high valuation customers and low valuation customers [25]. Based on this, most scholars assume that customers have a valuation price range for products in the pre-sale model and predict the customer demand function by utility function and probability distribution model. Liu et al. apply economic principles to explain the existence of the pre-sale mechanism in the context of pre-sale, and study the influence of consumers' time preference factors and social preference factors on the online pre-sale mechanism [26]. In addition to customer time preference studies, delivery time and service time also have an impact on customer time satisfaction. You and Grossmann, in a series of studies of multiple manufacturers, distribution centers and retailers, introduced the concepts of "guaranteed service time" and "order processing time" as the basis for calculating safety stock [27]. Of course, waiting time is also an important factor in customer time satisfaction. In their article, van Riel et al. suggest that the longer the customer wait time, the more likely it is to cause some negative emotions in the customer, which in turn leads to lower customer satisfaction [28]. Jiang and Rosenbloom investigate the role of price perception, service attribute level performance and satisfaction unfolding over time and their effect on customer retention [29].

This section introduces several aspects of enterprise pre-sale mode, related commodity inventory management in pre-sale mode and customer satisfaction in pre-sale mode, and introduces the existing related theoretical research. Although the existing research has achieved certain theoretical research results and practical results, and constructed a relatively perfect research framework, there are still some deficiencies, and there are no multiple ways to consider returns in the context of e-commerce pre-sale mode and Therefore, in this paper, we take the maximization of enterprise profit and customer time satisfaction as the relevant decision objectives, and consider two types of returns in the context of pre-sale mode, in order to carry out the research.

3 Problem Description and Model Construction

3.1 Problem Description

Most of the e-commerce enterprise's distribution system is usually composed of three parts: warehouse - distribution center - terminal distribution station. Distribution center is usually set up by e-commerce enterprises in a region, used for warehousing and distribution of integrated logistics center. The terminal distribution station is the next part of the distribution center, which is usually close to the consumers and can realize the last-loop distribution. The distribution structure considered in this paper is simplified to a distribution center based on the system, i.e., a regional distribution center for e-commerce enterprises, and the terminal distribution stations below are the front warehouses, as shown in figure. 1.

The consumer demand in each region can be divided into two parts: consumers who are aware of the pre-sale activity and those who are not, denoted as X and Y respectively. The number of consumers within the service area of the jth terminal is the quantity of consumers in the first stage who know the pre-sale activity, denoted as X_j, and the number of consumers who actually make a purchase decision is N_{j1}. The number of consumers who do not know the pre-sale activity is the quantity of consumers in the second stage, denoted as Y_j, and the number of consumers who actually make a purchase decision is N_{j2} [12]. The number of potential consumers in the two stages, X_j and Y_j, follows a normal distribution of $\left(\mu_{\mathrm{Xj}}, \delta_{Xj}^2\right)$ and $\left(\mu_{\mathrm{Yj}}, \delta_{Yj}^2\right)$ respectively [14].

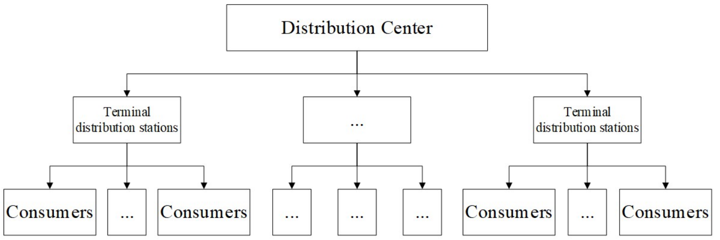

Figure 1. Distribution system structure of e-commerce enterprises

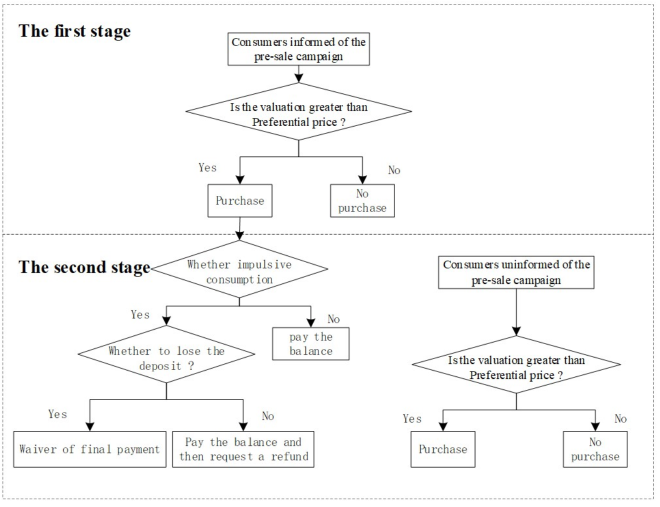

Figure 2. Consumer purchase decision flow chart

Whether the consumer will make a purchase decision depends on the comparison of the consumer valuation and the price of the good, and whether the order delivery time meets expectations. In this paper, we assume that the consumer valuation v obeys a uniform distribution on a to b, $v \in u[a, b]$, where u is the consumer's expectation of the value of the good [25]. Assume that the selling price of the good is p , the discount rate of the good in the pre-sale model is $\xi(0 < \xi \leq 1)$, and the deposit is $\lambda(\lambda \leq p - \xi p)$. For consumers in the first stage, consumers make a purchase decision when their valuation $v > \xi p + \eta t_h$. For consumers in the second stage, consumers make a purchase decision when their valuation $v > p + \eta t_h$. Considering that consumers have different time sensitivities, the consumer's time sensitivity is assumed to be uniformly distributed from 0 to 1, $\eta \in u\,[0, 1]$.

Two return methods have been described in the previous Section, suppose that the percentage of consumers served by the jth terminal distribution station who return goods using the first method is α_{j}, and the percentage of consumers who return goods using the second method is β_{j} [23]. Since this paper gives priority to the optimal inventory allocation problem under the pre-sale model, which means that only the impact of the first-stage consumer return rate on the optimal inventory allocation is considered, in the first stage under the jth terminal distribution station, the actual demand $n_{j1} = N_{j1} * (1 - \alpha_j - \beta_j)$.

Table 1. Description of model parameters

	Parameter of model	Interpretation of model parameter
Decision Variables	q_{cj}	The Inventory front quantity of the *j*th terminal distribution station, *j*=1,2... k
	Q_c	Inventory quantities in distribution centre.
	Q	Total inventory of e-commerce companies.
Consumer-related parameters and variables	N_{j1}	The number of consumers who pay a deposit at the deposit payment stage within the service area of the *j*th terminal distribution station follows a distribution with mean μ_j^1 and standard deviation δ_j^1.
	n_{j1}	The first phase of the actual need to remove the return rate. $n_{j1} = N_{j1} * (1 - \alpha_j - \beta_j)$
	N_{j2}	The number of consumers within the service area of the *j*th terminal distribution station who purchase goods at full price in the second stage obeys a distribution with mean μ_j^2 and standard deviation δ_j^2.
	α_j	The ratio of the number of consumers who canceled the final payment to the number of consumers who paid the deposit in the service area of the *j*th terminal distribution station.
	β_j	Ratio of the number of consumers within the service area of the *j*th terminal distribution station who are refunded the full amount after the final payment.
Parameters and variables related to e-commerce companies	p	Selling Price.
	c	Purchase cost per item.
	h	Out-of-stock cost per item.
	λ	Deposit.
	ξ	Pre-sale price as a percentage of normal sales price
	M_j	Expected profit within the service area of the *j*th terminal distribution station
	M	Total profit expected by e-commerce companies
Parameters and variables related to consumer time satisfaction	t_h	Average order delivery time.
	t_w	Average waiting time for orders.
	t_o	Average order processing time.
	t_d	Average delivery time of orders.
	θ_j	Weighted average time satisfaction for the *j*th terminal distribution station.
	θ	Average consumer time satisfaction of e-commerce companies.

In summary, the consumer purchase decision flow chart under the two-stage pre-sale model is shown in figure. 2.

3.2 Variable Definition

According to the above content analysis, this paper involves numerous parameters related to e-commerce companies, consumers and the market, and the symbolic descriptions of the parameters to be used in the model are shown in table 1 below:

There are two decision variables in this model, the first one is placed in the amount of front inventory at each terminal distribution station, and the second one is the amount of inventory at the distribution center Q_c. The two together make up the total optimal inventory of the e-commerce enterprise, $Q = Q_c + \sum_{j}^{k} q_{cj}$, where *j*=1,2,...k.

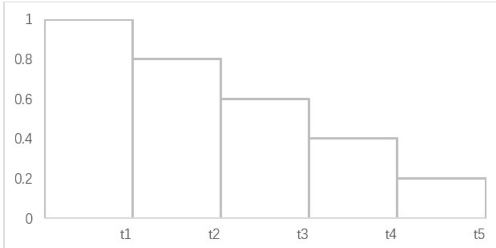

Figure 3. Graph of consumer time satisfaction function

3.3 Model Building

(1)Time satisfaction function

Considering the complexity of directly converting consumers' order delivery length into time satisfaction, this paper uses discrete consumer time satisfaction. The expected order delivery time of consumers is divided into five intervals equally, and the time satisfaction is the same in the same interval range, and each interval corresponds to a different time satisfaction [28].

Order delivery time t_h is divided into three parts: average order response time t_w, average order processing time t_o and average order delivery time t_d [27], $t_h = t_w + t_o + t_d$. The order delivery time t_d is also divided into three categories: the delivery time from the distribution center to the terminal $t_d{}^{\mathrm{cs}}$, the delivery time from the terminal to the consumer $t_d{}^{\mathrm{so}}$, and the delivery time from the distribution center to the consumer $t_d{}^{\mathrm{co}}$, $t_d^{co} = t_d^{cs} + t_d^{so}$. Under the condition of inventory frontage, consumers can avoid the delivery time from the distribution center to the terminal distribution station, i.e., the consumers can get higher time satisfaction by this method. The average order processing time t_o is related to the number of orders, which is measured in minutes per order.

Time satisfaction $\theta$is defined as the affiliation function of the delivery time of their orders, i.e., the individual consumer time satisfaction is $\theta(t_h)$. The expression of the consumer time satisfaction function is shown in Eq. (1).

$$\theta(t_h) = \begin{cases} 1 & t_h \le t_1 \\ 0.8 & t_1 < t_h \le t_2 \\ 0.6 & t_2 < t_h \le t_3 \\ 0.4 & t_3 < t_h \le t_4 \\ 0.2 & t_4 < t_h \le t_5 \\ 0 & t_5 < t_h. \end{cases} \tag{1}$$

The images of the functions corresponding to the different order delivery time t_h of the respective satisfaction are shown in figure. 3.

In the calculation of time satisfaction, the average time satisfaction of consumers is calculated in this paper, so the satisfaction of all consumers is calculated by taking the summed average. The time satisfaction $\theta(\mathrm{t_{h}}{}_{\mathrm{j}}^{i})$ is calculated by averaging the order delivery time of all consumers at the jth terminal in stage i. The time satisfaction is calculated by averaging the order delivery time t_{jh}^{i} of all consumers at the jth terminal in stage i.

Since consumers making purchase choices at each stage have different expectations of order delivery length, the average satisfaction for a single terminal delivery station is a weighted average of the time satisfaction of consumers who choose to purchase at both stages. That is:

$$\theta_j = \left[n_{j1} * \theta(t^1_{jh}) + N_{j2} * \theta(t^2_{jh})\right] / (n_{j1} + N_{j2}). \tag{2}$$

Since inventory lead time enables the consumer to reduce the waiting time ${t_d}^{cs}$ from the distribution center to the terminal distribution station, the order delivery time for goods dispatched from the *j*th terminal distribution station $t^{s1}_{jh} = t_h - {t_d}^{cs}$. However, there are two possibilities: the inventory lead time at the terminal distribution station is insufficient to meet the first stage order demand and the inventory lead time at the terminal distribution station exceeds the first stage order demand, so the two cases are discussed separately.

(a) When the inventory lead time of the *j*th terminal is not enough to meet the demand of the first stage orders, the missing part needs to be insufficient from the distribution center, then the order delivery time of this part $t^{c1}_{jh} = t_h$. Since this part of the order requires background processing time, it leads to an increase in the order waiting time t^{ps1}_o for the second stage, $t^{ps1}_o = \max(N_{j1} - q_{cj}, 0) * t_o$, so the delivery time of the second stage order $t^{c2}_{jh} = t_h + t^{ps1}_o$.

(b) When the inventory lead at the *j*th terminal delivery station exceeds the first stage order demand, its excess can be used to satisfy the second stage order demand. Then the delivery time for that part of the second stage order $t^{s2}_{jh} = t^{c2}_{jh} - {t_d}^{cs}$.

Summarizing the above yields the time satisfaction objective function:

$$\theta_j = \left[n_{j1} * \theta(t^{ps1}_{jh}) + N_{j2} * \theta(t^{ps2}_{jh})\right] / (n_{j1} + N_{j2}). \tag{3}$$

In this formula:

$$t^{ps1}_{jh} = [t^{s1}_{jh} * \min(n_{j1}, q_{cj}) + t^{c1}_{jh} * \max(n_{j1} - q_{cj}, 0)]/n_{j1}, \tag{4}$$

$$t^{ps2}_{jh} = t^{ps2'}_{jh} / N_{j2}, \tag{5}$$

$$\begin{aligned} t^{ps2'}_{jh} &= t^{s2}_{jh} * \min\{max(q_{cj} - n_{j1}, 0), N_{j2}\} + t^{c2}_{jh} * \{[N_{j2} - (q_{cj} - n_{j1})] * \\ &\min[\max(q_{cj} - n_{j1}, 0), \max(n_{j1} + N_{j2} - q_{cj}, 0), 1]\}. \end{aligned} \tag{6}$$

As for e-commerce enterprises, their total average time satisfaction is the weighted average of the time satisfaction of each terminal distribution station, that is:

$$\theta = \sum_{j=1}^{k} \theta_j * (n_{j1} + N_2) / \sum_{j=1}^{k} (n_{j1} + N_{j2}). \tag{7}$$

(2)Profit function

According to the relationship between the two levels of inventory, we assume that the inventory in the *j*th terminal distribution station is q_{cj}, and no horizontal transfer is allowed between each terminal distribution station, and the inventory in the distribution center is Q_c, then the total inventory of the e-commerce enterprise is. When the inventory is not enough to meet the demand, there will be out-of-stock cost, each unit of out-of-stock cost is recorded as h, the purchase cost of each piece of goods is recorded as c.

In addition to this, the flow of goods between stations will also generate logistics costs, we can assume each piece of goods sent from the distribution center to the *j*th distribution station logistics costs is g_{cj}, each piece of goods sent from the th distribution station to the demand point logistics costs is g_{jo}, each piece of goods sent from the distribution center to the demand point logistics costs is g_{co}, and according to the actual situation $g_{co} < g_{jo} + g_{cj}$ can be assumed. When the inventory exceeds the demand, the reverse logistics will be generated, and the reverse logistics cost is assumed to be equal to the normal logistics cost.

In the service area of the jth terminal distribution station, the number of consumers who make a purchase decision in the first stage is N_{j1}, and the number of consumers removing the people who apply for return after the later stage is n_{j1}. According to the uniform distribution of consumer valuation, the number of actual purchase users should be $\frac{b-(\xi p+\eta t_h)}{b-a} * n_{j1}$; the number of consumers involved in the second stage of purchase is N_{j2}.

In the service area of the jth terminal distribution station, the revenue of the first stage is the price of goods sold in the first stage and the deposit, that is $\xi * p * n_{j1} + \alpha_j N_{j1} \lambda$; the revenue of the second stage is $p * N_{j2}$, the purchase cost of goods sold is $c * (q_{cj} + q_{co})$; the terminal distribution station logistics cost is $(g_{cj} + g_{co}) * \min(n_{j1} + N_{j2}, q_{cj})$; the distribution center logistics cost is $g_{co} * \max(n_{j1} + N_{j2} - q_{cj}, 0)$; the out-of-stock cost is $h * \max(n_{j1} + N_{j2} - q_{cj}, 0)$; the reverse logistics cost of unsold goods is $g_{cj} * \max(q_{cj} - n_{j1} - N_{j2}, 0)$.

In summary, the profit function of the jth terminal distribution station can be obtained as follows:

$$\begin{aligned} M_j = \xi * p * n_{j1} + \alpha_j N_{j1} \lambda + (p * N_{j2}) - c * (q_{cj} + q_{co}) - \\ (g_{cj} + g_{co}) * \min(n_{j1} + N_{j2}, q_{cj}) - h * \max(n_{j1} + N_{j2} - q_{cj}, 0) \\ -g_{co} * \max(n_{j1} + N_{j2} - q_{cj}, 0) - g_{cj} * \max(q_{cj} - n_{j1} - N_{j2}, 0). \end{aligned} \tag{8}$$

The profit of each terminal distribution station is summed to obtain the total profit function of the e-commerce enterprise:

$$\begin{aligned} M = & \sum_{j=1}^{k} [\xi * p * n_{j1} + \alpha_j N_{j1} \lambda + (p * N_{j2})] - c * (\sum_{j=1}^{k} q_{cj} + Q_c) \\ & - \sum_{j=1}^{k} (g_{cj} + g_{co}) * \min(n_{j1} + N_{j2}, q_{cj}) - g_{co} * min(Q_c, \sum_{j=1}^{k} \max(n_{j1} + N_{j2} - q_{cj}, 0)) \\ & - \sum_{j=1}^{k} g_{cj} * \max(q_{cj} - n_{j1} - N_{j2}, 0) - h * \max(\sum_{j=1}^{k} \max(n_{j1} + N_{j2} - q_{cj}, 0) - Q_c, 0). \end{aligned} \tag{9}$$

(3)Dual objective decision model

The final bi-objective optimal function is constructed with the highest objective of profit maximization and time satisfaction. The objective function of two-level inventory allocation for e-commerce enterprises is as follows:

$$\max \theta, \max(M). \tag{10}$$

The constraints are as follows:

$$\begin{gathered} \lambda \leq p - \xi p \\ \sum_{j=1}^{j} q_{cj} + Q_c \geq \sum_{j=1}^{j} n_{j1} \\ \theta(t_h) = \begin{cases} 1 & t_h \leq t_1 \\ \ldots & \ldots \\ 0 & t_5 \leq t_h. \end{cases} \end{gathered} \tag{11}$$

Constraint 1 indicates the deposit limit in the pre-sale model; constraint 2 indicates that the inventory of the front-end stock and the distribution center can guarantee the supply of demand in the first stage; and constraint 3 indicates the subordinate function of time satisfaction.

4 Algorithm Solving

4.1 Conditional Assumptions

Assumption 1. No lateral transfers are allowed between terminal distribution stations.

Assumption 2. Each consumer can be allowed buy only one item.

Assumption 3. There is no replenishment within the sales period, without considering the order lead time and emergency orders.

Assumption 4. Logistics costs are only related to the quantity of goods, without considering the series of storage costs, transportation costs and loading and unloading costs.

Assumption 5. The considered consumer returns occur before the consumer receives the goods, i.e., returns after the consumer receives the goods are not considered.

Assumption 6. The impact of the duration of the pre-sale phase on the order delivery duration is not considered.

4.2 Algorithm Introduction

The main difference between NSGA and the simple genetic algorithm is that the algorithm is stratified according to the dominance relationship between individuals before the selection operator is executed. Its selection operator, crossover operator and variation operator are not different from the simple genetic algorithm [30].

Before the selection operation is executed, the population is sorted according to the dominance and non-dominance relationships between individuals.

First, all the non-dominated individuals in this population are identified, and they are assigned a shared virtual fitness value. The first non-dominated optimal stratum is obtained.

Then, ignoring this group of stratified individuals, the other individuals in the population continue to be stratified according to the dominance and non-dominance relationship and are given a new virtual fitness value that is smaller than the value of the previous stratum, and the above operation is continued for the remaining individuals until all individuals in the population are stratified.

4.3 Step of Algorithm

The decision objective of the two-level inventory model considering two return methods is the bi-objective optimum of consumer time satisfaction and profit maximization of the e-commerce enterprise, so a non-dominated sorting genetic algorithm is used to solve it using MATLAB, and the solution steps are as follows:

Step 1. real number coding, the model has a total of k+1 decision variables, each chromosome coded as k+1 genes, the first k genes are the number of stock in the terminal distribution station, the (k+1)th gene is the number of stock in the distribution center.

Step2. randomly generating a certain number of individuals as initialized populations according to the range of values of the variables.

Step 3. calculating the objective function value and virtual fitness of each individual, using non-dominated ranking with calculation of crowding, and ranking each individual according to its merit.

Step 4. selection: using the binary tournament method to select half of the individuals within the population as parents.

Step 5. crossover and mutation, randomly selecting two individuals to break the chromosomes to form new individuals, continuously outputting new individuals of half the population size and combining them with the parent to become new offspring; mutation operation on individuals by probability, i.e. randomly selecting a gene for random assignment.

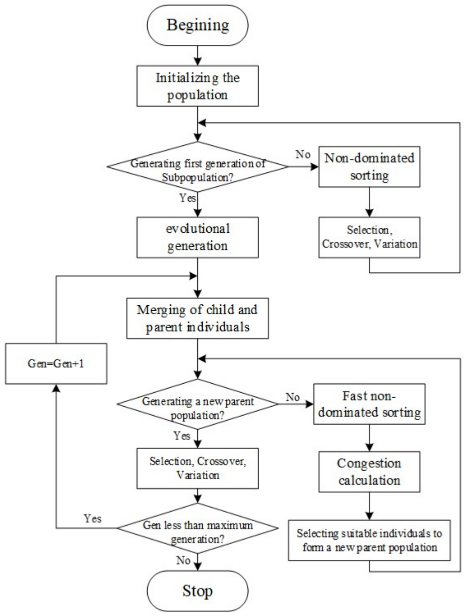

Figure 4. Algorithm step flow chart

Step 6. elite strategy, calculate the fitness of the population, and then retain half of the number of individuals with higher fitness as the final offspring.

Step 7. Store the mean value of the objective function for the individuals with current Pareto frontier of 1, and the frontier face of the population, i.e., the individuals in the first stratum.

Step 8. The children become the parents for the next iteration.

The flow chart of the solution steps is shown in figure. 4.

5 Example Analysis and Sensitivity Analysis

In this subsection, a brand of basketball shoes is used as an arithmetic example to analyze how e-commerce companies should control the inventory quantity of each terminal distribution station and distribution center under the pre-sale mode considering the double return method. A sensitivity analysis is also performed on some relevant parameters to derive the impact of their value changes on the two objective functions.

5.1 Step of Algorithm

This paper takes a two-stage pre-sale of a brand of basketball shoes during the “Double Eleven” period as a case study. Consumers who participate in the pre-sale can get a certain price discount after paying a deposit. Considering the characteristics of basketball shoes, the returned goods can still be used for later sales. According to Tmall’s “Double Eleven” analysis of pre-sale history data, the ratio between the first and second stage sales was set at about 4:1. The basketball shoes were priced at 599 RMB, and after the user paid a 40 RMB deposit, he or she could pay a final payment of 499 RMB to complete the purchase of the product. Consumers who did not pay a deposit could purchase the product directly for

599 RMB in the second phase. The consumer's valuation of the sneaker follows a uniform distribution of (540,800), and if the consumer who paid the deposit does not pay the final payment in the second stage, the deposit is not refundable. According to the study of Shen et al. [31], the return rate of the sneaker is usually in the range of 10% to 30%, so the percentage of consumers who pay the final payment and then apply for refund is set to 20%, while the number of consumers who directly give up the deposit is usually very small, so the percentage of consumers in this part is set to 3%. That is p = 599, $\lambda \approx 0.9$, $a = 540$, b = 800, $\alpha_j = 3\%$, $\beta_j = 20\%$.

The purchase cost of the sneaker is 200 RMB, and the cost of out-of-stock is set at 20% of the selling price in accordance with the penalty for late delivery on the e-commerce platform, i.e., the cost of out-of-stock is 120 RMB. That is $c = 200$, $h = 120$.

The e-commerce company has a distribution center K in a city and three terminal distribution stations under the distribution center. The consumers who know about the pre-sale activities in the first stage in the service area of the terminal distribution station K_1 obey the normal distribution $(800, 20^2)$, and the consumers who know about the pre-sale activities in the second stage obey the normal distribution $(200, 10^2)$; the consumers who know about the pre-sale activities in the first stage in the service area of the terminal distribution station obey the normal distribution $(600, 20^2)$, and the consumers who know about the pre-sale activities in the second stage obey the normal distribution $(180, 20^2)$. The consumers within the service area of the terminal distribution station K_3 who know about the pre-sale activities in the first stage obey the normal distribution $(1200, 10^2)$, and the consumers who know about the pre-sale activities in the second stage obey the normal distribution $(350, 10^2)$. That is $\mu_{\mathrm{X1}} = 800$, $\delta^2_{X1} = 20^2$, $\mu_{\mathrm{Y1}} = 200$, $\delta^2_{Y1} = 10^2$; $\mu_{\mathrm{X2}} = 600$, $\delta^2_{X2} = 20^2$, $\mu_{\mathrm{Y2}} = 180$, $\delta^2_{Y2} = 20^2$; $\mu_{\mathrm{X3}} = 1200$, $\delta^2_{X3} = 10^2$, $\mu_{\mathrm{Y3}} = 350$, $\delta^2_{Y3} = 10^2$.

The logistics cost of goods from the distribution center K directly to the hands of consumers g_{oo}= 12 RMB, from the distribution center K to the terminal distribution station K_1, K_2, K_3 the logistics cost g_{cj}= 8 RMB, from the terminal distribution station to the consumer logistics cost g_{jo}= 6 RMB. According to the data of the State Post Bureau, the average time limit of express delivery is 56.74 hours, the average time limit of processing at the sending place is 7.93 hours, the average time limit of transportation is 34.62 hours, the average time limit of processing at the sending place is 10.52 hours, and the average time limit of delivery is 3.66 hours[1]. Accordingly, the delivery time from the distribution center K to the terminal distribution stations K_1, K_2, K_3, and the delivery time from the terminal distribution stations to the consumers t_d^{cs} =35 hours, and the delivery time from the terminal distribution stations to the consumers t_d^{so}=15 hours, and the delivery time from the distribution center to the consumers t_d^{co}=50 hours. The average order waiting time t_w=8 hours, and the average order processing time is 0.02 hours per order[2]. Consumer expectations for the delivery time of an order are between (0, 96) hours.

5.2 Solution Results and Comparative Analysis

The optimal results for the weighted time satisfaction and total profit components corresponding to different inventory quantities at each terminal distribution station and distribution center based on two return methods of inventory frontage strategies in pre-sale mode, solved in MATLAB based on genetic algorithms, are shown in table 2.

The weighted time satisfaction and total profit analysis is shown in figure. 5.

[1]National Post Office "Notice on the Results of the Third Quarter 2022 Express Service Satisfaction Survey and Time Frame Punctuality Test"
https://www.spb.gov.cn/gjyzj/c100015/c100016/202211/b9647afcb995450996ccc46d2a928d7d.shtml

[2]Refer to the average order processing time of JingDong self-operated

Table 2. E-commerce pre-sale inventory allocation solution

Number	K_1	K_2	K_3	K	Total Inventory	Time satisfaction	Total profit
1	1010	699	375	142	1010	0.73	854863.8
2	1014	475	763	414	1014	0.72	856117.8
3	1015	477	436	508	1015	0.68	858155.8
4	1015	508	436	477	1015	0.68	859323.8
5	1015	1495	1593	479	1015	0.66	860905.2
6	1031	508	134	436	1031	0.64	861296
7	1066	740	410	1659	1066	0.63	863182
8	1123	213	475	118	1123	0.62	867841.8
9	1134	189	658	101	1134	0.72	868379
10	1150	1944	1366	550	1150	0.61	872196
11	1340	596	470	1059	3465	0.6	872589
12	1393	148	102	184	1827	0.59	878395.8
13	1429	401	656	364	2850	0.58	881236
14	1488	737	629	114	2968	0.56	957605
15	1593	373	738	419	3123	0.52	964281.8

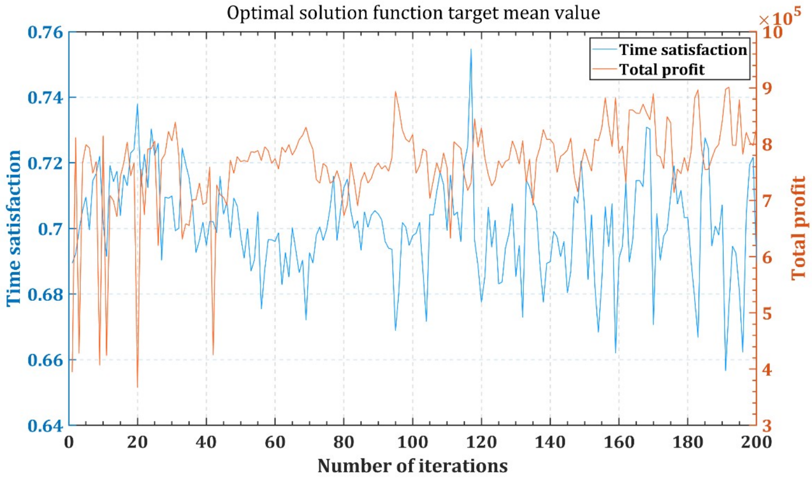

Figure 5. E-commerce pre-sale time satisfaction - profit graph

According to the table results can be seen, from the top to the bottom of the e-commerce business profits are rising, while the overall time satisfaction trend is declining. But the process of slowly increasing profits, the ninth line of data time satisfaction also steeply increased, after this and then the pursuit of greater profits, its time satisfaction declining trend is more obvious, so the program can be considered the optimal configuration program. That is, the time satisfaction is 0.68, the total profit is 964301.5, the inventory quantity of three terminal distribution stations are 522, 480, 364, and the inventory quantity of distribution center is 2569. The optimal configuration scheme of e-commerce pre-sale based on two return methods is shown in table 3 below:

Compared with the single return method considering only lost deposit in existing studies, the optimal results for the weighted time satisfaction and total profit components of the pre-

Table 3. Optimal inventory allocation solution for e-commerce pre-sales

K_1	K_2	K_3	K	Total Inventory	Time satisfaction	Total profit
1203	522	480	364	2569	0.68	964301.5

Table 4. E-commerce pre-sale inventory allocation scheme for a single return method

Number	K_1	K_2	K_3	K	Total Inventory	Time satisfaction	Total profit
1	1063	257	1235	158	2713	0.67	949561.5
2	1073	452	56	22	1603	0.66	953624.7
3	1086	482	583	1679	3830	0.65	954649.5
4	1113	518	850	823	3304	0.64	956255.5
5	1126	503	586	110	2325	0.63	958529.5
6	1154	1477	542	110	3283	0.59	960384.7
7	1158	1575	484	87	3304	0.58	961105.5
8	1191	855	836	311	3193	0.56	963715.5
9	1203	522	480	364	2569	0.68	964301.5
10	1262	588	716	381	2947	0.59	967049.5
11	1340	596	470	1059	3465	0.57	971933.5
12	1393	148	102	184	1827	0.58	976525.5
13	1429	401	656	364	2850	0.58	983597.5
14	1488	737	629	114	2968	0.56	989425.5
15	1593	373	738	419	3123	0.53	1001223

sale e-merchants for the single return method considering only the lost deposit are shown in table 4:

As can be seen from the data in the table, when the second method of returning goods without considering consumption, the profits of e-commerce enterprises appear to rise more significantly, and in that case will have some influence on e-commerce enterprises to make optimal decisions.

5.3 Sensitivity Analysis

To analyze the impact of the return rate on the two-level inventory allocation study of e-commerce pre-sales, since most consumers choose the option of paying the final payment before applying for the return, only the change of the return rate β_j is considered as the impact of the results, and the values of β_j are 0.15,0.18,0.21,0.24,0.27,0.30,0.33,0.36. The above data are substituted to solve the analysis, and the results are shown in table 5:

Different return rates imply differences in consumer demand, and at this point there will be some variation in the amount of inventory at each terminal distribution station and distribution center. When the consumer refund rate continues to increase, the profits of e-commerce companies will also continue to decline. Therefore, in order to obtain higher profits, e-commerce companies need to improve their service level and other methods to reduce consumer dissatisfaction with the goods, thereby reducing the return rate.

6 Summary

A study of two-level inventory configurations of pre-sales e-commerce firms under two return methods leads to the following conclusions:

Table 5. E-commerce pre-sale inventory allocation solutions for different return rates

β	K_1	K_2	K_3	K	Total Inventory	Time satisfaction	Total profit
$\beta = 0.15$	1380	928	754	339	3401	0.68	940169.2
$\beta = 0.18$	1360	437	655	567	3019	0.68	916877.1
$\beta = 0.21$	1572	1255	1045	1589	5461	0.68	898124.7
$\beta = 0.24$	1653	369	563	178	2763	0.68	866781.9
$\beta = 0.27$	1477	224	445	432	2578	0.68	827300.7
$\beta = 0.30$	1529	747	771	165	3212	0.68	782138.8
$\beta = 0.33$	1229	268	1410	1218	4125	0.68	694183.7
$\beta = 0.36$	1234	272	1795	1033	4334	0.68	676652.3

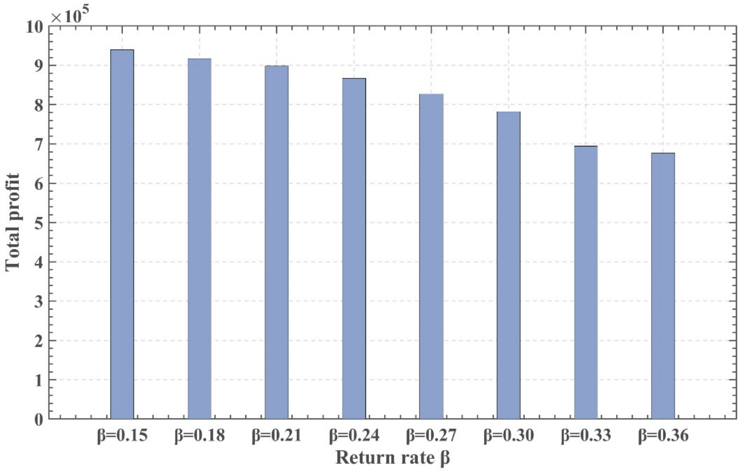

Figure 6. E-commerce pre-sale profit chart with different return rates

(a) Order delivery length is the main factor affecting consumer time satisfaction, but in the process of pursuing consumer time satisfaction, e-commerce companies need to continuously increase the amount of inventory pre-positioning, which in turn leads to a greater possibility of having the supply exceed the demand at the terminal distribution stations and generating more reverse logistics costs leading to a decrease in the total profit of e-commerce companies. Therefore, when developing the optimal configuration of the two levels of inventory, the two goals of total profit and time satisfaction should be taken into account.

(b) The evidence has a greater impact on the bi-objective optimal decision of e-commerce enterprises when considering two return methods, and the existence of two return methods is more consistent with showing the reality. Therefore, the model is more practical for e-commerce enterprises to do two-level inventory allocation research under pre-sale mode.

With the continuous research on two-level inventory allocation for pre-sale e-commerce companies, the limitations of the existing research are found, which can be continued in the future, and the limitations are as follows:

(a) This paper only considers the study of individual commodities of e-commerce enterprises, and does not consider the impact of the correlation between multiple commodities on the demand for that commodity. For example, when a certain sportswear and basketball shoes belong to the same series of products, consumers who buy the clothing may also choose to buy the shoes, and this part of demand is not considered at this stage.

(b) This paper currently only considers one-time orders, which incur out-of-stock costs when there is a shortage of goods. However, in reality, e-commerce companies have the possibility of emergency replenishment, which can avoid out-of-stock costs in this case.

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