A Comparative Study of Preventive Maintenance Thresholds for Deteriorating Systems

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Abstract. Degradation of engineering systems is a phenomenon that affects the reliability and the lifetime of systems. Maintenance of deteriorating systems has been widely studied over the past 50 years. Numerous maintenance models have been proposed in the literature to manage degradation. In this paper, we focus on a condition-based maintenance strategy for those systems suffering a continuous deterioration process. We propose the use of a non-constant maintenance threshold to determine when a preventive action is required. The main goal is to analyse the effect of non-constant threshold in the long-term maintenance cost rate. We demonstrate numerically the convergence of the long run cost rate and some statistical properties of the proposed model. We compare the use of a constant threshold with a non-constant threshold through two different simulated case studies. The simulations provide some statistical results that reveals interesting advantages and drawbacks of using a non-constant threshold.

Keywords: Deteriorating systems, Gamma process, Preventive maintenance, Maintenance management.

1 Introduction

Maintenance optimization is gaining importance due to the significative operation and maintenance (O&M) costs, required to ensure the availability of production systems. Modern systems demand sophisticated maintenance strategies to ensure reliability and availability, without increasing the O&M costs. Maintenance strategies can be classified into three main groups: corrective maintenance (CM), preventive maintenance (PM) and condition-based maintenance (CBM). Among these strategies, CBM has demonstrated to be an efficient way to reduce the lifetime cycle cost by performing maintenance activities at the best time before a system failure [1]. Researches has been developed CBM for numerous systems, e.g. in the field of nuclear industry [2]; renewable energy systems such as wind farms [3], solar power plants [4, 5] or photovoltaics [6]; in railway transportation [7], maintenance of infrastructure [8], etc.

In this paper, we focus on those systems which undergo a continuous degradation process over time. Consequences of degradation can cause inoperability of systems, downtimes, costs, etc. or even catastrophic consequences in safety-critical systems, such as in the fields of nuclear technology, aviation, or medicine. In general, two types of stochastic deterioration models can be distinguished, discrete-state and continuous-state deterioration models.

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Discrete state-deterioration is usually modelled through Markov [9] or Semi-Markov processes [10]. On the other hand, continuous-state deterioration is commonly modelled through gamma [11], process; Wiener [12], or inverse Gaussian processes [13]. These models are used to understand deterioration processes in single and multi-unit systems.

Regarding the maintenance strategy, an essential characteristic to classify the model is how the information of the system is obtained. Continuous monitoring is the most effective method [13], since the system condition in real time is provided, and alarms can be triggered immediately after the detection of anomalies. However, all deteriorating systems cannot be equipped with continuous monitoring due to excessive cost, inaccessibility, technological limitations, legal issues, or others measurement limitations. In these cases, planned inspections, which have demonstrated to be high cost effective, provide a valuable alternative. Some research works in literature assume variable time between consecutive inspections and determine the optimum times to perform such inspections [14]. Most of maintenance models described in the literature are based on perfect maintenance, i.e., maintenance actions recover the deteriorating systems back to the as-good-as-new (AGAN) state. Other studies propose a more realistic scenario based on imperfect maintenance, where maintenance actions are partial repairs that cannot lead the system to the AGAN state. Moreover, some imperfect CBM models consider past dependent partial repairs [11].

In this study, we focus on single unit systems with continuous state degradation where the degradation is modelled through a Gamma process. In the recent years, numerous models based on Gamma deteriorating systems have been developed with different specificities: Reference [15] developed a maintenance strategy considering performance based maintenance, Reference [16] considers a similar model but with imperfect maintenance; Reference [17] includes some economic aspects such as the market price volatility to determine the best maintenance strategy; Reference [18] considers a system with multiple Gamma degradation processes; Reference [19] developed a maintenance strategy for a system with random failure thresholds. As can be observed in the literature, the maintenance of Gamma modelled deteriorating systems is widely studied. Currently, this research line tends to analyse systems with particular features, e.g. centrifugal pumps [20], heavy machines tools [21], micro-engine systems [22] where the maintenance must be adapted to some specificities.

The main novelty of this research work is the use of a non-constant preventive maintenance threshold for continuous deteriorating systems with perfect maintenance. Deterioration is progressive over time; therefore, a time dependent preventive maintenance threshold can be useful not only for determine the best time to carry out a preventive action, but also to identify an abnormal deterioration of the system. This analysis, which has not been found in the literature, provides important results that can be useful to design optimum maintenance strategies. Specifically, this study contributes to determine if non-constant maintenance thresholds can improve any aspect of maintenance.

The remainder of this study is structured as follows: Section 2 presents the proposed degradation model, detailing the mathematical fundamentals and assumptions; Section 3 shows the behaviour of the model when both constant and non-constant maintenance thresholds are considered; Section 4 explains the calculation of the long run cost rate, which will be used to evaluate the goodness of the maintenance strategy. Section 5 shows some statistical results for different simulated scenarios. Finally, Section 6 presents some conclusions and proposes future works.

## 2 The Proposed Model

In this study, we use a stochastic degradation process model based on reference [23]. This model considers a single unit deteriorating system with the following assumptions:
Let the random variable $X_t$ stands for the deterioration state of the system at time $t$, being $X_0 = 0$ and $t \geq 0$. The degradation increment $\Delta X(t, s) = X_t - X_s$, with $0 \leq s \leq t$, follows a Gamma distribution with shape parameter $\alpha = (t - s)\gamma$ and scale rate $\beta$. Then: $\Delta X \sim \Gamma((t - s)\gamma, \beta)$ whose probability density function (pdf) is defined by:

$$f(s, t, x) = \frac{x^{(t-s)\gamma-1}}{\Gamma((t-s)\gamma)} e^{-(t-s)\gamma x} e^{(-\beta s)}, \text{ for } x \geq 0. \quad (1)$$

The cumulative density function (cdf) is:

$$F(s, t, x) = \frac{\gamma((t-s)\gamma, (\beta x))}{\Gamma((t-s)\gamma)}.$$

where $\gamma(\cdot)$ is the lower incomplete gamma function. The survival function can be defined by:

$$\bar{F}(s, t, x) = 1 - F(s, t, x) = \frac{\Gamma((t-s)\gamma, (\beta x))}{\Gamma((t-s)\gamma)}.$$

In terms of degradation, the system is assumed to be As Good As New (AGAN) after each maintenance action, i.e. perfect maintenance is considered. However, in the proposed model, the degradation only affects system reliability and does not impact the system performance.

To distinguish between the application of a preventive and a corrective action, we consider that only corrective actions will reset the system to an AGAN condition. The maintenance activity is modelled with the following characteristics:

- The state of the system is observed by inspections. These inspections are perfect, i.e., they reveal the state of the system without uncertainty. Inspections, which are assumed to be instantaneous, are carried out at times $(T_n)_{n \in \mathbb{N}}$ with $(T_0) = 0$. Let $T^-_n$ and $X^-_T$ be the time and the state of the system just before an inspection, respectively. Inspections are regularly carried out at times $T_n = T_{n-1} + \lambda k$, where $k = 1, 2, 3, \cdots$ and $\lambda$ is the time between two consecutive inspections.

After each inspection, depending on the state of the system, a decision is made. Two thresholds are employed:

- The threshold $L$ determines a critical deterioration level above which the system is failed. If $X_T^- > L$ then, a corrective maintenance (CM) action will be made.

- The threshold $M(t)$ determines whether a preventive action must be performed, such that if $M(T^-_n) < X_T^- < L$, then a preventive maintenance action is required. In this paper, we propose a novel non-constant preventive maintenance threshold. The main objective is to assess the effect of a non-constant preventive threshold in the long run cost rate of maintenance. This preventive maintenance threshold is defined by a linear equation as:

$$M(t) = m(t - T_{CM}) + b, \quad m, \quad b \in \mathbb{R}. \quad (4)$$

where $T_{CM}$ is the time when the last corrective maintenance activity was performed.

- If $X_T^- > M(T^-_n)$, then no maintenance is required.

- If $X_T^- > M(T^-_n)$, then maintenance is required.

- After each maintenance action, the system is returned to an AGAN state. Therefore, the degradation immediately after a corrective action ($X^+_{T_{CM}}$) and the degradation immediately after a preventive maintenance action ($X^+_{T_{PM}}$) are equal to 0.

3
Table 1. Parameter configuration of scenarios

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$m$</th>
<th>$b$</th>
<th>$\lambda$</th>
<th>$L$</th>
<th>$C_i$</th>
<th>$C_p$</th>
<th>$C_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant Preventive maintenance</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>35</td>
<td>3</td>
<td>60</td>
<td>0</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>Threshold</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Scenario 2</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Constant Preventive maintenance</td>
<td>1</td>
<td>5</td>
<td>0.3</td>
<td>50</td>
<td>3</td>
<td>60</td>
<td>0</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>Threshold</td>
<td></td>
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</tbody>
</table>

3 Scenarios for Simulation: Case Studies

The main goal of this work is to compare the system behaviour under two different maintenance policies. For this purpose, two scenarios will be considered: on one hand, a scenario with a constant maintenance threshold and, on the other hand, a scenario with a non-constant maintenance threshold. These scenarios have been created using the parameter configuration in table 1.

Figure 1 shows a simulation of the proposed model using Scenario 1. This figure shows the deterioration of the system (blue line) following the gamma distribution. Inspections are regularly done every 3 time units. It is important to mention that we do not consider a specific time unit since, depending on the type of system, these units can vary. For example, some systems should be inspected every 3 months, and others can be inspected every 3 years. Additionally, the constant preventive and corrective maintenance thresholds are indicated with yellow and red dashed lines, respectively.

Figure 2 shows a simulation of the model using Scenario 2. In this case, the preventive threshold is not constant but linearly increasing during each corrective renewal cycle.

The non-constant preventive threshold forms a regenerative process with variable regeneration period. Due to perfect maintenance, the regenerative period is completed each time that a maintenance action is performed. However, hereinafter the period between two consecutive corrective actions will be called renewal cycle.

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**Figure 1.** System degradation with constant preventive maintenance threshold. Scenario 1
4 Calculation of Long Run Costs Rate

This paper is oriented to determine the effect of non-constant maintenance threshold in the long run cost rate of the maintenance policy. The cumulative maintenance costs over time are defined by:

\[ C(t) = C_i(t) + C_c(t) + C_p(t) = C_i N_i(t) + C_c N_c(t) + C_p N_p(t). \]  

(5)

Being \( C_i(t), C_c(t) \) and \( C_p(t) \) the cumulative costs of the inspections, corrective actions, and preventive actions, respectively. These cumulative costs are given by some fixed costs \( (C_i, C_c, C_p) \) and \( N_i(t), N_c(t), N_p(t) \) which are the random number of inspections, corrective actions, and preventive actions, respectively, in the period \([0, t)\).

The long run expected cost rate can be defined as the expected cumulative cost at time \( t \) divided by the time period \( t \).

\[ EC_\infty = \lim_{t \to \infty} \frac{E[C(t)]}{t}. \]  

(6)

In this paper, we develop a numeric simulation to figure out the main statistical properties of Scenario 2. Monte Carlo analysis has been employed by considering different time periods. Each box of Figure 3 is formed by 50 Monte Carlo iterations for different simulation time periods, with increments of 1500 time units. For instance, the first box is formed by 50 resulting cost rates for a simulation period of 500 time units, the second one has a simulation period of 2000 time units, and so on. The inspection cost \( (C_i) \) has been set to 0 since inspections are regularly performed and therefore, they do not affect neither the convergence of the long run cost rate nor the comparison with the Scenario 1.

Figure 3 demonstrates the convergence of the long run expected maintenance cost rate. As the simulation period increases, the resulting cost rate follows a normal distribution. The normality of this variable has been verified through the Anderson-Darling Test [24]. As can be observed in Figure 3, the long run cost rate tends to its expected value with a decreasing variance, what demonstrates the consistency of the long run cost rate estimate.

![Figure 2. System degradation with a non-constant preventive maintenance threshold. Scenario 2](image-url)
The study of long run cost rate can be simplified by applying the (semi)regenerative properties of the system. Based on renewal theory, the expected cost rate can be estimated by considering a single renewal cycle.

\[ EC_{\infty} = \lim_{t \to \infty} \left( \frac{E[C(t)]]}{t} \right) = \frac{E[C(S_1)]}{E[S_1]} \, . \] (7)

Hence, considering only one renewal cycle, the long run cost rate can be defined as:

\[ EC_{\infty} = \frac{C_i E[N_i(S_1)] + C_c E[N_c(S_1)] + C_p E[N_P(S_1)]}{E[S_1]} \, . \] (8)

In literature, authors employ the embedded Markov chain to determine the probabilistic properties of the system [11].

5 Statistical Results of Preventive Maintenance Threshold

Once the consistence of the long run cost rate estimate has been demonstrated, we have performed an analysis of system behaviour for both scenarios.

Each proposed maintenance policy has been simulated during a period of 10^6 time units. figure. 4 shows a comparison of the renewal cycles duration (period between corrective actions) for both scenarios.

As can be observed, the duration of the renewal cycles becomes more regular when a non-constant threshold is considered. This behaviour increases the mean and reduces significantly the variance. Non-constant threshold leads the renewal cycles to be more regular and therefore, planning maintenance action becomes easier. Additionally, we have analysed the number of preventive maintenance actions that were performed in the cycles. This number of preventive actions is shown Fig. 5.

Finally, once the number of preventive and corrective actions has been obtained, the cost rate in the total simulation period (10^6 time units) can be calculated by:

\[ C_{10^6} = \frac{\sum_{i=1}^{R} N_{P_i} \cdot C_P + R \cdot C_c}{10^6} \, . \] (9)
Table 2. Statistical results for both scenarios

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Duration of the renewal cycles (S)</strong></td>
<td>Mean ((\bar{S}))</td>
<td>4.9E+08</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>Variance Var((S))</td>
<td>1.87E+08</td>
</tr>
<tr>
<td><strong>Preventive actions per cycle((N_P))</strong></td>
<td>Mean</td>
<td>3.46E+08</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Variance Var((N_P))</td>
<td>1.55E+08</td>
</tr>
<tr>
<td><strong>Total of renewal cycles in the simulation period ((R))</strong></td>
<td></td>
<td>20400</td>
</tr>
<tr>
<td><strong>Total of preventive actions in the simulation period</strong></td>
<td></td>
<td>70536</td>
</tr>
<tr>
<td><strong>Cost rate in the total simulation period</strong></td>
<td></td>
<td>4.6884</td>
</tr>
</tbody>
</table>

Table 2 collects the main statistical results for both scenarios. Considering 95% confidence intervals, the expected duration of the renewal cycles and the expected number of preventive actions per cycle are given by:

\[
E [S_i] = \bar{S} \pm Z_{0.025} \sqrt{\frac{Var(S)}{n}}.
\]  

(10)

Figure 4. Duration of renewal cycles

Figure 5. Preventive actions per cycle
Table 3. Expected results for each scenario

<table>
<thead>
<tr>
<th>Constant threshold</th>
<th>Non constant thresholds</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E [S_i]$</td>
<td>[48.42; 49.60]</td>
</tr>
<tr>
<td>$E [N_p (S_i)]$</td>
<td>[3.40; 3.51]</td>
</tr>
<tr>
<td>$EC_\infty$</td>
<td>[4.61; 4.64]</td>
</tr>
</tbody>
</table>

$$E [N_p (S_i)] = \tilde{N}_p \pm Z_{0.025} \sqrt{\frac{Var (N_p)}{n}}. \quad (11)$$

Where $n$ is the total number of renewal cycles of the simulation. Knowing that each renewal cycle starts after a corrective action, the expected number of corrective actions during a renewal cycle is $E [N_C (S_i)] = 1$. Assuming that the inspection costs are 0, the expected long run cost rate can be calculated as:

$$EC_\infty = \left[ \frac{C_c + C_p \tilde{N}_p - C_p Z_{0.025} \sqrt{Var(N_p)} n}{\tilde{S} + Z_{0.025} \sqrt{Var(S)} n} ; \frac{C_c + C_p \tilde{N}_p + C_p Z_{0.025} \sqrt{Var(N_p)} n}{\tilde{S} - Z_{0.025} \sqrt{Var(S)} n} \right]. \quad (12)$$

According to the previous equations, we obtain the results in table 3:

6 Discussion

The study of deterioration is essential to ensure the correct performance of deteriorating systems in the long term. Modelling the deterioration is an important challenge since each system presents specific features that affect the evolution of the deterioration. The literature in this area is vast, for instance Google Scholar provides more than 11 thousand of papers published in 2022 about maintenance of deteriorating systems, which reveals the current relevance of this topic in the field of maintenance.

The analysis conducted in this paper reveals that, for the given parameter configuration, the constant threshold minimises the long run cost rate. We have not found a linear threshold providing less long run cost rate than the constant one. However, we have found some advantages in the use of non-constant threshold. For example, non-constant thresholds facilitate to determine when the next corrective action will be required, since the variance of the renewal cycles is reduced. Additionally, an increasing non-constant threshold allows to act against excessive degradation in early stages of the system, which could be a symptom of anomalies.

Obtaining the optimum setting of non-constant thresholds is a real challenge. Authors consider that other kinds of non-constant thresholds can provide further advantages. The model presented in this paper will be used as a starting point to discover new maintenance thresholds that improve maintenance by using machine learning algorithms.

7 Conclusions and Future Work

In this work, we have developed a continuous state degradation process though a Gamma distribution and considered perfect maintenance actions. We pretend to open a new research line by considering variable thresholds to optimize maintenance. For this purpose, we have analysed two different maintenance strategies for a deteriorating system.

We propose a novel maintenance strategy which uses a time dependent linear preventive maintenance threshold. We have demonstrated that with this threshold, like with constant
thresholds, the degradation of the systems follows a regenerative process. Therefore, the long run cost rate is convergent and can be calculated by considering a single renewal cycle.

Two scenarios have been created to simulate both maintenance policies. Scenario 1 and Scenario 2 present constant and non-constant maintenance thresholds, respectively. These maintenance strategies are simulated during a simulation period of 106 time units. Some statistical results have been obtained to compare the behaviour of the system under both scenarios.

The main conclusion that we have extracted from our analysis is that we have observed that the constant threshold provides less expected long run cost rate, whilst the non-constant threshold provides less variance in the renewal cycles duration (time between two consecutive corrective actions), which can facilitate the maintenance planning. Therefore, we can conclude that non-constant thresholds can provide some advantages in terms of planification.

As future work, we propose to optimize the parameter configuration to obtain the optimum non-constant maintenance threshold. In addition, we propose studying the system reliability when non-constant maintenance threshold is considered.

References

[22] B. Wu, D. Ding, Reliability Engineering & System Safety 217, 108112 (2022)