Pile-slab foundation's settlement evaluation in pseudo-nonlinearity formulation

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Abstract. In this paper, an analytical solution to the problem of the interaction of a pile-slab foundation with a 2-layer elastic soil basement is proposed. A new method was proposed to take into account the nonlinearity of soils in an elastic formulation. This method consists in limiting the bearing capacity of the pile along the lateral surface by the strength of the adjacent soil and redistributing the applied load to the pile toe. In the article, a comparative analysis of the proposed analytical solution of the problem in the elastic formulation with numerical simulation in the elastic-plastic formulation is carried out and the scope of the proposed analytical solution, which taking into account the described mechanism of the nonlinearity of the soil, is investigated. Numerical simulation was carried out in the geotechnical software package *Plaxis 2D*. According to the results of analytical and numerical calculations, the graphs of the dependence of settlement on loads were constructed for various values of soil strength. The graphs showed significant differences in the rate of development of plastic zones of soil deformation obtained by analytical and numerical methods.

Key words: pile, pile-slab foundation, stress-strain state, pile-soil interaction, analytical solution, numerical solution, shear stress, pile bearing capacity, plastic deformations, pile settlement.

1 Introduction

It is known that the lateral surface of the pile is included in the work at the initial application of the load and the toe of the pile perceives a small value of the load. The shear stress along the lateral surface will increase with an increase in the load until the bearing capacity is exhausted in local areas. Then the load will begin to be gradually transferred to those sections of the pile where the shear stress has not reached its limit, as well as to the toe of the pile.

The mechanism of load transfer, which is described above, is implemented in computational models using elastic-plastic, elastic-viscous and elastic-viscous-plastic soil

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models. However, when using elastic soil models the bearing capacity on the lateral surface does not exhaust, and therefore, shear stresses continue to increase. Therefore, the use of elastic soil models has obvious limitations on the range of loads.

The existing calculation methods for determining the components of the stress-strain state (SSS), and the bearing capacity and settlement of piles can be divided into two types: analytical methods and numerical ones. The use of numerical methods does not make difficulties in the calculation with non-linear and viscous properties of soils. But the application of analytical methods makes manual calculation very laborious. Therefore, analytical solutions are developed in an elastic formulation and then become more complicated by applying more complex soil models [1-3].

However, some authors have made attempts to take into account the redistribution of the load on the toe of the pile after the exhaustion of the bearing capacity of the lateral surface in an elastic formulation. In the dissertation research of Sidorov V.V. [4] an analytical solution for determining the settlement of a single barret was obtained. When the mobilized shear stress τ_{mob} along the lateral surface, obtained with using the proposed analytical solution, reached the value τ_{max}, then further load application was carried out only on the soil under the toe of the pile. Comparison of the "load-settlement" graphs, obtained with using an analytical solution in an elastic formulation and a numerical solution in an elastic-plastic formulation, showed high convergence from 0 to 7000 kPa. Both graphs have areas of non-linear deformation because both methods take into account the possibility of reaching the limit state along the lateral surface of the pile. Similar conclusions were made in the dissertation research of Strunin P.V. [5] when studying the interaction of a jet grouting pile with an elastic soil mass. The convergence of the "load-settlement" graphs was obtained at loads from 0 to 4000 kPa.

The study of the load distribution between the lateral surface and the toe of pile is an important and difficult problem because the accuracy of determining the shear stresses makes it possible to design the optimal construction of the pile foundation by maximizing the bearing capacity of the soil under the toe of the pile [6, 7]. A number of proposed analytical solutions in the elastic formulation by different authors can be used in a wide range of loads. At the same time, they have a high convergence with the results of numerical simulation even in the area where small zones of soil plastic flow appear [8–10]. Some researchers have also developed analytical solutions for cyclic and dynamic loads because in the process of changing the load over time, the stress-strain state of the “soil-foundation” system changes significantly [11, 12]. It is important to notice that when a pile interacts with soft clays which have creep properties the stress-strain state changes over time. This leads to a redistribution of the external load along the lateral surface and under the pile toe [13, 14]. In addition, the results of the analytical solution of the problem are significantly affected by the chosen rheological model of soil behavior [15, 16]. In the case of pile drains, around which a complex SSS of the soil is formed, the distribution and redistribution of the load between the pile and the soil will additionally depend on the rate of consolidation and the geometric parameters of the pile and the soil cylinder around it [17-19].

A comparative analysis with using numerical simulation and experimental studies should be carried out to assess the accuracy of analytical solutions. For example, studies of Gotman A.L. and Gavrikov M.D. [20] showed that due to the high bearing capacity of the soil under the toe of a pile 65 meters long the “load-settlement” graph continues to grow without signs of a limit state. In fact, the pile can take much larger loads than the design loads. At the same time, the bearing capacity is exhausted along the lateral surface, respectively, the entire load is transferred to the pile toe.

In this paper, an analytical solution for a pile-slab foundation interacting with a 2-layer basement in an elastic formulation with limiting the bearing capacity of the lateral surface of the pile and redistributing the load between its lateral surface and the toe to describe the nonlinearity of soils is proposed. In the first approximation, the distribution of shear stresses along the pile and settlement in depth are taken according to a linear law. The purpose of the article is a comparative analysis of the proposed analytical solution of the problem with taking into account the nonlinear properties of soils in an elastic formulation with numerical simulation in an elastic-plastic formulation and a study of the limitation of the scope of the proposed analytical solution.

2 Methods

The assumption that the sections do not influence each other was taken to solve the problem with a pile-slab foundation. Therefore, the calculation scheme of the analytical problem is a cell in a pile-slab foundation with a width of *2b*. Cell height is *L*, pile length is *l*, pile diameter is *2a*. The problem is solved for a two-layer basement where the thickness of the 1st soil layer is *l* and all soil characteristics are indicated with index 1. The 2nd soil layer has a thickness *(L-l)* and, accordingly, its characteristics are indicated with index 2. The design scheme of the problem is shown in Figure 1.

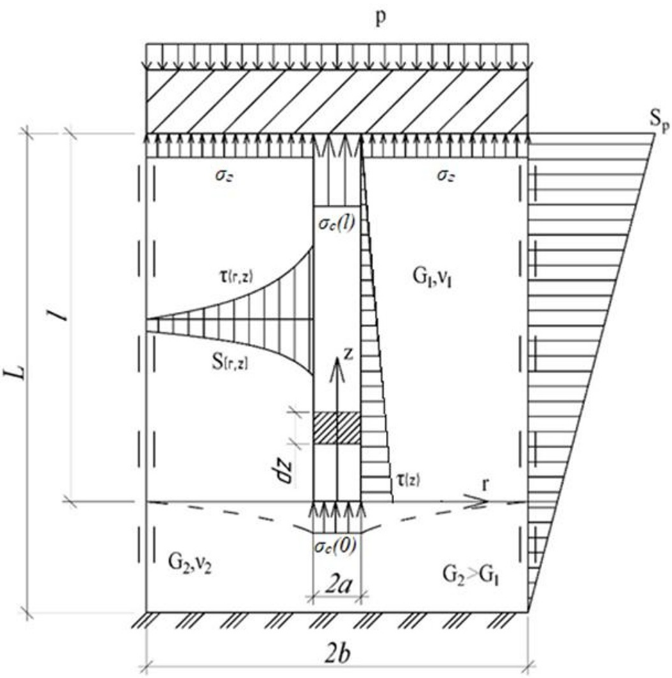


Fig. 1. The design scheme of the problem.

In the problem there are four unknown SSS components: reactive stress under the pile foundation σ_z, reactive stress at the pile head $\sigma_c(l)$, reactive stress under the pile toe $\sigma_c(0)$ and shear stress along the lateral surface of the pile τ_0. By solving a system of four equilibrium equations, it is possible to determine all unknown components of the SSS:

$$\begin{cases} \pi b^2 p = \pi a^2 \cdot \sigma_{\text{c}}(l) + \pi(b^2 - a^2) \cdot \sigma_{\text{г}} \\ \pi a^2 \cdot \sigma_{\text{c}}(l) = \pi a^2 \cdot \sigma_{\text{c}}(0) + \pi a \cdot \tau_0 \cdot l \\ \tau_0 \cdot \frac{(b-a)}{3G_1} + \sigma_{\text{г}} \cdot \frac{\beta}{E_2} \cdot (L-l) \cdot \frac{(L-l)}{L} = \sigma_{\text{c}}(0) \cdot \frac{\pi a(1-\nu_2)}{4G_2} \cdot K \\ \frac{2\tau_0 l^2}{3aE_c} + \frac{\sigma_c(0) \cdot l}{E_c} + \sigma_{\text{c}}(0) \cdot \frac{\pi a(1-\nu_2)}{4G_2} \cdot K = \sigma_{\text{г}} \cdot \frac{\beta}{E} \cdot L \end{cases} \quad (1)$$

The settlement of a pile-slab foundation consists of the sum of the soil settlement under the pile foundation, the compression of the pile shaft and the settlement of the soil under the pile toe and is determined by the formula:

$$S = \frac{p \cdot \beta_g \cdot L}{E}\left(1 - \frac{l}{L}\right) + \frac{(\sigma_c(l) - \sigma_c(0)) \cdot \beta_c \cdot l}{E_c} + \sigma_{\text{c}}(0) \cdot \frac{\pi a(1-\nu_2)}{4G_2} \cdot K, \quad (2)$$

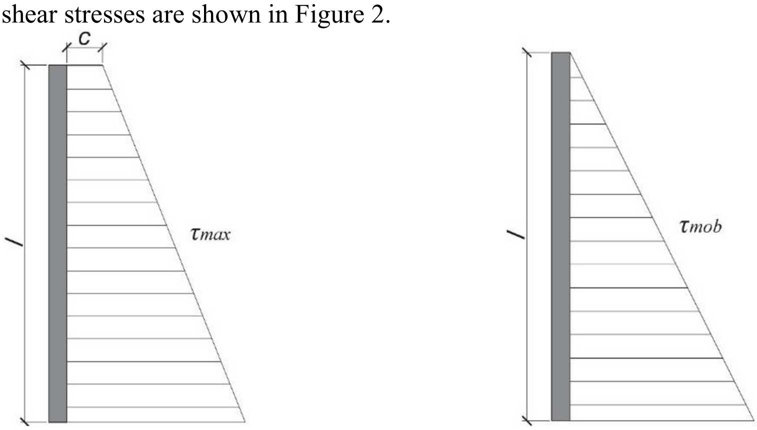
where β_g - the coefficient of impossibility of lateral expansion of the soil;
β_c - coefficient of impossibility of lateral expansion of the pile material.

A graphical-analytical method was used to take into account the exhaustion of the bearing capacity of the lateral surface of the pile and the transfer of further load to its toe. The limiting value of shear stress τ_{max} was determined according to the Coulomb law reaching its maximum value at the level of the pile toe:

$$\tau_{max} = \sigma_x \tan\varphi + c, \quad (3)$$

where φ, c - the strength characteristics of the soil;
σ_x - the horizontal stress at the considered level acting perpendicular to the shear stress.

When the effective (mobilized) value of the shear stress τ_{mob} which is obtained from the system of equations (1) reaches the limit value equal to τ_{max} a further increase in the load will be perceived by the soil under the toe of the pile. Thus, the settlement in equation (2) will increase due to the compression of the soil under the pile tip. Diagrams of the limit and mobilized shear stresses are shown in Figure 2.



(a) (b)

Fig. 2. Diagrams of shear stresses along the lateral surface of the pile: (a) - for given strength characteristics of the soil, (b) - obtained by solving the system of equilibrium equations (1).

3 Results

For a quantitative assessment of the SSS of a cell using analytical equations (1) and (2) and a comparative analysis of the results with numerical simulation the following initial data were taken:

1. Geometric characteristics of the cell: a = 0.5 m; b = 2.5 m; L =40 m; l = 30 m.

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2. Pile material properties: Ec = 30000 MPa; βc = 0.8.
3. Soil properties: E_1 = 12.5 MPa; v_1 = 0.35; E_2 = 70 MPa; v_2 = 0.25; β_g = 0.8.
4. Coefficient taking into account the depth of the stamp: K = 0.7.
5. Load p = 0…400 kPa.

Numerical modeling in a two-dimensional formulation was performed in the geotechnical software package *Plaxis 2D* to determine the accuracy of the proposed analytical solution taking into account the limitation of the bearing capacity of the lateral surface of the pile. The Mohr-Coulomb model was used to describe the behavior of soils. Several variants of problems with different strength properties of soils: at $\varphi = 10°$, $\varphi = 15°$, $\varphi = 20°$ were considered in numerical simulation.

To limit the bearing capacity of the lateral surface of the pile and redistribute the load the load p was determined at which τ_{mob} from equation (1) reaches the limit value τ_{max} at φ = 10°, φ = 15° and φ = 20° respectively:

$$\tau_{max,1} = 540 \cdot \frac{0,35}{1-0,35} \cdot tan\,10 + 1 = 52,3\;kPa;$$
$$\tau_{max,2} = 540 \cdot \frac{0,35}{1-0,35} \cdot tan\,15 + 1 = 78,9\;kPa;$$
$$\tau_{max,3} = 540 \cdot \frac{0,35}{1-0,35} \cdot tan\,20 + 1 = 106,8\;kPa.$$

Accordingly, the maximum values of the load p, perceived by the side surface of the pile, are 177, 267 and 361 kPa. The load applied to the pile-slab foundation exceeding these values was transferred to the toe of the pile.

As a result, graphs of the dependence of settlement on load were obtained and are shown in Figure 3. At φ = 10° convergence of the analytical solution with the numerical solution is observed in the load range from 0...100 kPa, while the graph obtained by numerical simulation has a pronounced non-linearity. At φ = 15° the convergence is higher and is achieved in the range from 0…200 kPa. In this case, the numerical solution gives linear deformation under loads up to 125 kPa, which coincides with the analytical solution. At φ = 20°, convergence is observed in the load range from 0...350 kPa, and the graph obtained by numerical simulation has a linear section at a load of up to 200 kPa. Thus, the analytical solution (2) in the elastic formulation gives good qualitative and quantitative convergence of the results in a certain range of loads depending on the strength of the soil.

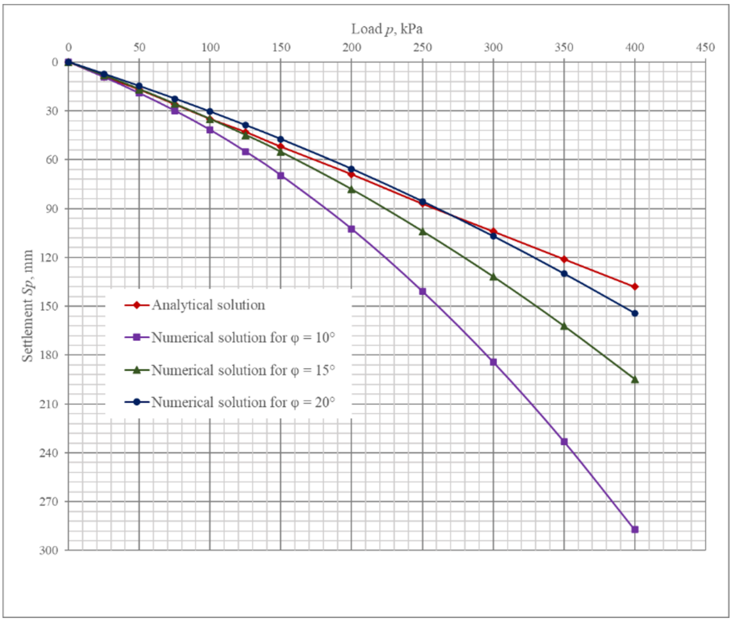

Fig. 3. Graphs of the dependence of settlement on load obtained by analytical and numerical methods.

Graphs of the dependence of settlement on load at various values of the angle of internal friction were built using the proposed modified analytical solution. Figure 4 shows the results for φ = 10°. The modified analytical solution in the elastic setting does not give good convergence with the numerical solution due to strong nonlinearity.

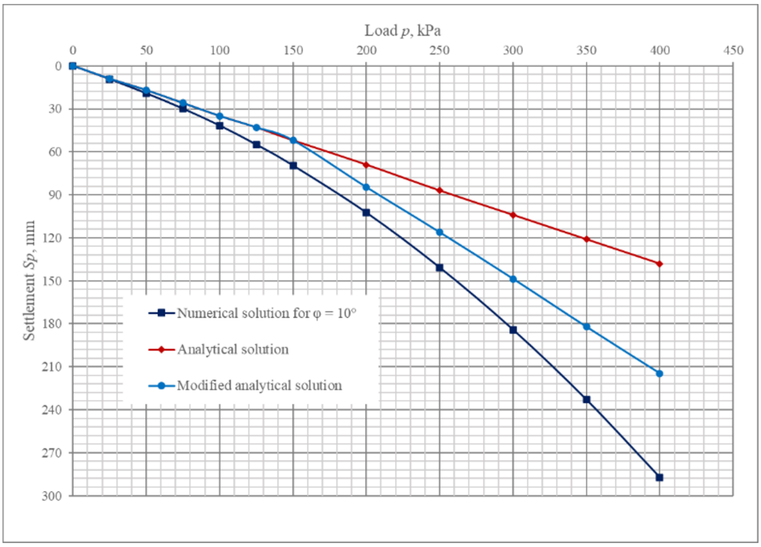

Fig. 4. Graphs of the dependence of settlement on load obtained by the analytical method without and taking into account the limitation of the bearing capacity of the lateral surface of the pile and by the numerical method at φ = 10°.

The results at φ = 15° are shown in Figure 5. The proposed method shows high quantitative and qualitative convergence with the numerical solution in the field of nonlinear deformation. At the same time, the graph shows that the proposed method for calculating the settlement takes into account the nonlinearity of the soil.

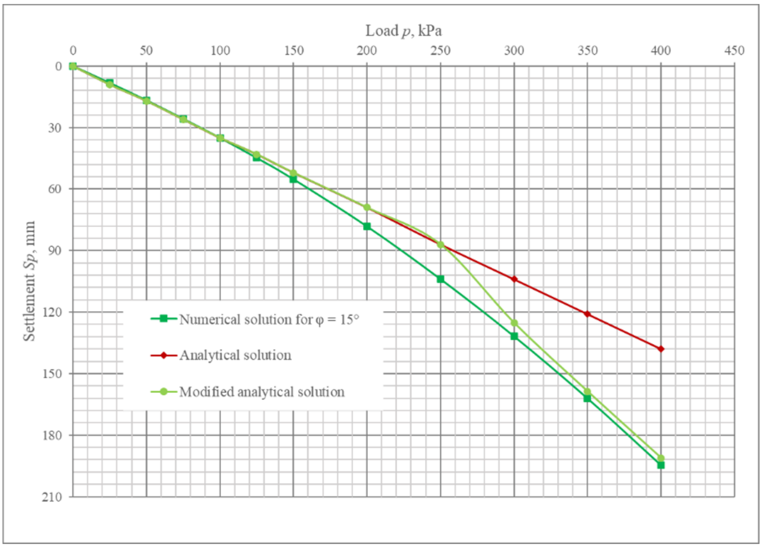

Fig. 5. Graphs of the dependence of settlement on load obtained by the analytical method without and taking into account the limitation of the bearing capacity of the lateral surface of the pile and by the numerical method at φ = 15°.

Graphs of the dependence of settlement on load at φ = 20° are shown in Figure 6. The modified analytical solution shows low convergence with the numerical solution due to the strong nonlinearity of the graph obtained by the analytical method. The numerical solution has a flatter graph corresponding to the linear deformation of the soil because with increasing strength, the process of soil plastic flow occurs more slowly.

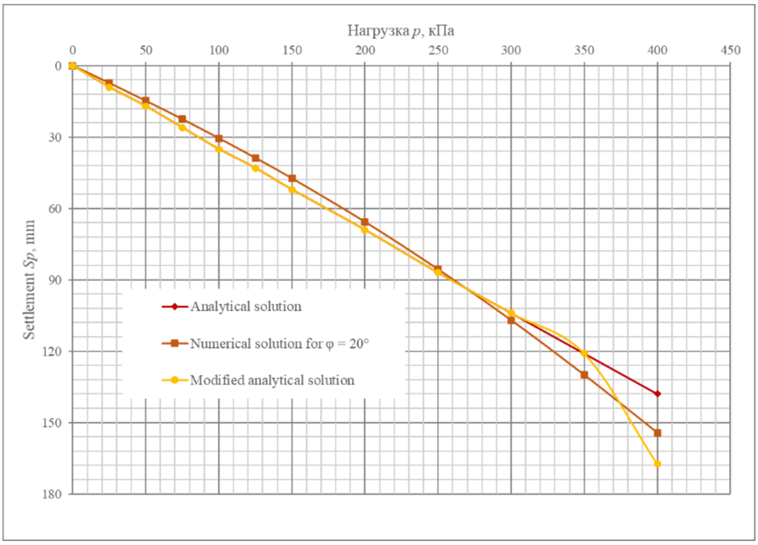

Fig. 6. Graphs of the dependence of settlement on load obtained by the analytical method without and taking into account the limitation of the bearing capacity of the lateral surface of the pile and by the numerical method at φ = 20°.

4 Discussion and conclusions

1. In the article an analytical solution of the problem of the interaction of a pile-slab foundation with a 2-layer elastic soil base was proposed with taking into account the linear distribution of shear stresses along the pile shaft and settlement in depth. The non-linearity of soils in the elastic formulation was taken into account by limiting the bearing capacity of the pile along the lateral surface and redistributing the load on the pile toe.

2. Comparison of graphs of the dependence of settlement on load, obtained by the analytical method in the elastic formulation and by the numerical method in the elastoplastic formulation, showed that the qualitative and quantitative convergence of the results is influenced by the strength of the soil. The lower the strength, the more pronounced the nonlinearity and the less convergence. The higher the strength, the better the convergence of the analytical solution with the numerical one. But with taking into account high soil strength settlements in the initial section will be much less which will show poor convergence with the analytical solution even at small values of loads.

3. The solution of the problem by the modified analytical method showed poor convergence with the numerical solution at φ = 10° due to the possible exhaustion of the bearing capacity of the soil under the toe of the pile which is not taken into account in the modified analytical solution. High qualitative and quantitative convergence of the results was obtained at φ = 15° both in the area of linear and non-linear soil deformation over the entire considered load range from 0...400 kPa. At φ = 20° the proposed method gives a strong nonlinearity in contrast to the numerical solution which limits the use of this method in more durable soils and requires its refinement.

4. To further study the increase in the range of applicability of the modified analytical solution to soils of different strengths it is necessary to introduce a limit on the bearing capacity under the pile toe, consider the nonlinear distribution of shear stresses along the pile shaft and also consider the gradual depletion of the bearing capacity of soils along the lateral surface with the transfer of load to the sections where the tensile strength of the soil is not reached.

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